# Light-Cone Analysis of Massive $\mu$-Pair Production*

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An operator representation for the behavior of the product of two electromagnetic currents near the light cone is used to discuss massive $\mu$-pair production in hadron-hadron interactions. Restrictions from the Stanford Linear Accelerator Center electroproduction experiments and from Regge theory are incorporated. Our predictions are in agreement with the recent Columbia–Brookhaven National Laboratory experiment.

The interesting Stanford Linear Accelerator Center (SLAC) experimental results on deep inelastic electroproduction[1] have been discussed from a number of different theoretical frameworks, including those based on "almost equal-time" commutators,[2] asymptotic infinite-momentum sum rules,[3] vector-meson dominance (VMD),[4] "parton" models,[5] and light-cone commutators.[6,7] Needless to say, the several numbers obtained from SLAC are insufficient really to test these ideas or distinguish between them.[8] The recent Columbia–Brookhaven National Laboratory experiment measuring massive muon-pair production from high-energy proton-proton collisions[9] is therefore extremely useful theoretically since it involves an initial state different from SLAC's and provides additional experimental constraints. Several theoretical investigations of this process have already been given.[10] In this note we shall apply some theoretical results[11] on the behavior of current products near the light cone, incorporating the SLAC experimental results, to study the Columbia experiment. Our predictions turn out to be in excellent agreement with experiment.

We consider the reaction $p+p\to\mu^++\mu^-+$anything and call $p$ and $p'$ the momenta of the initial nucleons ($p^2=p'^2=m^2$), and $q$ the momentum of the muon pair. We define the invariants $s=(p+p')^2$, $\nu=p\cdot q$, and $\nu'=p'\cdot q$. The cross section is (neglecting the muon mass)

$$\frac{d\sigma^{(r)}}{dq^2}=\frac{\alpha^2}{6\pi^3}\frac{1}{[s(s-4m^2)]^{1/2}}\int\frac{d^3q}{q_0}\frac{1}{q^2}W^{\mu\nu}\epsilon_\mu{}^{(r)}\epsilon_\nu{}^{(r)}, \tag{1}$$

where

$$W_{\mu\nu}=E_1E_2\int d^4x\,e^{-iq\cdot x}\,{}_{\rm in}\langle pp'|J_\mu(x)J_\nu(0)|pp'\rangle_{\rm in}. \tag{2}$$

In (1), $\epsilon_\mu{}^r(q)$ describes the polarization $r=T_1$, $T_2$, or $L$ of the $\mu$ pair, and the integration region in the center-of-mass (c.m.) frame is described by the inequality

$$(q^2)^{1/2}<q_0<(s+q^2-4m^2)/2\sqrt{s}\equiv\kappa_0(q^2,s). \tag{3}$$

In (2), $J_\mu$ is the electromagnetic current, a spin average is (here and everywhere) understood, and only the connected part of the matrix element occurs. The total cross section is

$$\frac{d\sigma}{dq^2}=-\frac{\alpha^2}{6\pi^3}\frac{1}{[s(s-4m^2)]^{1/2}}\frac{d^3q}{q_0}\frac{1}{q^2}W_\mu{}^\mu. \tag{4}$$

In the physical region for our reaction we must have $q^2=q_0{}^2-\vec{q}^2>0$. This is in contrast to the SLAC kinematics where $q^2<0$. Using current conservation and the reflection property $W_{\nu\mu}=W_{\mu\nu}{}^*$, we can write $W_{\mu\nu}=(q_\mu q_\nu-g_{\mu\nu}q^2)W_1(s,q^2,\nu,\nu')+\cdots$.

The SLAC experiment can be nicely described with the assumption that the appropriate dimensionless functions $F_i(q^2,\nu)$ become functions of only the ratio $\rho\equiv\nu/q^2$ in the limit $-q^2\to\infty$, $\nu\to\infty$, $\rho$ fixed. This corresponds to the expectation that a massive photon should only probe the short-distance (mass-independent) structure of the target. We should like to apply this same idea to the $\mu$-pair process but note that, because of purely hadronic scale-noninvariant effects,[2] this need not imply that the dimensionless structure functions $F_i=q^2W_i$ become functions of only the ratios $\rho\equiv\nu/q^2$, $\rho'\equiv\nu'/q^2$, and $\sigma\equiv s/q^2$ in the limit

$$q^2,s,\nu,\nu'\to\infty \text{ with } \rho,\rho', \text{ and } \sigma \text{ fixed}. \tag{5}$$

We shall rather implement the electromagnetic scale-invariance principle by assuming that the short-distance behavior of the product $J_\mu(x)J_\nu(0)$ is mass independent. This approach has led to an elegant analysis of the SLAC results[7] and an operator generalization will be shown below to be the relevant concept for understanding the $\mu$-pair results.

Combining (1) and (2), we obtain the expression

$$\frac{d\sigma^r}{dq^2} \propto \frac{1}{q^2}\int d^4x\, \Delta_R^+(-x;q^2)\langle pp'|J^\mu(x)J^\nu(0)|pp'\rangle \epsilon_\mu^r \epsilon_\nu^r, \tag{6}$$

where

$$\Delta_R^+(-x;q^2) \equiv \int_R d^4k\, e^{-ik\cdot x}\theta(k_0)\delta(k^2-q^2), \tag{7}$$

with the integration region $R$ specified by $|\vec{k}| < [\kappa_0^2 - q^2]^{1/2} \equiv \kappa$ in the c.m. frame. Now, when $s\to\infty$ and $q^2 \to \infty$, we see from (3) that $R\to$ (all space) and so $\Delta_R^+(x;q^2)\to\Delta^+(x;q^2)$, the ordinary free-field Wightman function. Thus, in the specified limit, only the integration region $x^2 \simeq 0$ is important in (6). More quantitatively, it can be shown that (7) has support essentially in the region

$$|x^2 - 1/\kappa^2| < 1/q^2. \tag{8}$$

Elsewhere $\Delta_R$ gives exponential decrease or rapid oscillation which damps the integrand in the physical region. This region is near the light cone provided $q^2$ and $\kappa$ are both big.[12]

Having established the relevance of the light cone $x^2 \sim 0$ to our problem, we proceed to develop the appropriate formalism. Wilson[13] has proposed that any product $A(x)B(0)$ of local fields has an expression for $x$ near zero of the form

$$A(x)B(0) \simeq \sum_i C_i(x)O_i(0), \tag{9}$$

where the functions $C_i(x)$ have singularities for $x^\mu \to 0$ (within logs) of the form $(x^2 - i\epsilon x_0)^{d_i - d_A - d_B}$, where $d_i$ is the dimension of the field $O_i$. Only the (finite number of) fields of dimension less than $d_A + d_B$ give singularities and need be kept in (9). We shall adopt Wilson's proposal since it leads to an understanding of many aspects of particle physics[13] and is correct to all order of (renormalizable) perturbation theory.[14] It can be shown[11] that the appropriate generalization of (9) for $x^2\to 0$ (but not necessarily $x^\mu \to 0$) has the form[15]

$$A(x)B(0) \simeq \sum_i C_i(x^2 - i\epsilon x_0)\sum_{n=0}^{\infty} x_{\alpha_1}\cdots x_{\alpha_n} O_{n,i}^{\alpha_1\cdots\alpha_n}(0), \tag{10}$$

where $O_{n,i}^{\alpha_1\cdots\alpha_n}(0)$ is a local field of dimension $d_{n,i} < d_i + n$ so that $C_i(x^2) \leqslant (x^2)^{d_i - d_A - d_B}$ within logs. For $x^\mu \to 0$, only a finite number of terms in (10) contribute and we get back (9), but for $x^2\to 0$, all terms must be kept. Equation (10) implies that the leading light-cone singularity of the one-proton matrix element has the form

$$\langle p|A(x)B(0)|p\rangle \simeq \sum_i C_i(x^2) f_i(x\cdot p),$$

where

$$f_i(\lambda) \equiv \sum_n a_{n,i}\lambda^n,$$

with

$$\langle p|O_{n,i}^{\alpha_1\cdots\alpha_n}(0)|p\rangle = a_{n,i}p^{\alpha_1\cdots\alpha_n} + b_{n,i}g^{\alpha_1\alpha_2}p^{\alpha_3}\cdots p^{\alpha_n} + .$$

Applying these results to the case of interest, using current conservation and the fact that the dimension of $J_\mu(x)$ is three,[13] one obtains (for the symmetric part in $\mu\nu$, apart from $c$-numbers),

$$J_\mu(x)J_\nu(0) \simeq -(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)O_1(x) - (g_{\mu\nu}\partial_\alpha\partial_\beta - g_{\alpha\mu}\partial_\beta\partial_\nu - g_{\alpha\nu}\partial_\beta\partial_\nu + g_{\alpha\nu}g_{\beta\mu}\Box)O_2^{\alpha\beta}(x), \tag{11}$$

where

$$O_1(x) = E_1(x^2 - i\epsilon x_0)\sum_n x^{\alpha_1}\cdots x^{\alpha_n} O_{1,\alpha_1\cdots\alpha_n}^{(n)}(0), \tag{12}$$

and

$$O_2^{\alpha\beta}(x) = E_2(x^2 - i\epsilon x_0)\sum_n x^{\alpha_1}\cdots x^{\alpha_n} O_{2,\alpha_1\cdots\alpha_n}^{(n)\alpha\beta}(0). \tag{13}$$

Taking the one-proton matrix element of (11) leads to the light-cone analysis of electroproduction given in Ref. 7. Exact scaling requires that $E_2(x^2) \leq \ln x^2$ and $E_1(x^2) \leq 1/x^2$.[7] The experimentally observed nontrivial scaling[1] implies that $E_2(x^2) = \ln x^2$ and the asymptotic vanishing of $\sigma_L^{ep}$ (presumably as $1/q^2$) suggests that also $E_1(x^2) = \ln x^2$. We shall assume here these simple logarithmic singularities. Singularities stronger by logarithmic powers would change our results only by logs.

We next define

$$\langle pp'|O_1(x)|pp'\rangle = E_1(x^2)f_0, \tag{14}$$

and

$$\langle pp'|O_2^{\alpha\beta}(x)|pp'\rangle = E_2(x^2)[f_1 g^{\alpha\beta} + f_2 p^\alpha p^\beta + f_3 p'^\alpha p'^\beta + f_4(p^\alpha p'^\beta + p'^\alpha p^\beta)], \tag{15}$$

where $f_i = f_i(s, x\cdot p, x\cdot p') + O(x^2)$, the $x^2$ dependence being irrelevant for our purposes since it leads to weaker singularities. Fermi statistics require that

$$f_i(s,x\cdot p, x\cdot p') = +f_i(s, x\cdot p', x\cdot p), \quad i=0,\ 1,\ 4; \quad f_2(s,x\cdot p,x\cdot p') = -f_3(s,x\cdot p', x\cdot p), \tag{16}$$

and crossing gives

$$f_i(s,x\cdot p,x\cdot p') = f_i(s,-x\cdot p,-x\cdot p'), \quad \text{all } i. \tag{17}$$

We thus obtain

$$W^{\mu\nu}\epsilon_\mu{}^r\epsilon_\nu{}^r \simeq \int d^4x\, e^{-iq\cdot x} E(x^2)\{-q^2(2f_0+f_1) + (-q_\alpha q_\beta + \epsilon_\alpha{}^r\epsilon_\beta{}^r q^2)[f_1 g^{\alpha\beta} + \cdots]\}, \tag{18}$$

where we have written $E_1(x^2) = E_2(x^2) = \ln x^2 \equiv E(x^2)$.

Consider the contribution of

$$\langle pp'|O_{1\alpha_1\cdots\alpha_n}{}^{(n)}(0)|pp'\rangle = F^{(n)}(s)(p_{\alpha_1}\cdots p_{\alpha_n} + p_{\alpha_1}{}'\cdots p_{\alpha_n}{}') + \cdots \tag{19}$$

to $f_0(s,x\cdot p, x\cdot p')$. We assume that the large-$s$ behavior of such amplitudes (corresponding to the emission of a zero four-momentum particle with Lorentz indices $\alpha_1\cdots\alpha_n$) is governed by Regge theory.[16] Then $F^{(n)}(s) \to c_n s^\alpha$, whereas the omitted terms (involving the mixed polynomials $p_{\alpha_1}\cdots p_{\alpha_m} p_{\alpha_{m+1}}{}' \times\cdots p_{\alpha_n}{}'$ behave like $s^{\alpha-(n-m)}$, where $\alpha$ is the $t=0$ intercept of the leading contributing Regge trajectory (presumably the Pomeranchuk with $\alpha=1$). Thus we can write

$$f_0(s,x\cdot p,x\cdot p') \to s^\alpha[f_0(x\cdot p) + f_0(x\cdot p')], \tag{20}$$

where $f_0(x\cdot p) \equiv \sum_n c_n (x\cdot p)^n$. Similarly, considering (15) leads to the behavior (20) for $f_1$ and $f_2 = f_3$ and to (20) but with $s^{\alpha-1}$ for $f_4$. We thus see that the leading light-cone singularity carries a large-$s$ behavior equal to the maximum allowed by Regge theory for the full amplitude. This result, incidentally, also justifies our use of the weak expansions (11)-(13) in (6) even though $s\to\infty$. Any nonleading contribution can grow no faster with $s$ but must fall faster with $q^2$ for fixed $\rho$ and $\rho'$.

Returning to (18), we are led to consider the behavior of the pole contributions to integrals of the form $I \equiv \int d^4x\, e^{-iq\cdot x} E(x^2 - i\epsilon x_0) f(x\cdot p)$ in the limit (5). We obtain $I \simeq (1/m)(1/\eta)(\partial/\partial\eta)(1/\eta)\tilde{f}((q_0-\eta)/m)$, where $\tilde{f}(\omega)$ is the Fourier transform of $f(\lambda)$ and $\eta \equiv |\vec{q}|$. In the limit (5), we thus obtain $I \simeq (1/q^4)(1/\rho^2)f'(1/\rho) \equiv (1/q^4)g(\rho)$. In this way (18) is seen to have the asymptotic form

$$W^{\mu\nu}\epsilon_\mu{}^r\epsilon_\nu{}^r \to (s^\alpha/q^4)\{q^2 g_0(\rho) + [\nu^2 - (p\cdot\epsilon^r)^2 q^2]g_1(\rho) + 2s^{-1}[\nu\nu' - (p\cdot\epsilon^r)(p'\cdot\epsilon^r)q^2]g_2(\rho) + (p\to p')\}. \tag{21}$$

The form (21) which we have obtained has a simple physical interpretation in terms of the Regge picture which accounts well for the SLAC data.[7] The SLAC results for $\rho \geq 2$ can be described by the assumption that they correspond to (Pomeranchuk) Regge-pole–dominated behavior with the $q^2$ dependence on the photon-Pomeranchukon-photon vertex given by scale invariance. If we adapt this picture for the present situation, and further use Regge theory to conclude that the $(pp')$-Pomeranchukon-$(pp')$ vertex has the large-$s$ behavior $s^\alpha = s^1$ [thus obtaining a Regge-squared description (see Fig. 1) corresponding to the two large subenergies $\nu$ or $\nu'$ and $s$ (with $q^2$ fixed)], we obtain precisely the form (21) for large $\rho$ with the further information that $g_0(\rho) \to A_0\rho$, $g_1(\rho) \to A_1/\rho$, and $g_2(\rho) \to A_2/\rho$ for some constants $A_i$. Our final assumption will be that these asymptotic behaviors set in at the SLAC points $\rho \sim 2$. Then we can neglect $g_2$ in (21) and obtain for (1)

$$\frac{d\sigma^r}{dq^2} \to \text{const}\frac{1}{q^6}\int_R \frac{d^3q}{q_0}\left\{(A_0 + q^2 A_1)(P\cdot q) + q^2 A_1\left[\frac{(p\cdot\epsilon^r)^2}{\rho} + \frac{(p'\cdot\epsilon^r)^2}{\rho'}\right]\right\}, \tag{22}$$

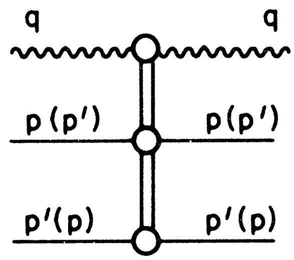


FIG. 1. The "Regge-squared" model.

and for (4)

$$\frac{d\sigma}{dq^2} \to \text{const}\frac{1}{q^6}(3A_0 + 2A_1q^2)\int \frac{d^3q}{q_0}(P\cdot q), \qquad (23)$$

where $P = p + p'$.

We have compared our prediction (23) with the experimental results in Fig. 2. The experimentalists do not measure the total unconstrained cross section (23), but have an angle cut $\cos\theta \geq 0.998$ and a momentum cut $12 < p_{\text{lab}} < 29$ GeV/$c$ for $E_{\text{lab}} = 29.5$ GeV. It can be shown that these cuts do not significantly affect the relevance of the light cone. The theoretical curve we have plotted corresponds to performing the integral (22) over the cut region and taking $3A_0/2A_1 \sim 20$ GeV$^2$. Our result is seen to be in good agreement with experiment.[17] We can obtain an even better fit by suitably adjusting the functions $g_i(\rho)$ below the value ($\rho \sim 2$) suggested by SLAC for the onset of the asymptotic behavior. The origin of the shoulder in Fig. 2 comes from an interplay between the phase-space control of the integration region in Eq. (23) and the $q^2$ dependence of the coefficient.

We conclude from this analysis that our configuration-space techniques are very useful for understanding the interactions of massive photons with hadrons. The powerful operator expansion (11) should be equally useful for studying related processes such as $e^+e^- \to$ hadron + anything. Work in this direction is in progress.

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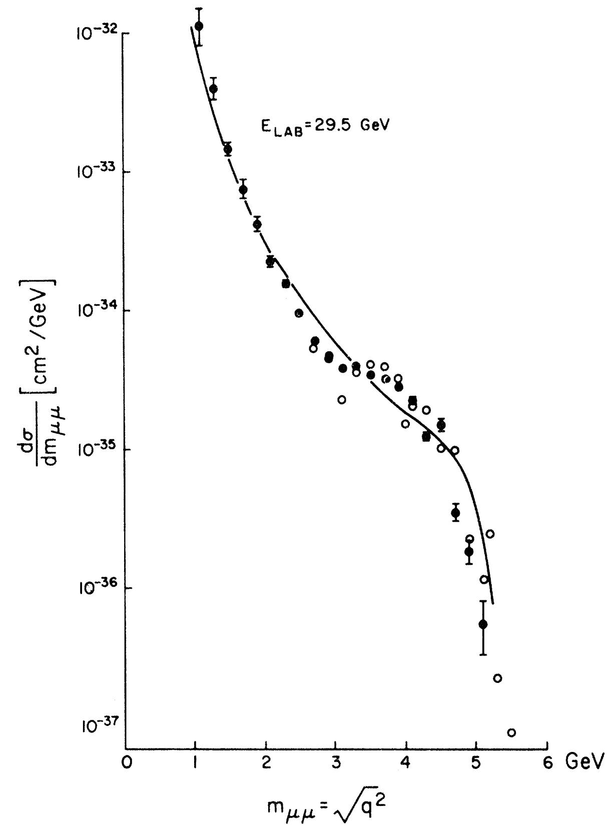


FIG. 2. Experimental cross section of Christenson *et al.*, Ref. 8.

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[12]At this point we make the important observation that,

because of the spectral properties of $\langle pp'|JJ|pp'\rangle$, the $x_0$ integration in Eq. (6) can be closed in the upper half-plane.

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[15]In principle, the singular functions $C_i(x)$ could vary with $n$. We assume the simpler expression (10), however, in accordance with the SLAC results discussed below.

[16]We are using the easily derived fact that $\langle \mathrm{in}|O|\mathrm{in}\rangle$ has the same asymptotic behavior as the scattering amplitude $\langle \mathrm{in}|O|\mathrm{out}\rangle$ if $O$ satisfies the asymptotic condition.

[17]Our results are, however, in disagreement with all of those of Ref. 10. Matveev *et al.* assume the $F_i$ are functions of the ratios or use a VMD model which predicts that $d\sigma/dq^2 \sim s/q^6$. As can be seen from (21), our $F_i$ differ from functions of the ratios by powers of $s$ [and have the striking form $g(\rho)+g(\rho')$]. By explicitly evaluating a particular Feynman diagram, Berman, Levy, and Neff obtain a special case of (21) corresponding to $g_0=1, g_1=g_2=0$. Sakurai, of course, uses a VMD model and concludes that there is predominantly longitudinal polarization (Ref. 8). Our prediction for $\sigma_L/\sigma_T$ can be obtained from (22) by using the $A_0$ and $A_1$ which fit $d\sigma/dq^2$. It is seen to be a function of $g^2$ which does not make $\sigma_L/\sigma_T$ particularly large or small. This prediction should be testable in future experiments. Finally, the model of Drell and Yan predicts that $d\sigma/dq^2 \propto (1/q^4)F(s/q^2)$, whereas we predict that $d\sigma/dq^2 \propto (1/q^2)F_1(s/q^2)+F_2(s/q^2)$.

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# Observation of Muon Trident Production in Lead and the Statistics of the Muon*

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We have observed the production of muon tridents in lead with an effective cross section of $51\pm 7$ nb per nucleus, in agreement with the predictions of quantum electrodynamics. This measurement is sufficiently accurate that the interference term due to the presence of two identical muons in the final state is seen. The size of the measured interference term is $1.15\pm 0.25$ times the value predicted for Fermi statistics.

In an experiment performed at the alternating-gradient synchrotron of Brookhaven National Laboratory, we have studied the direct production of muon pairs by incident muons in the field of a lead nucleus, or muon tridents:

$$\mu^{\pm}+\mathrm{Pb}\rightarrow\mu^{\pm}+\mathrm{Pb}+\mu^{+}+\mu^{-}.$$

Because of the two identical muons in the final state, this reaction is sensitive to the statistics of the muon.

Many attempts have been made to understand what causes the muon-electron mass difference, including a proposal that the muon might have anomalous statistics.[1] The question of statistics is interesting in its own right because the principles which relate the statistics of a particle to its spin are fundamental to the understanding of quantum field theory.

Bose statistics is excluded for the muon by the spin-statistics theorem since the spin is well known.[2] Similarly, the simpler forms of generalized statistics are excluded by the measured cross section of the photoproduction of muon pairs.[3] However, no direct evidence exists on the statistics of the muon. In fact, the experiment reported here is the first quantitative measurement of any process with two identical muons in the final state.[4]

The experiment reported here[5] was run for about 200 h in the 10.5-GeV muon beam designed by Columbia[6] and modified by Harvard.[7] The beam had an intensity of 12 000 muons per 0.4-sec pulse and a measured pion contamination of 0.01 %. The momentum spectrum of the beam was studied by triggering the detection apparatus on one thousand beam muons of each polarity and