Table II. Resulting quark-production cross sections for 70-GeV protons on copper for different assumed quark masses ($\hbar = c = 1$, values in GeV). The last column gives the upper limits for the quark-production cross section consistent with the Serpukhov data (Ref. 3).

M_Q	$E_{\text{c.m.}}$	P	$\sigma_A/\sigma_{\text{free}}$	$\overline{\sigma}$
5.5	12.9	0.26	2.5	1×10^{-37}
6.0	13.9	0.44	3×10^{-1}	7×10^{-35}
7.0	15.9	0.86	2.5×10^{-2}	1×10^{-35}
8.0	17.9	1.29	4×10^{-3}	7×10^{-33}
9.0	19.9	1.89	5×10^{-4}	3×10^{-32}
10.0	21.9	2.39	2×10^{-4}	1×10^{-32}

Table II gives the resulting upper limits for the quark-production cross sections in nucleon-nucleon collisions using the Serpukhov data.[3,4] The numbers for the cross sections include a factor $A^{-1/3}$ to account for the attenuation of the incoming proton beam during its passage through the nucleus (the cross sections are proportional to $A^{2/3}$ rather than to A). The listed values were obtained by matching to the upper limits shown in Fig. 5 of Ref. 4 for $M_Q \leq 4.8$ GeV. They thus provide a continuation of the above curve labeled $N+N \to N+N+Q+\overline{Q}$, $q=-\frac{1}{3}$, to higher quark masses. Considering that the data of McCusker and Cairns[1] imply a cross section of the order 10^{-31} cm^2, one sees that in order for the different data not to be contradictory one has to postulate either that the cross section remains small near the threshold energy (in agreement with Ref. 1), or that the quarks are produced at such angles that they would not have been detected in the Serpukhov setup, or both.

We would like to thank S. Meshkov for useful discussions.

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Calculation of the Total Cross Section for Double Compton Scattering*

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(Received 30 November 1970)

We report the results of a calculation of the *total* cross section for double Compton scattering ($\gamma+e\to\gamma+\gamma+e$). At very high incident photon energies the *total* cross section for double Compton scattering becomes larger than that for the usual Compton effect ($\gamma+e\to\gamma+e$) even though the latter is a lower order process.

The possible existence of double Compton scattering,

$$\gamma+e\to\gamma+\gamma+e, \tag{1}$$

was first conjectured by Heitler and Nordheim.[1] This reaction has since been observed in a number of experiments,[2] and some differential cross-section measurements have been carried out.[3]

The original calculations of Heitler and Nordheim were just order-of-magnitude estimates in which they assumed that in the majority of events, the available energy is shared equally by the two photons. With this assumption, they argued that the order of magnitude of the double Compton total cross section is smaller than that for the single Compton scattering

$$\gamma+e\to\gamma+e \tag{2}$$

by a factor of $1/\alpha \simeq 137$ for incident photon energies $\omega_1 \gg m$, where m is the electron mass. (We use natural units with $\hbar = c = 1$.) They also "showed" that for incident photon energies $\omega_1 \ll m$, double scattering is even more depressed relative to single Compton scattering.

An exact calculation of the differential cross section for double Compton scattering to lowest order in α was later carried out by Mandl and Skyrme[4] using the methods of relativistic quan-

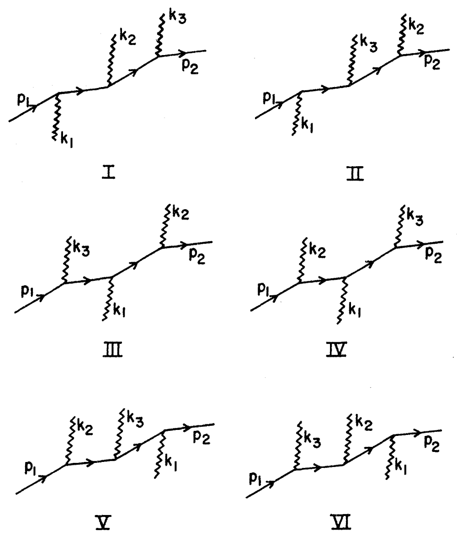

FIG. 1. Feynman diagrams for double Compton scattering to lowest order in α. The incident photon four-momentum is k_1, and k_2 and k_3 are the final photon four-momenta. p_1 and p_2 are the initial and final electron four-momenta, respectively.

tum electrodynamics developed by Feynman and Dyson. Their calculations cast grave doubt on the estimates of Heitler and Nordheim since they showed, among other things, that the assumption of equipartition of available energy among the two photons in the final state is quite unjustified.

In this Letter we wish to report the results of an exact calculation of the total cross section for double Compton scattering to lowest order in α. The total cross section has the following notable advantages over the differential cross section:

(i) It can be meaningfully compared with the total cross section for single Compton scattering.

(ii) The total cross section is considerably larger than the differential cross section and should therefore be, in principle, more readily observable. This advantage is somewhat offset, however, by the fact that in the case of total cross-section measurements it is experimentally more difficult to discriminate legitimate double Compton events from spurious double coincidences.[5]

We first evaluated the differential cross section for Reaction (1) to lowest order in α and verified the Mandl-Skyrme result.[4] To this order, the Feynman diagrams that contribute to the cross section are shown in Fig. 1. We then proceeded to evaluate the total cross section σ_D for Reaction (1) by integrating the differential cross section over all possible final photon and electron momenta consistent with energy and momentum conservation. The photon integrations were restricted to final photon energies larger than ϵ, where ϵ represents the minimum photon energy detectable in an actual experiment. The phase-space integration was carried out using a method first suggested by Sirlin.[6] The final two integrations were carried out numerically on a CDC-6400 computer. Figures 2 and 3 summarize some of the results of our calculations. In Fig. 2 we have also plotted the total cross section σ_S for single Compton scattering

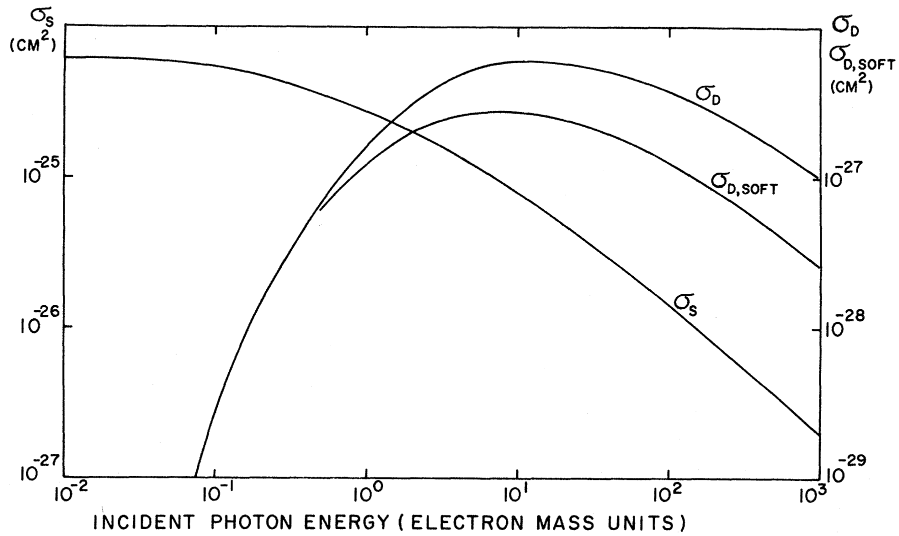

FIG. 2. Total cross sections for single Compton scattering (σ_S) and double Compton scattering (σ_D) as a function of the incident photon energy in electron mass units. It is important to note that the scale used for σ_D and $\sigma_{D,\mathrm{soft}}$ is given on the right-hand side of the graph and is different from the one used for σ_S which is given on the left-hand side.

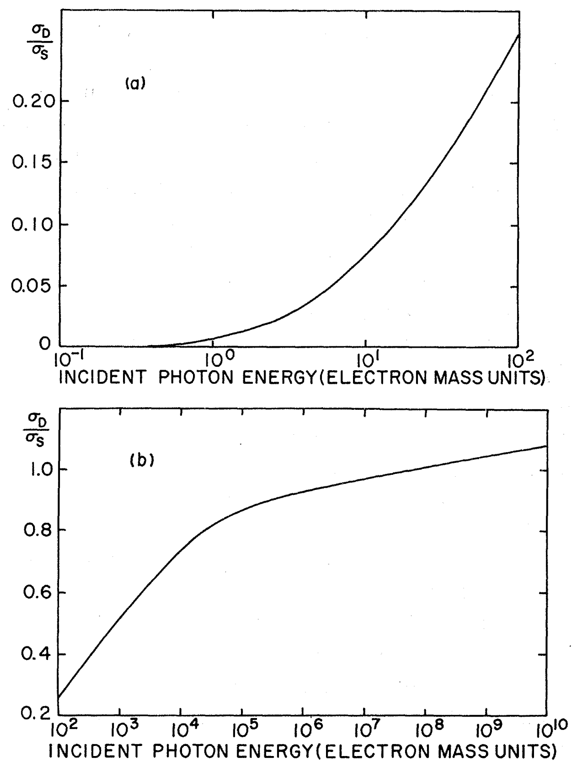

FIG. 3. Ratio of total cross sections for double and single Compton scattering as a function of the incident photon energy.

[Reaction (2)].[8] In addition, for comparison purposes, in Fig. 2 we have plotted $\sigma_{D,\,\mathrm{soft}}$ which, up to a normalization factor, corresponds to the total cross section for double Compton scattering when one of the final photons is soft. (The expression on page 503 of Ref. 4 was used for this purpose.[7]) The curve for the total double Compton cross section σ_D in Fig. 2 corresponds to a low-energy photon cutoff $\epsilon = 0.01m$. One of the important points to note is that σ_D and $\sigma_{D,\,\mathrm{soft}}$ have the same shape. In fact, both curves seem to develop maxima at approximately the same incident photon energy, which, of course, backs up our calculation of σ_D. As additional consistency checks of our calculation, independent asymptotic calculations of σ_D were carried out for incident photon energies $\omega_1 \ll m$ and $\omega_1 \gg m$ and were found to be in complete agreement with the exact expression. Such tests are essential in view of the complicated nature of the calculation.

We can summarize our findings as follows:

(a) Our most interesting result is, of course, the fact that at very high incident photon energies the total cross section for double Compton scattering σ_D becomes larger than that for the single Compton effect σ_S [see Fig. 3(b)]. This is very reminiscent of the results of two other recent calculations[9,10] in quantum electrodynamics. In the first,[9] the total cross section for a certain process involving the exchange of two photons was shown to be larger than that for a lower order process involving the exchange of one photon at high center-of-mass energies. In the other calculation[10] Serbo showed that the total cross section for electron-positron annihilation into three electron-positron pairs becomes larger than that for the lower order annihilation into two electron-positron pairs at very high center-of-mass energies. These examples indicate that naive power counting does not always work when estimating the relative orders of magnitude of various effects at very high energies.[11] At such energies the relative energy dependence becomes extremely important.

(b) The cross section σ_D vanishes for a certain minimal incident photon energy. This is due to the fact that we have assumed that the outgoing photons have energies larger than ϵ, and emission of such photons below that minimal incident photon energy becomes kinematically impossible. For larger incident photon energies, however, Fig. 2 shows that σ_D grows quite rapidly. At low energies this growth is enhanced by the emission of a low-energy photon. In fact, one can see that for incident photon energies of the order of $5m$, the ratio of the double Compton total cross section to the single Compton cross section is of the order of $\alpha \ln(m/\epsilon)$ [see Fig. 3(a)]. In contrast, the single Compton total cross section σ_S is a monotonically decreasing function of the incident photon energy.

(c) We have also examined the dependence of σ_D on ϵ. As is to be expected, σ_D decreases with increasing ϵ for fixed incident photon energy. Furthermore, we have found that at very high incident photon energies (for example, $\omega_1 > 10^6 m$), the dependence of σ_D on ϵ is greatly reduced. This was also to be expected since at such high energies $\sigma_{D,\mathrm{soft}}$ falls off considerably faster than σ_D with increasing incident photon energy, indicating that at such high energies the probability of low-energy photon production is considerably damped.

The total cross section σ_D which we have calculated includes the contribution of final photons emitted in all possible directions. In comparing our results with experiment, a small correction will have to be made to exclude the case when one or both final photons are emitted in small forward and/or backward cones and therefore escape observation.

The authors would like to thank the computing center at the State University of New York at Buffalo, which is partially supported by National Institutes of Health Grant No. FR-00126 and National Science Foundation Grant No. GP-7318, for providing part of the machine time.

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*Research supported in part by the National Science Foundation under Grant No. GP-9536.

†Based on a thesis to be submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Faculty of Natural Sciences and Mathematics at the State University of New York at Buffalo.

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