Spectroscopic Factors for Stripping to Unbound Isobaric Analog States*

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The reaction ^{92}Mo(^{3}He,d)^{93}Tc leading to unbound isobaric analog states (IAS) has been studied. A distorted-wave Born-approximation analysis of the IAS using unbound single-particle form factors is shown to provide good fits and reasonable spectroscopic factors for (^{3}He,d) data from this and other work in the $A=90$ region. Calculations for available (d,n) reaction data result in spectroscopic factors which are lower than the (^{3}He,d) values.

Unbound isobaric analog states have been traditionally studied in resonance scattering reactions. They have only recently been the subject of intense investigation in proton stripping reactions, and apparent anomalies have been reported in the (^{3}He,d)[1] and (d,n)[2] cross sections for these levels. In particular, it has been suggested that the low (^{3}He,d) cross sections observed for $3s_{1/2}$ transitions in the $A=90$ region are due to the special structure of the isobaric analog states, rather than to the features of the reaction mechanism.[1] Unfortunately, a distorted-wave Born-approximation (DWBA) analysis may not be performed in the usual manner due to the unbound nature of the final states. We have studied the reaction ^{92}Mo(^{3}He,d)^{93}Tc leading to isobaric analogs of low-lying levels in ^{93}Mo and have performed a DWBA analysis of our data, as well as that of Refs. 1 and 2, using a technique previously developed to describe stripping to unbound $T_<$ levels.[3] A wave function was calculated for a proton "quasibound" by the combined Coulomb and centrifugal barrier in a Woods-Saxon well:

$$V(r)=V_0 f(r)+\frac{V_0\lambda_{s.o.}}{a}\left(\frac{h}{2M_pc}\right)^2\left(\frac{df}{dr}\right)\vec{l}\cdot\vec{\sigma}+V_C(r),$$

where

$$f(r)=\{1+\exp[(r-r_0A^{1/3})/a]\}^{-1},$$

with $r_0=1.25$ F, $a=0.65$ F, and $\lambda_{s.o.}=25$. The depth V_0 was varied to minimize the ratio of the exterior to interior amplitude of the wave function for a particle of given J^π at the actual binding energy (negative for these levels). The wave function was then normalized by setting

$$\int_0^{R_{t.p.}}U_{lj}^2(r)dr=1,$$

where $U_{lj}(r)$ is the radial form factor and $R_{t.p.}$ is the exterior classical turning radius. This normalization is continuous across zero binding energy, and is consistent with the normalization used for bound states where $R_{t.p.}$ is infinite. The DWBA calculation is then performed in the same manner as for bound states, although upper cutoff radii of several hundred fermis are required to obtain convergence of the DWBA radial integrals if the exterior amplitude is large ($\gtrsim$0.01 times the interior amplitude). An analysis of the reaction ^{40}Ca(^{3}He,d)^{41}Sc[3] has shown that calculations of this type are relatively insensitive to changes in scattering phase shifts, and that the spectroscopic factors are in agreement with those obtained from elastic scattering experiments. Further tests of the technique is the subject of another communication.[4] Huby[5] has recently shown that a "pseudo-bound" technique similar to the one used here (although differing somewhat in detail) may be derived from the "faithful" calculation of Huby and Mines.[6]

The experimental and DWBA (^{3}He,d) cross sections are related (for spin-zero targets) by[7]

$$d\sigma/d\Omega=4.42C^2S\sigma_{lj}(\theta),$$

where $\sigma_{lj}(\theta)$ is the DWBA cross section and C^2 is the isotopic spin Clebsch-Gordan coefficient [$C^2=(2T_0+1)^{-1}$ for a $T=T_z+1$ final state and target isospin T_0]. The spectroscopic factor S thus extracted for an analog state should be comparable to that measured in a neutron stripping reaction to the corresponding parent state.

Data were obtained with the 35.0-MeV ^{3}He beam from the cyclotron using a 1-mg/cm^2 Mo foil enriched to 98.3% ^{92}Mo. Dueteron pulses were selected by standard particle-identification techniques using a ΔE-E silicon-detector telescope and stored as 1024-channel spectra in the IBM 7094 computer. The data were reduced on line to absolute, center-of-mass differential cross sections with the aid of the display oscilloscope. Angular distributions were measured for laboratory angles between 8.5° and 45.5° for three deuteron groups corresponding to isobaric analog states. The overall energy resolution was typically 80 keV, mostly the result of target-thickness effects. Figure 1(a) shows angular distributions observed for the unbound isobaric ana-

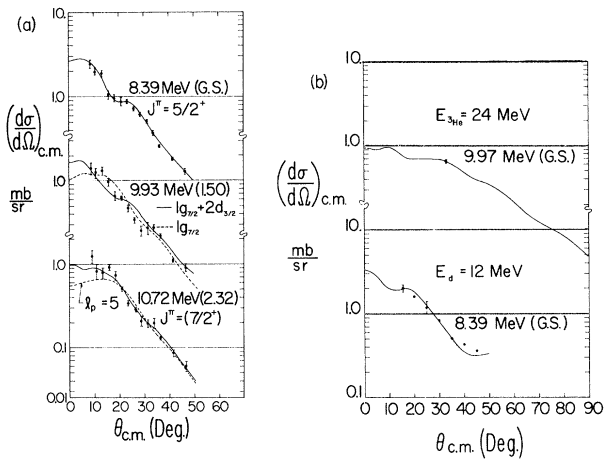

FIG. 1. (a) Deuteron angular distributions for IAS excited in the reaction ^{92}Mo(^{3}He,d)^{93}Tc. The curves are DWBA calculations using unbound form factors. Excitation energies of the parent states are shown in parentheses. (b) DWBA calculations for IAS excited in the reactions ^{90}Zr(^{3}He,d)^{91}Nb (upper curve) and ^{92}Mo(d,n)^{93}Tc (lower curve). The (^{3}He,d) curve has the general features of the data described in Ref. 1. The optical parameters chosen in Refs. 1 and 2 were used in the calculations.

log states, along with DWBA fits to the data.[8,9] The spectroscopic factor obtained for the ground-state analog, 0.72, is in good agreement with that obtained for the parent state, 0.84, by Moorhead and Moyer[10] via the reaction ^{92}Mo(d,p)^{93}Mo.

The analog of the 0.95-MeV $3s_{1/2}$ level in ^{93}Mo is observed very weakly at only a few angles and no distribution was obtained. Several closely spaced levels have been observed near 1.5-MeV excitation in ^{93}Mo, and the deuteron group corresponding to the analogs at 9.93-MeV excitation in ^{93}Tc appears to contain at least two unresolved levels. The solid curve in Fig. 1(a) represents a mixture of $2d_{3/2}$ and $1g_{7/2}$ strengths calculated by assuming the parent-state spectroscopic factors[10]; however, the fit to the data is relatively poor. Spin and parity assignments of $\frac{7}{2}^+$ and $\frac{11}{2}^-$ have been made for the parent of the 10.72-MeV level by Hjorth and Cohen[11] and Moorhead and Moyer,[10] respectively. The angular distribution of the analog state appears to be best described by an $l_p = 4$ calculation, and the spectroscopic factor is in reasonable agreement with the value of 0.37 reported in Ref. 11 for the parent state.

Calculations have also been performed for some of the 24-MeV (^{3}He, d) data of McGrath *et al.*,[1] and the ^{92}Mo(d, n)^{93}Tc data of Zaidi *et al.*[2] The angular distribution for the $2d_{5/2}$ analog state excited in the reaction ^{90}Zr(^{3}He, d)^{91}Nb is described in Ref. 1 as being relatively structureless and decreasing by about a factor of 14 over the angular interval of 10.8° to 85° c.m. Our calculation for this transition [Fig. 1(b)] has those general features, and a normalization to the one experimental value reported yields a spectroscopic factor of 0.71, in fair agreement with the ^{90}Zr(d, p)^{91}Zr(g.s.) value of 0.89 obtained by Cohen and Chubinsky.[12] Also shown in Fig. 1(b) is

Table I. DWBA spectroscopic factors for $2d_{5/2}$ isobaric analog states and $3s_{1/2}$ IAS stripping cross sections.

	$2d_{5/2}$	(^{3}He,d)		(d,n)	
Target	$S_n(d,p)$[a]	S_p[b]	S_p[c]	S_p[d]	S_p[e]
^{90}Zr	0.89		0.71		
^{92}Mo	0.84	0.72	0.75	0.33	2.88

		$\sigma(^3\text{He},d)$[f] (μb/sr)		$\sigma(d,n)$[g] (μb/sr)	
	$3s_{1/2}$				
Target	$S_n(d,p)$[a]	Exp.	Calc.	Exp.	Calc.[d]
^{90}Zr	0.72	<23[c]	30[c]		
^{92}Mo	0.64	50 ± 25[c]	56[c]	100 ± 30	218
		<100[b]	47[b]		

[a] See Refs. 10 and 12.
[b] Present work.
[c] Calculated for data of Ref. 1.
[d] Calculated for data of Ref. 2.
[e] See Ref. 2.
[f] Average of cross sections at 30° and 35° lab.
[g] Cross section at 15.5° (see Ref. 2).

a DWBA fit to the ground-state analog distribution from the reaction ^{92}Mo$(d,n)^{93}$Tc.[2] Although the shape of the distribution is reasonably well predicted, the spectroscopic factor is lower than the (^{3}He, d) value by more than a factor of 2. Similar discrepancies have been observed between the (^{3}He, d) and (d,n) reactions in lighter nuclei.[13] Table I lists the spectroscopic factors obtained for the unbound $2d_{5/2}$ analog states in ^{91}Nb and ^{93}Tc. The last column shows a value calculated by Zaidi *et al.*[2] using an isospin lowering operator on the form factor for the bound-neutron parent state.

The $3s_{1/2}$ analog states are weakly excited in both the (^{3}He, d) and (d,n) reactions, and no complete angular distributions are presently available. Experimental cross sections have been measured at a few angles however, and Table I shows a comparison of experimental $3s_{1/2}$ cross sections with DWBA predictions using the parent-state spectroscopic factors. The DWBA calculations predict very small cross sections for unbound $l_p=0$ levels in reasonable agreement with experiment, although the experimental errors are large.

In summary, it is apparent that the (d,n) and (^{3}He, d) angular distributions for stripping to unbound isobaric analog states can be adequately described by the DWBA without ascribing any special structure to the IAS. It is necessary, however, that the DWBA form factor used in the calculation properly describe the actual binding energy of the transferred particle. Although the absolute (d,n) spectroscopic factors are lower than expected, the relative values for the $2d_{5/2}$ and $3s_{1/2}$ transitions are reasonable. Our calculations indicate that the previously reported $3s_{1/2}$ inhibitions[1] and $2d_{5/2}$ enhancements[2] are in fact a consequence of the reaction mechanism.

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Effects of Nonlocalities in Nuclear Optical-Model Potentials

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It is shown that a precise account of the nonlocal aspects of the low-energy nucleon-nucleus interaction can be made by means of a simple effective-mass approximation provided that one invokes the correct effective mass of the problem. Unlike conventional descriptions with energy-dependent local potentials, the effective-mass description is seen to involve the true geometric parameters of the interaction.

In the present note we attempt to clarify certain aspects of the role and the description of nonlocalities in nuclear optical-model potentials. In particular, we show that a simple effective-mass approximation to the nonlocal nucleon-nucleus optical potential[1] provides a precise and instructive description of the scattering problem provided that we invoke the correct effective mass to be associated with the nonlocality. From this vantage, the wave effects peculiar to the nonlocality (for instance, the