Experimental Demonstration of the Riedel Peak*

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Careful measurements have been made of the power and frequency dependence of the ac Josephson steps in a superconducting tunnel junction exposed to microwave radiation. At values of $2\alpha = 2eV_{rf}/hf > 25$, significant deviations from Bessel function behavior $[J_N(2\alpha)]$ begin to occur. These deviations are correctly predicted in the detailed microscopic theory of Werthamer and are a consequence of the logarithmic singularity in the ac Josephson current at $f_J = 4\Delta/h$, i.e., the Riedel peak.

It is well known that a Josephson tunnel junction exposed to microwave radiation will exhibit a series of current steps in its I-V curve. These steps, known as the Josephson steps, occur at the discrete voltages given by $Nhf/2e$ and are described in the phenomenological theory by the equation

$$I_{dc}(V_{dc}) = I_J(0)\sum_{N=0}^{\infty} |J_N(2\alpha)|\delta(V_{dc} \pm Nhf/2e), \tag{1}$$

where $I_J(0)$ is the zero-voltage Josephson current, J_N is the Nth-order Bessel function, and $\alpha = eV_{rf}/hf$. The predicted Bessel-function dependence of the step heights has been observed by several researchers.[1,2]

The derivation of Eq. (1) assumes that the Josephson current $I_J(f_J)$ is constant with frequency. Riedel[3] and Werthamer[4] have shown that $I_J(f_J)$, for the case of two identical superconductors at $T = 0°K$, is actually given by

$$\begin{aligned} I_J(f_J) &= [2I_J(0)/\pi]K(hf_J/4\Delta), \quad hf_J/4\Delta \leq 1, \\ &= (2I_J(0)/\pi)(4\Delta/hf_J)K(4\Delta/hf_J), \quad hf_J/4\Delta \geq 1, \end{aligned} \tag{2}$$

where K is a complete elliptic integral of the first kind. Thus, as shown in Fig. 1, the Josephson current exhibits a peak at $f_J = 4\Delta/h$. Since the Josephson frequency is given by $f_J = 2eV/h$, the peak occurs at a bias voltage of $2\Delta/e$. By including this frequency dependence of $I_J(f_J)$ in the microscopic theory, Werthamer[4] has shown that the ac Josephson step heights should be given by

$$I_{dc}(V_{dc}) = \sum_{N=0}^{\infty} |\sum_n J_n(\alpha)J_{N-n}(\alpha)I_J((n-\tfrac{1}{2}N)2f)|\delta(V_{dc} \pm Nhf/2e). \tag{3}$$

In this Letter we show that the predictions of Eq. (3) can be verified experimentally and that these measurements may be used to obtain data on the shape of the Riedel peak.

It is a general property of Bessel functions that $J_N(\alpha) \simeq 0$ for all $\alpha < N$. Thus the significant terms in the summations of Eq. (3) are those for which $|n| < \alpha$ and $|N-n| < \alpha$. Using this fact together with the identity[5]

$$\sum_n J_n(\alpha)J_{N-n}(\alpha) = J_N(2\alpha), \tag{4}$$

it follows that for $V_{rf} < 2\Delta/e$, the argument of I_J in Eq. (3) is less than $4\Delta/h$ in all of the significant terms. Thus, from Fig. 1 we see that $I_J \simeq I_J(0)$, and Eq. (1) and Eq. (3) are nearly identical. Experimentally this means that normal Bessel function behavior should be observed for all values of α for which the highest observable Josephson step occurs at a voltage $V_{dc} < 2\Delta/e$.

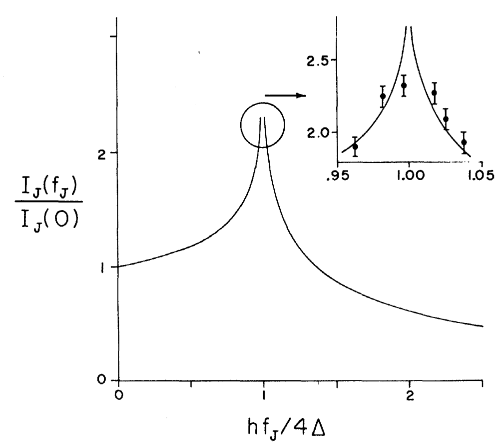

FIG. 1. Normalized Josephson current as a function of frequency from Eq. (2). The insert shows a series of points calculated from the experimental data. The peak height appears to be about 2.4 at 1.3°K.

Previous observations of the power dependence of the step heights have been in this region.

If the applied rf voltage is large ($V_{rf} > 2\Delta/e$), some of the contributing terms in the n summation of Eq. (3) will contain values of $I_J \neq I_J(0)$, and significant deviations from Bessel-function behavior will occur in every step. An approximate rule for the value of V_{rf} at which the first departure from Eq. (1) occurs is the following: If the Mth step falls nearest the gap, then the first deviation of the Nth step occurs when $2eV_{rf}/hf = N+M$.

From Fig. (1) it can be seen that these deviations will be most pronounced when

$$(n-\tfrac{1}{2}N)2f \simeq 4\Delta/h. \qquad (5)$$

This occurs when one of the Josephson steps falls near the gap voltage. If an even numbered step falls near the gap, Eq. (5) can be satisfied only for N even, and thus the even numbered steps show the strongest deviations. A frequency shift sufficient to bring an adjacent odd-numbered step in line with the gap shifts the strongest deviations to the odd steps. The existence of these unusually large and highly frequency-dependent deviations is a direct consequence of the sharp peak in $I_J(f_J)$. At reasonably high frequencies ($f > 15$ GHz for tin), the largest such deviations arise principally from just the one term for which n satisfies Eq. (5). A measurement of the step heights over a range of frequency and rf voltage thus provides a good method for determining the value of $I_J(f_J)$ near $f_J = 4\Delta/h$. We have performed an experiment to make these measurements and to provide data for a quantitative comparison with the theory.

A frequency range of 20-26 GHz was chosen for the experiment. These frequencies are high enough to make the effects of the Riedel peak clearly evident, and yet are not so high as to cause a spatial variation of V_{rf} over the junction area ($\simeq 10^{-6}$ cm^2).[6] Such a spatial variation would violate the conditions under which Eqs. (1) and (3) are derived. Evaporated film junctions of the point overlap configuration were used.[6] These junctions had a tunneling resistance of 3-30 Ω and a very high sensitivity to the applied rf field. The large number of data points required were obtained by taking a motion picture of the I-V curve (displayed on an oscilloscope) as the rf voltage was continuously varied. The data were reduced by examining the film frame by frame. dc bias was provided by a constant-current source and the I-V curve was measured with the conventional four-wire circuit.

A precision attenuator, whose scale was projected along the side of the movie film, was used to measure the relative value of V_{rf} and hence the value of 2α. The attenuator reading A (in dB) was converted to a value of V_{rf} using the relation $V_{rf} = C10^{-A/(20\ \mathrm{dB})}$. The constant C was chosen to obtain an overall best fit to the theory using data on the first twelve steps. In a similar fashion all of the step heights were divided by a single normalization constant I_0, again chosen for a best fit. I_0 was approximately equal to the dc Josephson current with $V_{rf} = 0$.

In Fig. 2, we show a representative sample of

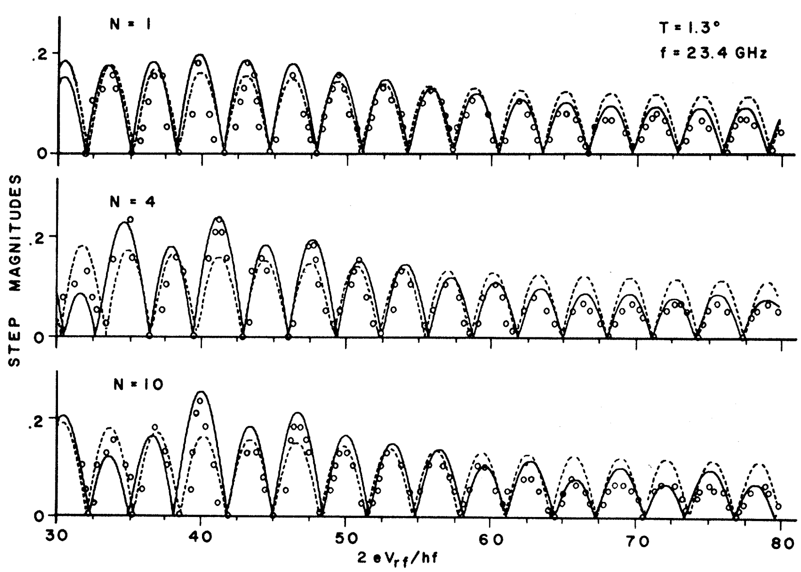

FIG. 2. Josephson step magnitudes from Eq. (1) (dashed line) and Eq. (3) (solid line), and experimental data points (open circles).

our data on a 3.6-Ω junction at 23.4 GHz. The dashed lines are the value of $|J_N(2\alpha)|$ for the $N = 1, 4$, and 10 steps. The solid lines are computed from Eqs. (2) and (3) using the values $2\Delta = 1.163$ meV and $f = 23.40$ GHz. At this frequency, the 24th Josephson step falls near the gap. Thus we expect to see the largest deviations from the $|J_N(2\alpha)|$ curve in the even numbered steps. Our data strongly support Eq. (3). The close agreement between experiment and theory especially at the unusually large maxima (for instance, those at $N=4$, $2\alpha=42$ and $N=10$, $2\alpha=40$) is strong evidence for the existence of the Riedel peak. Although we have shown the results for only three of the steps, we find similar agreement for all twelve steps measured. This experiment has been performed with several junctions over a range of frequencies with equally good results.

A somewhat more quantitative determination of $I_J(f)$ near the peak may be made. To do this, we choose a series of data points for a fixed N and α and a small range of frequency for which there is a particularly strong and frequency-dependent deviation. For each datum point, we select the one term (n') in the n summation of Eq. (3) which most nearly satisfies Eq. (5), i.e., $n' = 2\Delta/hf + N/2$. Using the datum point, and the theoretical values for the remaining terms of Eq. (3), we solve for $I_J((n'-\frac{1}{2}N)2f)$. A series of points obtained in this way is plotted on the insert of Fig. 1. The error bars indicate the spread which results from selecting different data points at the same frequency. A more complete discussion of these results will be presented elsewhere.

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Kondo Effect in Superconductors*

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A self-consistent treatment of the Kondo scattering of conduction electrons from magnetic impurities is presented in order to treat finite concentrations of such impurities in superconductors. The present theory leads to a concentration dependence for the transition temperature which differs markedly from the Abrikosov-Gor'kov result. The theory appears to account nicely for various experimental results. We also find that superconductivity, under certain circumstances, does not exist at all temperatures below T_c.

In this Letter, we present a theory of the Kondo effect in superconductors with finite concentrations of magnetic impurities. Recently, there has been some progress in extending to superconductors[1-4] the Nagoaka approximation,[5] as well as the Suhl approach,[6] treating the s-d model for magnetic impurities in metals. However, these theories confine themselves to a single impurity alone, i.e., to the lowest order in impurity concentration, while the finite-concentration case has not been seriously attacked in this context. One knows, however, that nonlinear effects can arise for even very low impurity concentrations, so that this case is of considerable interest. There is also an experimental challenge to the existing theory, since a number of systems investigated demonstrate a variation of the transition temperature T_c with impurity concentration which deviates in a characteristic and often inexplicable fashion from the Abrikosov-Gor'kov (AG) prediction.[7]

Our treatment of finite concentrations is based