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†Present address: Physics Department, Ohio Northern University, Ada, Ohio 45810.

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Axisymmetric Black Hole Has Only Two Degrees of Freedom

B. Carter
Institute of Theoretical Astronomy, University of Cambridge, Cambridge CB3 0EZ, England
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A theorem is described which establishes the claim that in a certain canonical sense the Kerr metrics represent "the" (rather than merely "some possible") exterior fields of black holes with the corresponding mass and angular-momentum values.

The purpose of this Letter is to give the statement (with a very brief outline of the proof) of a theorem which shows the crucial significance of the Kerr metrics[1] for the study of the ultimate state of a collapsing star, from the point of view of an external observer, in terms of a model based on classical (i.e., unquantized) matter fields obeying Einstein's equations of general relativity in an asymptotically flat space-time manifold. Penrose's theorem[2,3] shows that in such a problem one cannot expect that the manifold will be able to be extended so as to be complete in the strong sense. It is, however, reasonable to suppose that, at least in some cases, the manifold can be extended sufficiently for it to contain the Schmidt completion[4] of its domain of outer communications, i.e., of the set of points lying on timelike curves coming from and returning to asymptotically large distances. This is a mathematically precise formulation of what is usually meant by the statement that collapse takes place in such a way that "no naked singularities occur" or, equivalently, that "all singularities are hidden in black holes" (the "black holes" being regions of space-time beyond the domain of outer communications) since it automatically ensures that the domain of outer communications is nonsingular in the sense of Schmidt.[4] (The extent to which singularities really would be hidden has been much debated[5,6] and the balance of informed opinion seems inclined towards the idea that, at least in a wide class of cases—including those which do not differ too greatly from spherical symmetry—and conceivably in all cases, naked singularities will not occur.) When the singularities are hidden, it seems reasonable to suppose that the observable regions of space-time will (under physically realistic conditions) tend asymptotically with time towards a pseudostationary final state (i.e., one in which space-time is invariant under an isometry group generated by a Killing vector field which is timelike at least at sufficiently large asymptotic distances), since one would expect that nonstationary motions will in general be damped out (by gravitational radiation, viscosity, etc.), and since there is only a finite time during which the energy of such motions can be replenished from the interior parts of the star before they disappear through the horizon bounding the domain of outer communications. It also seems reasonable to suppose that such a final state will be a vacuum under most natural conditions, i.e., to suppose that in the long run all matter will have either fallen through the horizon or been ejected to asymptotically large distances. The theorem presented here shows that, subject provisionally to certain qualitative simplifying conditions (all having direct physical interpretations) of which many, perhaps all, may turn out to be redundant, such a final vacuum state must belong to the family of solutions found by Kerr.[1]

The likelihood that such a theorem existed became apparent[7] immediately after the publication of a somewhat analogous theorem by Israel.[8] Both Israel's theorem and the present theorem apply only to cases where the topological complications (e.g., the possibility that the black hole be toroidal as opposed to spheroidal) are explicitly ruled out. More precisely they apply to pseudostationary, asymptotically flat, vacuum (i.e., zero Ricci tensor) spaces which contain the Schmidt boundaries of their domains of outer communications, these domains having no subspaces with the same property, and being each topologically the product of the two-sphere and the two-plane, their Schmidt boundaries each consisting of a future and a past Killing horizon[9] (each topologically the product of the two-sphere and the real line), together with a (topologically spherical) bifurcation axis.[10] Let us refer to the domain of outer communications of such a space as a simple black-hole exterior solution, and let us use the symbols m and h to denote its asymptotically defined mass and angular momentum, respectively, in units where $c=G=1$. Then Israel's theorem may be expressed conveniently as follows: The class of simple black-hole exterior solutions which are static (rather than merely pseudostationary), and which satisfy the subsidiary condition that the gradient of the squared magnitude of the static Killing vector is nowhere zero, consists exclusively of the one-parameter family of spherical (i.e., Schwarzschild) solutions with m varying over the range $0<m<\infty$.

The new theorem which we wish to present here is as follows: The class of simple black-hole exterior solutions which are axisymmetric, and which satisfy the subsidiary condition that the causality axiom (i.e., no closed timelike or null curves) holds, consists of a discrete set of continuous families, each depending on at least one and at most two independent parameters. Among these, the two-parameter family of Kerr solutions with h and m in the range $-\infty<h<\infty$, $|h|^{1/2}<m<\infty$, is unique in admitting the possibility of zero angular momentum. Any other of these families (if they exist) must occur in congruent pairs, with h varying over the ranges $-\infty<h<0$ and $0<h<\infty$.

Since our result implies that Israel's subsidiary condition is redundant in the axisymmetric case, one is led to suspect that it may be redundant in general; analogously one may wonder whether the causality axiom (which holds automatically in the static case) is really necessary either, despite its essential role in the author's present proof. Moreover, just as Hartle and Thorne's perturbation analysis[11] (showing that within the class of simple black-hole exterior solutions, small continuous axisymmetric perturbations of the spherical solutions must agree with the Kerr solutions to first order in the dimensionless parameter h/m^2) foreshadows the present theorem (which shows their conclusion to be valid to all orders), so also the fact that their axisymmetry condition can now be seen (from Vishveshwara's more general perturbation analysis[12]) to be redundant suggests that the axisymmetry condition should be able to be eliminated from the present theorem as well. Thus we are further encouraged in conjecturing that the only simple black-hole exterior solutions are the Kerr solutions with $m>|h|^{1/2}$.

The proof of the theorem stated above consists of three main stages. In the first it is shown (using the results of Boyer,[10] Carter,[9] Morse and Heins,[13] and Papapetrou[14,15]) that any axisymmetric simple black-hole exterior solution can be covered in a canonically defined way by a standard Weyl-Papapetrou coordinate system with

$$ds^2=W(d\rho^2+dz^2)+Hd\varphi^2+2Q\,d\varphi dt-Pdt^2,$$

where the nonignorable coordinates are z with range $-\infty<z<\infty$, and $\rho\equiv(Q^2+PH)^{1/2}$ with range $0\le\rho<\infty$ ($|z|>b$) and $0<\rho<\infty$ ($|z|\le b$), where b is a positive parameter. (The missing coordinate range $|z|\le b$, $\rho=0$ corresponds to the Schmidt boundary.) By a method precisely analogous to that of Earnst[16] but with the role of H (which is strictly positive—except on the axis $\rho=0$, $|z|>b$ —by the causality condition) interchanged with that of F (which can become negative), the independent field equations reduce to

$$G_1=G_2=0,$$

where

$$G_1\equiv H\nabla\cdot(\rho\nabla H)-\rho|\nabla H|^2+\rho|\nabla\Psi|^2$$

and

$$G_2\equiv H\nabla\cdot(\rho\nabla\Psi)-2\rho\nabla H\cdot\nabla\Psi,$$

where Ψ is an auxiliary potential and ∇ is defined with respect to the flat metric $d\rho^2+dz^2$.

In the second (and longest) stage, the methods developed by Boyer and Lindquist[17] and by Carter[7,9] are used to determine the boundary conditions on H and Ψ as $\rho \to 0$ which are necessary and sufficient for the axis coordinate degeneracy to be removable and for there to exist a generalized Kruskal transformation to a new coordinate system extensible beyond the Schmidt boundary. Also the lowest order asymptotic boundary conditions are worked out. The former depend only on b, the latter only on h. (The Kerr solutions satisfy these conditions with $b^2 = m^2 - h^2/m^2$).

In the final stage of the proof it is shown that a smooth one-parameter variation of solution functions H and Ψ is uniquely determined by the path of variation of b and h. This is done by establishing that, subject to the boundary conditions, $\dot{H}$ and $\dot{\Psi}$ are uniquely determined as solutions of the linear system $\dot{G}_1 = \dot{G}_2 = 0$ by the values of $\dot{b}$ and $\dot{h}$, where a dot denotes differentiation with respect to the variation parameter. This step—the crux—depends on the existence of the key identity

$$\rho|\nabla(H^{-1}\dot{H}) + H^{-2}\dot{\Psi}\nabla\Psi|^2 + \rho|\nabla(H^{-1}\dot{\Psi}) - H^{-2}\dot{H}\nabla\Psi|^2 + \rho H^{-4}|\dot{H}\nabla\Psi - \dot{\Psi}\nabla H|^2$$
$$-H^{-3}(\dot{H}\dot{G}_1 + \dot{\Psi}\dot{G}_2) + H^{-4}(2\dot{H}^2 G_1 + \dot{H}\dot{\Psi}G_2 + \dot{\Psi}^2 G_1) \equiv \nabla[\rho H^{-1}\dot{H}\nabla(H^{-1}\dot{H}) + \rho H^{-1}\dot{\Psi}\nabla(H^{-1}\dot{\Psi})]$$

which enables us to obtain a standard kind of uniqueness proof by integrating over the space, using the fact that the boundary conditions turn out to be such as to eliminate the divergence contributions, while the field equations reduce the other side to a sum of positive terms. When $h = 0$, relatively trivial identities imply, in the same way, first that Ψ must vanish, and then that H is unique.

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Evidence for a Narrow $N^*(1470)$ †

Z. Ming Ma
Department of Physics, Michigan State University, East Lansing, Michigan 48823

and

Eugene Colton
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

(Received 8 September 1970)

A narrow $I = \frac{1}{2}$ enhancement at a mass of 1.462 ± 0.006 GeV/c^2 with a width of 0.049 ± 0.012 GeV/c^2 has been observed in $p\pi^0$ and $n\pi^+$ modes produced in pp collisions at 6.6 GeV/c. The effect, having a 6-standard-deviation significance, is narrower in width than has been observed before in missing-mass—spectrometer experiments.

Since its discovery in phase-shift analyses,[1] the $N^*(1470)$ state has been one of the most controversial, if not the most uncertain, resonances reported.[2] Early experiments reporting observations of its existence have been done using a one-armed proton spectrometer with counters and spark chambers.[3] Among these various experiments, general agreement was reached on