by the inserts in Fig. 2(b). Without a quenching field the $4D_{3/2}$-$4P_{1/2}$ resonance is merely an elongation of the right wing of the $4P_{3/2}$-$4S_{1/2}$ resonance; with a quenching field set at 1225 MHz, the $4D_{3/2}$-$4P_{1/2}$ resonance is a distinct, symmetrical resonance centered at 1375 MHz. A similar elucidation of the $4F_{7/2}$-$4D_{5/2}$ resonance is obtained by using the $4S_{1/2}$-$4P_{1/2}$ transition at 140 MHz to quench $4S$ states.

Several methods were used to reduce the data. The method employed was determined by the number of data points, the statistical errors, and the separation of the hyperfine components. Detailed resonances with small statistical errors and large hyperfine separations were fitted with a fitting function involving five free parameters and three Lorentzian components. The five parameters were the half-width, the resonance frequency for one component, and the amplitude of each of the three components. For resonances where there was only a small hyperfine structure or severe overlap of fine-structure components, the center of the line was located by measurements at symmetrical points on each side of the line, and the limits of error were determined by computer simulation of the resonances. Calculations were made for different choices of the initial hyperfine-state amplitudes in order to determine the effect of different weighting schemes on the measured frequency. The data were separately corrected for variations in the rf power, for the first-order Doppler shift, and for Stark shifts.

Table I summarizes the measured fine-structure intervals. It also gives the results of earlier experiments[3,4] and the theoretical values for the fine-structure intervals.[1,5,6] The agreement between experiment and theory is quite satisfactory. Work is now in progress to understand in detail the signal amplitudes and to use this information to study the relative populations of the angular momentum states produced in the collisions which form the hydrogen atoms.

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†National Science Foundation Predoctoral Fellow.

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Enhanced Damping of Large-Amplitude Plasma Waves

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The collisionless damping of small-amplitude plasma waves is independent of the exciting voltage V_{ex}. For large-amplitude waves, the damping rate increases linearly with V_{ex} and amplitude oscillations are observed. A phase transition appears at amplitude minima.

The amplitude oscillation of an electron plasma wave has been studied experimentally by Malmberg and Wharton.[1] In their experiment where the wave potential φ is within the range $e\varphi/T_e \lesssim 0.3$ (where T_e is the electron temperature in eV), the damping length of the amplitude seems to be independent of the exciting voltage. Theoretically, Armstrong[2] and Dawson and Shanny[3] showed that the initial temporal damping rate of the wave increases rapidly with amplitude. For ion acoustic waves, Sato et al.[4] have observed enhanced damping followed by growth when $e\varphi/T_i$

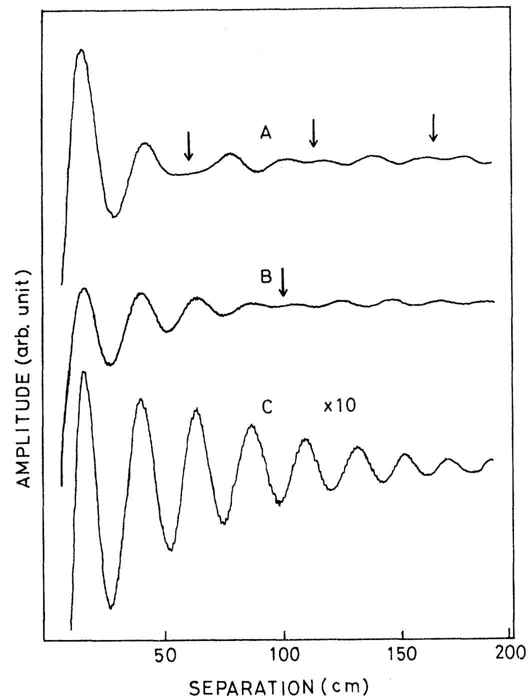

FIG. 1. Raw data. The transmitter voltage is 7.1, 3.7, and 1.4 V for curves A, B, and C, respectively.

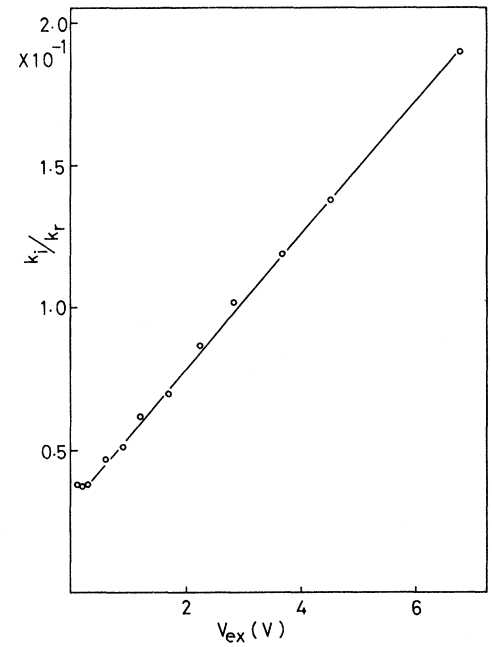

FIG. 2. The damping rate normalized by wave number as a function of the exciting voltage V_{ex}.

$\gtrsim 0.4$, where T_i is the ion temperature. This Letter reports the preliminary results of a transmission experiment on longitudinal electrostatic waves under an extended range of the potential, $e\varphi/T_e \lesssim 2$, using low-density and quiescent plasma. The purpose of this experiment is to measure the spatial damping rate as a function of transmitting voltage.

The experiment was performed using the space chamber at the Institute of Space and Aeronautical Science, University of Tokyo. The stainless-steel chamber is 2 m in diameter and about 4 m long. At both ends of the chamber plasma sources of back-diffusion type, 15 cm in diameter, are set up face to face. A receiving probe made of 1-mm diameter and 10-cm long tungsten wires is placed 1 m from one of the plasma sources. A transmitting disk grid 15 cm in diameter, consisting of 0.1-mm diameter molybdenum wires spaced 2 mm apart, is moved along the plasma column. The grid is connected by coaxial cable to a signal generator. The probe is connected to a wide-band amplifier and a balanced mixer. A reference signal from the transmitter is phase shifted and also added to the mixer; i.e., the system is used as an interferometer. The position of the transmitter grid is taken as the x axis of an x-y recorder, and the mixer output is applied to the y axis. The plasma is dark, its density is about 3.2×10^4 electrons/cm,^{3} and the electron temperature is about 0.9 eV. Hence the Debye length is about 4 cm. The background gas is argon and its pressure is 5×10^{-6} Torr. The electron mean free path for electron–neutral-atom collisions is of the order of 20 m. For the present electron-wave experiments, extending over a length of about 2 m, the plasma is considered as collisionless. A longitudinal magnetic field of 60 G is applied by a Helmholz coil of 2.5 m diameter. The electron-cyclotron frequency (170 MHz) is much larger than the electron plasma frequency (1.6 MHz).

When an rf voltage is applied to the grid inserted into the plasma, electron plasma waves are excited which propagate in both directions along the plasma column. The grid is held at floating potential. The dispersion of the waves is measured by varying a transmitting frequency from 1.2 to 9 MHz. The resultant dispersion relation is similar to that for finite dimension, finite temperature, and strong magnetic field. This relation already has been studied experimentally by several workers.[5,6]

Typical raw data drawn on the x-y recorder are shown in Fig. 1. The transmitting frequency is 6 MHz. The noise in the curves is caused by the vibration of the motor-driven grid. For curve C of this figure the peak-to-peak rf voltage applied to the transmitting grid, V_{ex}, is 1.4 V. When the amplitude of the oscillation is plotted on a logarithmic scale, it is found that the damping rate changes at the separation of 150 cm. On both sides of this critical separation the change of amplitude can be represented by straight lines. The initial damping length, corresponding to the first of the straight lines, is 58 cm and the final damping length, corresponding to the next, is 26 cm. For curve B the exciting voltage is 3.7 V. Initially the damping length is 32 cm, and beyond the separation of about 68 cm it is 13 cm. The amplitude reaches a minimum at the separation of 100 cm. For curve A the exciting voltage is 7.1 V. Its initial damping length is 18 cm. The reason for the change of damping rate in the course of transmission is not clear. The initial damping rate k_i divided by the wave number k_r of the wave is shown in Fig. 2 as a function of V_{ex} for the case where the transmitting frequency is 7 MHz and the wavelength is 19 cm. The wavelength is independent of V_{ex}. The damping rate k_i is constant if V_{ex} is smaller than about 0.3 V. If the exciting voltage V_{ex} exceeds 0.3 V, the damping rate increases linearly with V_{ex} and the amplitude oscillations are observed as shown by curves A and B in Fig. 1. Since the wave potential φ is proportional to V_{ex}, it is written as $\varphi = \frac{1}{2}\alpha V_{ex}$. With the value of α estimated below and with 0.3 V for V_{ex}, the critical potential φ_{cr} of amplitude oscillations is given by $e\varphi_{cr}/T_e \simeq 0.1$. For an ion acoustic wave, Sato et al.[4] obtained the relation $e\varphi_{cr}/T_i \simeq 0.4$.

When $V_{ex} = 0.3$ V, the measured value of k_i/k_r is nearly equal to 0.04 which is one order of magnitude smaller than the critical value (0.4) calculated by Hirshfield and Jacob[7] as a criterion for free streaming. So the decay of the wave is dominated by a spatial Landau damping, rather than by the free streaming of noninteracting particles. Defining λ_{osc} as the distance between amplitude minima, the following relation has been introduced:

$$\frac{\lambda}{\lambda_{osc}} V_p = \frac{k_{osc}}{k_r} V_p = \left(\frac{e\varphi}{m}\right)^{1/2}, \quad (1)$$

where λ and V_p are the wavelength and the phase velocity of the wave, respectively, $k_{osc} = 2\pi/\lambda_{osc}$, and m and e are the electron mass and charge.[1] Instead of λ_{osc} the position D of the first minimum is put into the left-hand side of Eq. (1) and the result is plotted in Fig. 3 as a function of $V_{ex}^{1/2}$ for two transmitting frequencies. It can

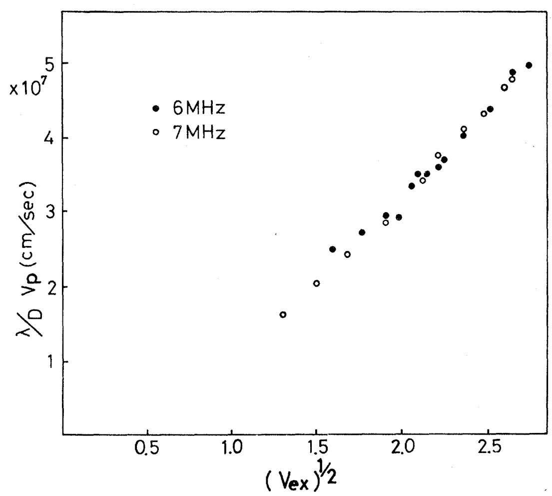

FIG. 3. Product of the inverse of the normalized distance with the wavelength and phase velocity as a function of the square root of the exciting voltage $V_{ex}^{1/2}$.

be seen that a relation of the form of Eq. (1) indeed holds independently of the transmitting frequency. Using the measured ratio D/λ_{osc} which is about 1.2, and using Eq. (1) and Fig. 3, we estimated the grid efficiency α at about 0.4. The damping length of the amplitude of the oscillation is somewhat larger than that of a small-amplitude wave.

Theoretically, for the case of small damping rate, amplitude oscillations should occur if $k_{osc}/k_s \gtrsim (2\pi)^{1/2}$, where k_s is the imaginary part of the wave number of a small-amplitude plasma wave.[8,9] As the measured k_{osc}/k_s is about 4.5 under the condition that $V_{ex} = 1.7$ V, it is in agreement with the above condition.

It should be noted that at the position of an amplitude minimum shown by an arrow in Fig. 1, a phase transition appeared. With an increase of V_{ex}, i.e., φ, this phase gap reaches the value π. This phase transition, which has not been reported in Refs. 1 and 4, seems to be an unpredicted result. At the position of the amplitude minimum, not only a decaying wave but also a growing wave seems to exist. That is, in the course of the damping of the decaying wave, a new growing wave appears, and the phase gap occurs between these two waves.

In summary, the damping of electron plasma waves is shown experimentally to be qualitatively dependent on the amplitude of the wave. Small-amplitude waves show linear Landau damping. Large-amplitude waves exhibit enhanced damping and amplitude oscillations. This behavior is different from that observed by Malmberg and

Wharton, but is similar to that of ion acoustic waves studied by Sato et al. At an amplitude minimum, a phase transition is observed.

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Anomalous $P_V(T)$ of Solid ^{3}He in High Magnetic Fields*

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The low-temperature variation of the pressure at constant volume of a 24-cm^3/mole solid-^{3}He sample has been measured at 40.3 and 57.2 kG. No difference was observed from the zero-field curve. This result is thermodynamically inconsistent with previous measurements on solid ^{3}He if the system is in equilibrium. Implications of this anomalous result are discussed.

The magnetic properties of solid ^{3}He are of interest for two reasons. First, because of the large zero-point motion, the nuclear spins experience a large exchange interaction J. This has been confirmed by NMR measurements of spin relaxation and spin diffusion[1], by measurements of pressure as a function of temperature at constant volume, $P_V(T)$[2], and by measurements of the nuclear susceptibility.[3,4] Only by the last method has the sign (antiferromagnetic) of J been determined. Secondly, because of the simple lattice and the apparent absence of magnetic asymmetry, ^{3}He is thought to be an excellent example of a Heisenberg antiferromagnet.[5] A precise comparison of the Heisenberg theory with experiment should be feasible for this substance in the paramagnetic range.[6] In an attempt to accomplish this we have used a capacitance–strain-gauge technique to measure $P_V(T)$ in a large magnetic field.

Our experiment consists of measurements of $P_V(T)$ at three fields, 1, 40.3, and 57.2 kG, of a 24-cm^3/mole, bcc solid-^{3}He sample. This is the first time, to our knowledge, that solid ^{3}He has been examined at such a high field. At 1 kG, as expected, we get very good agreement with the zero-field results of the Florida group.[2] However, contrary to all expectations we have observed no difference, within experimental accuracy, between $P_V(T)$ measured at low field and at high fields.

Adams et al.[2] have shown that at zero field and for temperatures between 15 and 200 mK the effect of the exchange interaction dominates $P_V(T)$. Their data can be described by the relation

$$P_V(T) = 3\frac{R\Gamma}{V}\left(\frac{J}{k}\right)^2\frac{1}{T}, \qquad (1)$$

where $\Gamma \equiv \partial \ln|J|/\partial \ln V$ is an "exchange Grüneisen constant." The effect of a magnetic field on $P_V(T)$ can be calculated from the susceptibility data by using the Maxwell relation $(\partial P/\partial H)_{V,T} = (\partial M/\partial V)_{H,T}$. The susceptibility data fit the form $M = CH/T(1+4J/kT)$ (where C is the Curie constant). Integrating the Maxwell relation we obtain

$$P_V(T) = P_0 + \frac{3R\Gamma}{V}\left(\frac{J}{k}\right)^2\frac{1}{T} + 2\frac{\Gamma}{V}\left(\frac{J}{k}\right)\frac{CH^2}{T^2} + \cdots. \qquad (2)$$

For bcc ^{3}He at 24 cm^3/mole and 60 kG the magnetic term becomes equal to the zero-field term at approximately 20 mK. The higher-order terms in Eq. (2), which reflect the model details, also become appreciable in this range of fields