developed by Boyer and Lindquist[17] and by Carter[7,9] are used to determine the boundary conditions on H and Ψ as $\rho \to 0$ which are necessary and sufficient for the axis coordinate degeneracy to be removable and for there to exist a generalized Kruskal transformation to a new coordinate system extensible beyond the Schmidt boundary. Also the lowest order asymptotic boundary conditions are worked out. The former depend only on b, the latter only on h. (The Kerr solutions satisfy these conditions with $b^2 = m^2 - h^2/m^2$).

In the final stage of the proof it is shown that a smooth one-parameter variation of solution functions H and Ψ is uniquely determined by the path of variation of b and h. This is done by establishing that, subject to the boundary conditions, $\dot{H}$ and $\dot{\Psi}$ are uniquely determined as solutions of the linear system $\dot{G}_1 = \dot{G}_2 = 0$ by the values of $\dot{b}$ and $\dot{h}$, where a dot denotes differentiation with respect to the variation parameter. This step—the crux—depends on the existence of the key identity

$$\rho|\nabla(H^{-1}\dot{H}) + H^{-2}\dot{\Psi}\nabla\Psi|^2 + \rho|\nabla(H^{-1}\dot{\Psi}) - H^{-2}\dot{H}\nabla\Psi|^2 + \rho H^{-4}|\dot{H}\nabla\Psi - \dot{\Psi}\nabla H|^2$$
$$- H^{-3}(\dot{H}\dot{G}_1 + \dot{\Psi}\dot{G}_2) + H^{-4}(2\dot{H}^2 G_1 + \dot{H}\dot{\Psi}G_2 + \dot{\Psi}^2 G_1) \equiv \nabla[\rho H^{-1}\dot{H}\nabla(H^{-1}\dot{H}) + \rho H^{-1}\dot{\Psi}\nabla(H^{-1}\dot{\Psi})]$$

which enables us to obtain a standard kind of uniqueness proof by integrating over the space, using the fact that the boundary conditions turn out to be such as to eliminate the divergence contributions, while the field equations reduce the other side to a sum of positive terms. When $h = 0$, relatively trivial identities imply, in the same way, first that Ψ must vanish, and then that H is unique.

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Evidence for a Narrow $N^*(1470)$ †

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A narrow $I = \frac{1}{2}$ enhancement at a mass of 1.462 ± 0.006 GeV/c^2 with a width of 0.049 ± 0.012 GeV/c^2 has been observed in $p\pi^0$ and $n\pi^+$ modes produced in pp collisions at 6.6 GeV/c. The effect, having a 6-standard-deviation significance, is narrower in width than has been observed before in missing-mass–spectrometer experiments.

Since its discovery in phase-shift analyses,[1] the $N^*(1470)$ state has been one of the most controversial, if not the most uncertain, resonances reported.[2] Early experiments reporting observations of its existence have been done using a one-armed proton spectrometer with counters and spark chambers.[3] Among these various experiments, general agreement was reached on its mass (~1.40 GeV/c^2), width (~0.20 GeV/c^2), and the slope of a rather steep differential cross section (of the order of 20 GeV^{-2}). Because the one-armed spectrometer is incapable of determining whether the observed proton comes from the decay of or is recoiling against a nucleon resonance, the signal of the $N^*(1470)$ was usually seen above a huge kinematical background. Thus, it has not been possible to study the properties of the enhancement in these experiments. Ellis et al.[4] reported a bubble-chamber study of proton-proton collisions at 28.5 GeV/c using a missing-mass technique in an attempt to identify the source of the enhancement near 1.4 GeV/c^2 as observed in Ref. 3. They reported that for $t < 0.12$ (GeV/c)^2 and missing mass between 1.25 and 1.55 GeV/c^2 the events were mostly from the two-prong topology, and the final state $pn\pi^+$ accounted for roughly 60% of the sample. They also suggested that the events within this band were consistent with being dominantly in the $I=\frac{1}{2}$ state.

In this Letter, we report the observation of an $I=\frac{1}{2}$ enhancement at 1.462 ± 0.006 GeV/c^2 with a full width of 0.049 ± 0.012 GeV/c^2. The experimental resolution in this mass region is 0.023 GeV/c^2 full width at half-maximum (FWHM) and has been unfolded from the experimental width. This effect, seen in its two-body decay modes, differs significantly from the $N^*(1400)$ as seen in counter-spark-chamber measurements in its mass and width, as well as the slope of the differential cross section. It will be substantiated in a later section that our data are inconsistent with a broad $N^*(1470)$ produced in the reaction $pd \rightarrow p_s pp\pi^-$ at 7 GeV/c as reported by Shapira et al.[5]

The experiment was performed using the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber exposed to an external proton beam of 6.6-GeV/c momentum. A total of 30 000 two-prong events were measured. The final states of interest in this study are as follows:

$$pp \rightarrow p(n\pi^+), \tag{1}$$

and

$$pp \rightarrow p(p\pi^0). \tag{2}$$

A study of Reaction (1) involving the production of $\Delta^{++}(1236)$ has been reported elsewhere,[6] and the cross section for Reaction (1) is 5.73 ± 0.35 mb. The cross section for Reaction (2) is 2.54 ± 0.16 mb.

Figure 1 shows the invariant-mass distributions for $n\pi^+$ and $p\pi^0$ combinations from Reactions (1) and (2), respectively. An enhancement near 1.46 GeV/c^2 can be seen in Reaction (1). To enhance this effect, we have attempted to minimize contributions from the charge-exchange diagram in which the proton and π^+ emerge from a common vertex. It has been shown in Ref. 6 that this mechanism plays a dominant role in Reaction (1) in the low $p\pi^+$ mass region. Kinematically, the Chew-Low boundary (the minimum $t_{p\rightarrow p\pi^+}$) rises with larger $p\pi^+$ mass, and thus the importance of the charge-exchange diagram is expected to diminish with higher $p\pi^+$ mass. Figure 2(a) shows the π^+n mass distribution for $M_{\pi^+p} > 2.4$ GeV/c^2. The background near 1.46 GeV/c^2 is reduced considerably and the enhancement stands clearly above the background. One notes that the mass cut kinematically restricts $t_{p\rightarrow\pi^+p}$ to greater than 0.26 GeV^2, thus serving to reduce the importance of the charge-exchange mechanism as mentioned previously. A similar criterion is applied to Reaction (2), and the resultant $p\pi^0$ mass distribution is shown in Fig. 2(b). The sum

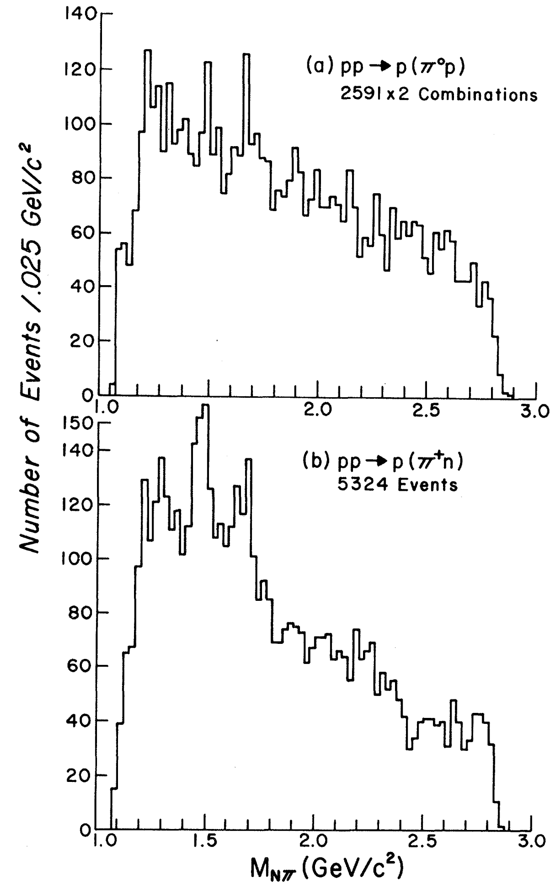

FIG. 1. Invariant-mass distributions of the bracketed pion-nucleon pair.

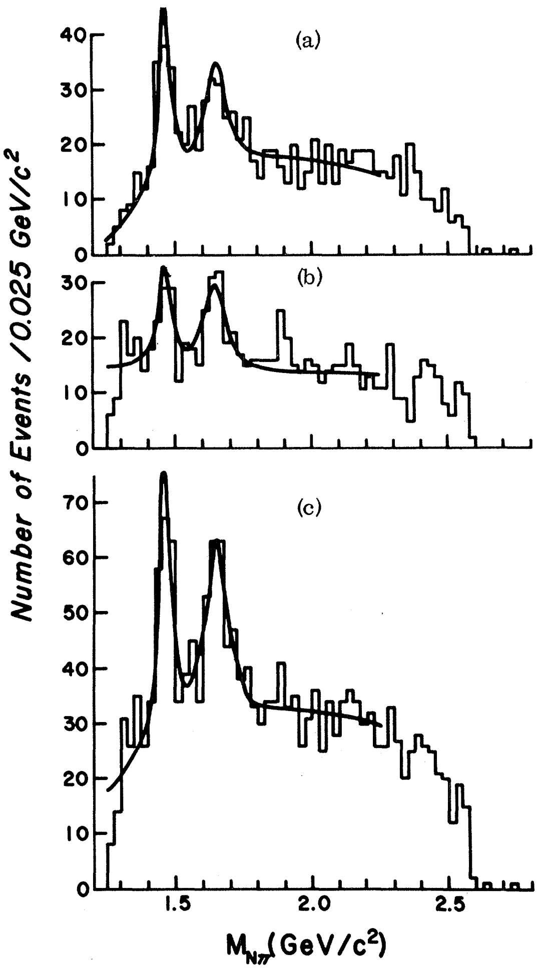

FIG. 2. (a) Invariant-mass plot for π^+n combinations for $M_{\pi^+p} > 2.4$ GeV/c^2. (b) Invariant-mass plot for π^0p with the other proton greater than 2.4 GeV/c^2. (c) The sum of (a) and (b). The solid curves represent fits by a quadratic polynomial background plus two Breit-Wigner functions as described in the text.

of Figs. 2(a) and 2(b) is displayed in Fig. 2(c). The combined signal in the mass region between 1.425 and 1.500 GeV/c^2 stands at the 6-standard-deviation level. To determine the masses and the widths of the enhancements, we use Fig. 2(c) and fit with an incoherent sum of a polynomial background and two S-wave Breit-Wigner functions, each multiplied by the same background function. The masses and widths thus obtained are

$$M_1 = 1.462 \pm 0.006 \text{ GeV}/c^2, \quad \Gamma_1 = 0.054 \pm 0.012 \text{ GeV}/c^2; \tag{3}$$

and

$$M_2 = 1.650 \pm 0.010 \text{ GeV}/c^2, \quad \Gamma_2 = 0.094 \pm 0.020 \text{ GeV}/c^2. \tag{4}$$

Table I. Branching ratios of the enhancements near 1.46 and 1.65 GeV/c^2.

m	$R(p\pi^0/n\pi^+)$	R expected $I=\frac{1}{2}$	$I=\frac{3}{2}$
1.462	0.50 ± 0.08	0.5	2
1.650	0.77 ± 0.12	0.5	2

The width parameters are found to be sensitive to the background level. The values of the parameters given in Eqs. (3) and (4) are obtained using a quadratic polynomial background, and the errors include a statistical error and an additional uncertainty due to background parametrization. The experimental resolution in this mass region is 0.023 GeV/c^2 FWHM, thus indicating that the natural widths for M_1 and M_2 are 0.049 ± 0.012 and 0.091 ± 0.020 GeV/c^2, respectively. The fitted values of M_1, M_2, Γ_1, and Γ_2 have been used to determine the amount of the resonances present in Reactions (1) and (2) separately. Since identical mass cuts have been made for both reactions, we may reasonably expect that the relative rates of a given resonance in Figs. 2(a) and (b) are the same as those without cuts. One notes that the mass cut discussed previously excludes any possibility of double counting for Reaction (2) in the region of the bumps since events with both $p\pi^0$ combinations below 2.4 GeV/c^2 are not included in the fits. The resultant branching ratios are given in Table I. Since identical sets of exchange diagrams are accessible for Reactions (1) and (2), the relative rate of production for a given resonance depends only on the isospin of the resonances in question. We conclude that the enhancements at 1.462 and 1.650 GeV/c^2 are $I=\frac{1}{2}$ states. It is interesting to point out that while the resonance parameters for the 1.650-GeV/c^2 enhancement agree well with other published bubble-chamber data in three-body modes,[7] our result on the 1.462-GeV/c^2 resonance is considerably narrower than other published results in this mass region.

The decay angular distribution of the $N^{*+}(1470)$ is distorted due to the mass cut mentioned earlier. Thus we have not been able to measure its spin and parity. However, if a $J^P=\frac{1}{2}^+$ assignment is assumed for this enhancement, we can estimate the cross sections for the processes

$$pp \to pN^{*+}(1470), \quad N^{*+}(1470) \to n\pi^+ \tag{5}$$

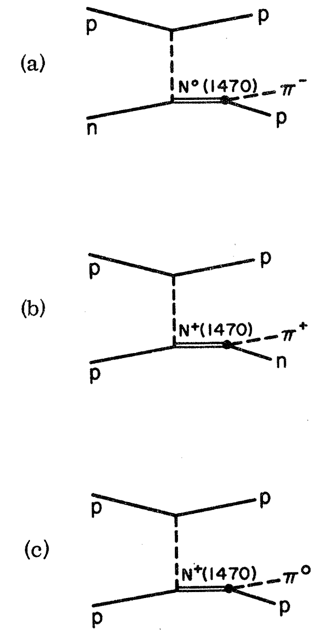

FIG. 3. (a) The exchange diagram responsible for the $N^*(1470)$ effect as reported in Ref. 5. (b), (c) The exchange diagrams for the production of $N^*(1470)$ for Reactions (5) and (6), respectively.

and

$$\begin{array}{l} pp \to pN^{*+}(1470) \\ \qquad \hookrightarrow p\pi^0. \end{array} \tag{6}$$

The loss of events due to the mass cut has been estimated using the Monte Carlo method and an isotropic decay angular distribution. The cross sections for Reactions (5) and (6) are estimated to be 0.29 ± 0.06 and 0.15 ± 0.03 mb, respectively, or a total cross section of 0.44 ± 0.07 mb for the process $pp \to pN^{*+}(1470)$, $N^{*+}(1470) \to N\pi$ (all charges).

Shapira et al.[5] reported an observation of the $N^*(1470)$ in the reaction

$$\begin{array}{l} pn \to pN^{*0}(1470) \\ \qquad \hookrightarrow p\pi^- \end{array} \tag{7}$$

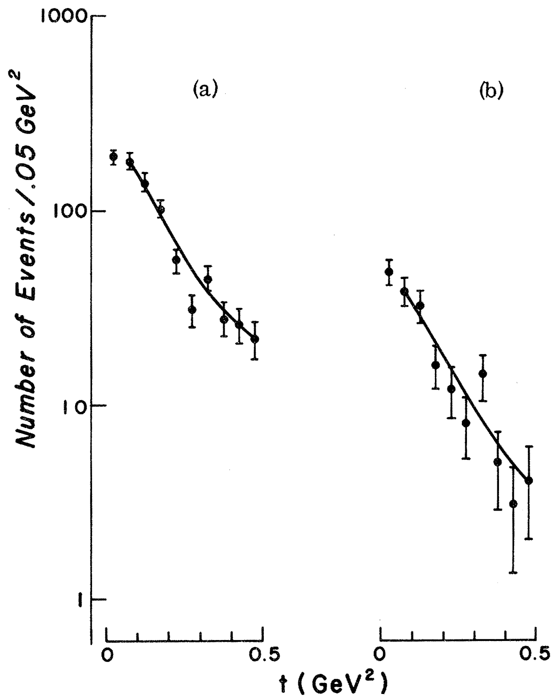

FIG. 4. Four-momentum–transfer to $N^*(1470)$, where $N^*(1470)$ is defined between 1.402 and 1.510 GeV/c^2, (a) using all events, (b) with a mass cut made on the other πN combination.

at 7.0 GeV/c. The enhancement has a mass of 1.446 ± 0.011 GeV/c^2 and a width of 0.198 ± 0.040 GeV/c^2. Furthermore, they reported that the effect is produced with a low momentum transfer from the target neutron as shown in Fig. 3(a). The absence of a similar effect from the incident proton which would require an $I = 1$ exchange suggests that $I = 0$ exchange is responsible for the enhancement.

We note that the exchange mechanisms available in that diagram are the same as those in our study [see Figs. 3(b) and 3(c)]. Assuming $I = 0$ exchange, the production cross sections of $N^*(1470)$ for the three reactions are related by the ratio

$$\sigma\left[\begin{array}{l} pn \to pN^{*0}(1470) \\ \qquad \hookrightarrow p\pi^- \end{array}\right] : \sigma\left[\begin{array}{l} pp \to pN^{*+}(1470) \\ \qquad \hookrightarrow n\pi^+ \end{array}\right] : \sigma\left[\begin{array}{l} pp \to pN^{*+}(1470) \\ \qquad \hookrightarrow p\pi^0 \end{array}\right] = 2:2:1.$$

The cross section for Reaction (7) calculated on the basis of Ref. 5 is 0.52 ± 0.07 mb and is inconsistent with our results of 0.29 ± 0.06 and 0.15 ± 0.03 mb for Reactions (5) and (6), respectively. Furthermore, we cannot reconcile the difference in the width of the enhancement in terms of physical arguments.

Next, we have examined the four-momentum–transfer distributions to the $N^{*+}(1470)$, which is defined by a band from 1.402 to 1.510 GeV/c^2. In Fig. 4(a) all events are used; samples from

Table II. Slope parameters for $N^{*+}(1470)$.

	b	c
Fig. 4(a)	-9.2 ± 1.5	6.7 ± 3.1
Fig. 4(b)	-7.7 ± 3.4	3.4 ± 7.0

Reactions (1) and (2) have been added after careful examination to ascertain that no statistically significant difference exists in their t distributions. Figure 4(b) shows the t distribution to a subsample of the events for which the mass of the pion with the unrelated nucleon is greater than 2.4 GeV/c^2. In both cases, the data are fitted by a phenomenological form of e^{bt+ct^2} for $0.05 \leq t \leq 0.5$ (GeV/c)^2. The lower limit is chosen to avoid a turnover of the t distribution due to the kinematical boundary. The resultant parameters are given in Table II. As may be seen, the slope parameter b is inconsistent with a value of ~20 as reported in counter and spark-chamber measurements. It has been pointed out in Ref. 5 that since the $N^{*+}(1470)$ and the proton have an identical set of quantum numbers, one may expect similarities between Reactions (5), (6), and the pp elastic scattering. Our slope parameter b agrees well with a value of 9.3 ± 0.6 for pp elastic scattering at 6.6 GeV/c.

In conclusion, we have observed an $I = \frac{1}{2}$ nucleon resonance with a mass of 1.462 ± 0.006 GeV/c^2 and a width of 0.049 ± 0.012 GeV/c^2 in the single-pion production channels of pp collisions at 6.6 GeV/c. The resonance, however, differs significantly from the $N^*(1400)$ reported in missing-mass–spectrometer experiments using counter and spark-chamber techniques. Our results are inconsistent with a broad $N^*(1470)$ effect as reported by Shapira et al.[5] in a $pd \to p_s pp\pi^-$ final state at 7 GeV/c.

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