tribution known, the model is sensitive to the type of nucleon-nucleon interaction incorporated.

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[1]G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. 171, 1115 (1968).

[2]A. Swift and L. R. B. Elton, Phys. Rev. Lett. 17, 484 (1966).

[3]C. J. Batty and G. W. Greenlees, Nucl. Phys. A133, 673 (1969).

[4]H. S. Kohler, Nucl. Phys. A139, 353 (1969).

[5]J. W. Negele, Phys. Rev. C 1, 1260 (1970).

[6]A. M. Green, Phys. Lett. 24B, 384 (1967).

[7]A. Kallio and K. Kolltveit, Nucl. Phys. 53, 87 (1964).

[8]G. W. Greenlees, W. Makofske, and G. J. Pyle, Phys. Rev. C 1, 1145 (1970).

[9]B. W. Ridley and J. F. Turner, Nucl. Phys. 58, 497 (1964).

[10]R. M. Craig *et al.*, Nucl. Phys. 58, 513 (1964).

[11]M. P. Fricke, E. E. Gross, B. J. Morton, and A. Zucker, Phys. Rev. 156, 1207 (1964).

[12]E. E. Gross *et al.*, Nucl. Phys. A102, 673 (1967).

[13]B. C. Sinha and V. R. W. Edwards, Phys. Lett. 31B, 273 (1970).

[14]D. Slanina and H. McManus, Nucl. Phys. A116, 271 (1968).

Small-Angle j Dependence of (α, p) Reactions*

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We have observed the small-angle j dependence of (α,p) reactions for $l=1$ and $l=2$ reactions. The small-angle effect appears to persist for cases where the large-angle j dependence is small or absent.

Both distorted-wave Born-approximation (DWBA) calculations and rather general geometrical considerations predict a small-angle j dependence for (α, nucleon) reactions.[1] The results of Ref. 1 are that an appropriately weighted average of the $j=l+\frac{1}{2}$ and the $j=l-\frac{1}{2}$ angular distributions will give the angular distribution expected for no spin-orbit interactions and, further, they predict that near 0° the $j=l+\frac{1}{2}$ cross section will drop relative to that for the $j=l-\frac{1}{2}$. The magnitude of the effect will depend on spin-orbit strengths but the sign is unique. A previous small-angle study of (α, t) reactions indicated small effects,[2] presumably due to a small spin-orbit force in the triton optical potential.[3,4]

To test the small-angle prediction we studied two reactions proceeding to $l=1$ states where both the $j=\frac{3}{2}$ and $j=\frac{1}{2}$ states were observable. These were ^{12}C$(\alpha,p)^{15}$N and ^{62}Ni$(\alpha,p)^{65}$Cu. Further, we studied the reactions ^{26}Mg$(\alpha,p)^{29}$Al and ^{28}Si$(\alpha,p)^{31}$P proceeding to levels with $l=2$, $j=\frac{5}{2}$ and $\frac{3}{2}$. These reactions were observed at bombarding energies of 28 and 35.5 MeV. Protons were detected by conventional ΔE-E counter-telescope methods at angles greater than 15° while small-angle data were obtained with an absorber foil in front of a single semiconductor detector. Overlapping points obtained with both methods gave good agreement. The absorber

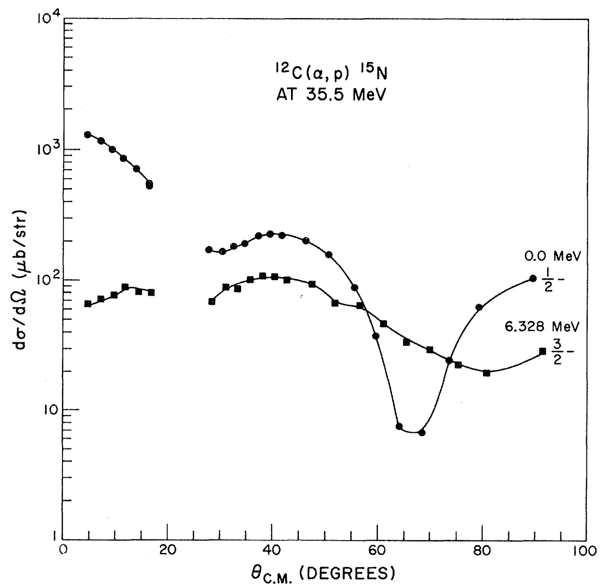

FIG. 1. Angular distributions for the $p_{1/2}$ (0.0-MeV) level and the $p_{3/2}$ (6.328-MeV) level of ^{15}N. The curves serve only to guide the eye. The upper j state shows filling in of minima and a significant drop (compared to the lower j state) at small angles.

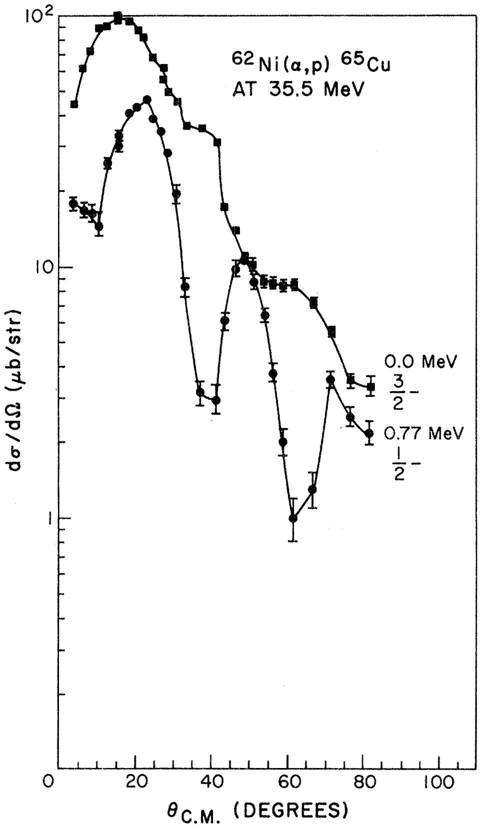

FIG. 2. Angular distributions for a pair of levels with $j=\frac{3}{2}$ and $j=\frac{1}{2}$ in ^{65}Cu. The curves serve only to guide the eye. The high-j state has little structure at large angles but a very pronounced drop at small angles.

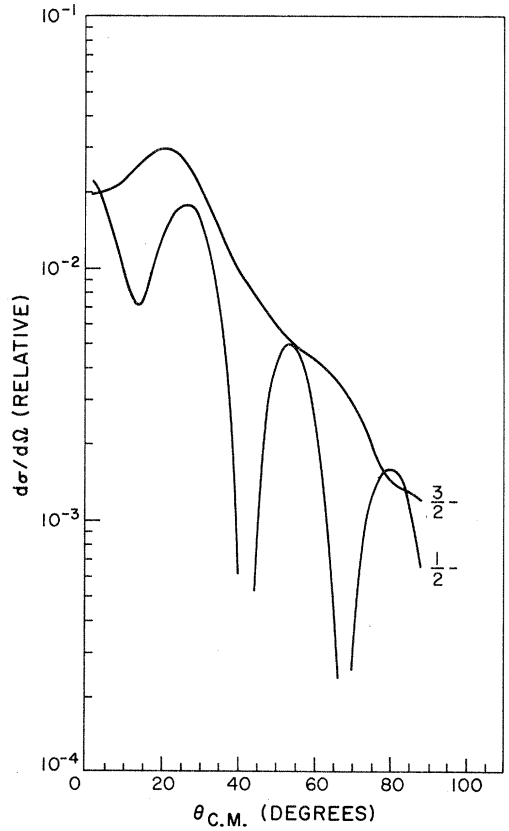

FIG. 3. DWBA calculations for ^{62}Ni$(\alpha,p)^{65}$Cu at 36 MeV. Proton parameters were obtained from Ref. 3. The α potential was of the 200-MeV well-depth family and the form factor was obtained by binding a triton cluster in a Woods-Saxon well.

allowed data to be obtained at lab angles of 2° although at such small angles a sizable background was observed which was caused by (α,p) reactions in the absorber foil initiated by Rutherford-scattered α particles from the target. Energy-loss straggling in the absorber limited the energy resolution at small angles to 300 keV full width at half-maximum. The four targets studied were chosen to allow unambiguous identification of the appropriate levels with this energy resolution.

The results at 35.5 MeV are shown for the $l=1$ cases in Figs. 1 and 2. The previously observed[5] large-angle j dependence, a "washing out" of minima for $j=l+\frac{1}{2}$, is clearly present. In both cases the cross section for the $j=\frac{3}{2}$ state falls at small angles while that for the $j=\frac{1}{2}$ state rises. This effect is particularly dramatic for the ^{62}Ni target since the large-angle $(\frac{3}{2})^-$ data are relatively smooth while an abrupt drop sets in below 20° (center-of-mass angle). This small-angle behavior is that predicted in Ref. 1 and appears in a DWBA calculation for ^{62}Ni$(\alpha,p)^{65}$Cu as shown in Fig. 3. This calculation utilized the code DWUCK[6] with proton parameters from Becchetti and Greenlees,[7] an α potential of the 200-MeV well-depth family, and a form factor obtained by binding a triton cluster in a Woods-Saxon well.

The results obtained at 35.5 MeV for $l=2$ cases are shown in Figs. 4 and 5. The large-angle j dependence for ^{26}Mg$(\alpha,p)^{29}$Al appears smaller than that for the $l=1$ cases previously mentioned. However, the small-angle result is still as predicted and quite dramatic.

The case of ^{28}Si$(\alpha,p)^{31}$P presents something of a puzzle. Previous studies[5] of large-angle j dependence for this reaction point out a strong energy dependence with j dependence vanishing at higher energies. Further, a study of the ^{30}Si$(\alpha,t)^{31}$P reaction at 42-MeV bombarding energy showed little large-angle j dependence but

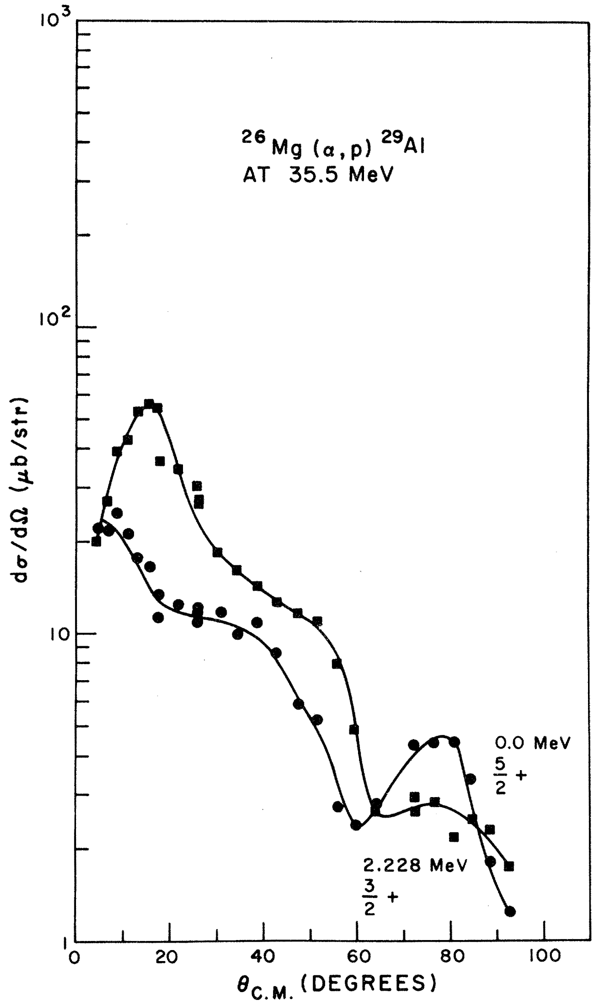

FIG. 4. Angular distributions for a $(\frac{5}{2})^+$, $(\frac{3}{2})^+$ pair of $l=2$ levels in ^{29}Al. The large-angle j dependence is less pronounced than in the $l=1$ cases but the small-angle effect persists.

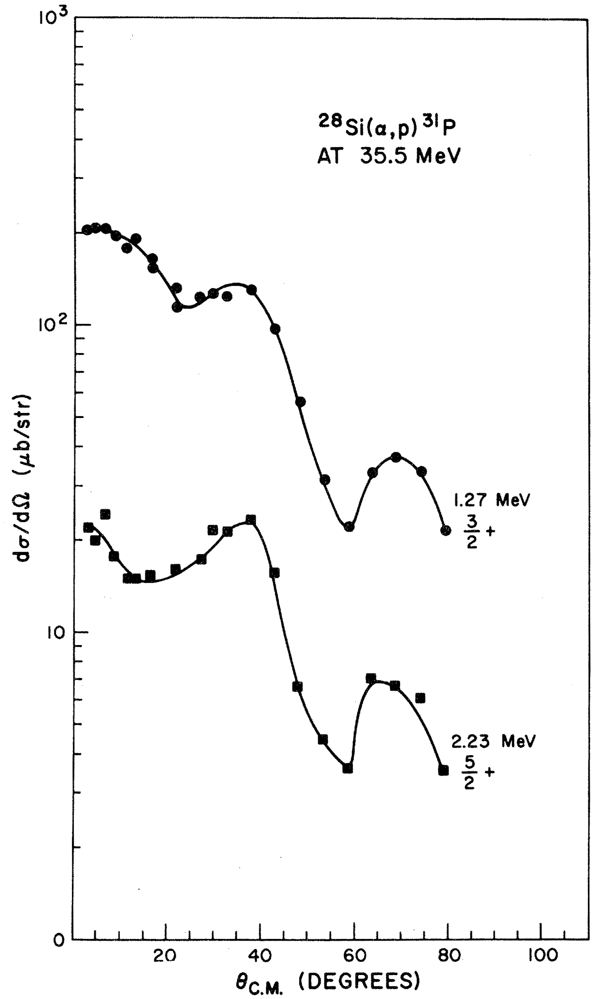

FIG. 5. Angular distributions for a $(\frac{5}{2})^+$, $(\frac{3}{2})^+$ pair of $l=2$ levels in ^{31}P. Large-angle j dependence is almost entirely absent (see Ref. 3) but at small angles the $(\frac{3}{2})^+$ cross section is 1.5 times that of the peak near 40°, while the $(\frac{5}{2})^+$ cross section is equal to the 40° value.

did show a small-angle effect as predicted in Ref. 1. The assignment of $(\frac{3}{2})^+$ and $(\frac{5}{2})^+$ to the ^{31}P levels at 1.27 and 2.23 MeV,[8] respectively, has been supported by recent particle-γ angular-correlation studies.[9] At 35.5 MeV large-angle j dependence is absent, as seen in Fig. 5. However, the small-angle effect, though weak, is still present. At small angles the $(\frac{3}{2})^+$ cross section is 1.5 times that of the 40° peak while that for the $(\frac{5}{2})^+$ level only equals the 40° value.

All of the above reactions were also studied at 28-MeV bombarding energy. Those results can be summarized by stating that the experimental data do not resemble DWBA calculations and do not show consistent small-angle j dependence of the sort predicted by Ref. 1.

In conclusion, it has been shown that small-angle j dependence of (α, p) reactions is a valuable spectroscopic tool at bombarding energies somewhat above 30 MeV. A magnetic spectrometer, rather than the absorber foil technique used in the present work, would allow the method to be applied to a wide variety of nuclear energy levels. Below 30 MeV it appears likely that compound-nucleus contributions prevent reliable application of the method. The precise range of bombarding energies which allow direct-reaction analysis has not been systematically defined in the present study and caution would be suggested in the application of direct-reaction theories in the low-energy region for (α, p) reactions.[10]

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[1]E. M. Henley and T. G. Kelley, Phys. Lett. 22, 495 (1966).

[2]P. F. Mizera, D. C. Shreve, and D. W. Storm, Phys. Lett. 25B, 400 (1967).

[3]R. L. Hutson, S. I. Hayakawa, M. Chabre, J. J. Kraushaar, B. W. Ridley, and E. T. Boschitz, Phys. Lett. 27B, 153 (1968).

[4]E. R. Flynn, D. D. Armstrong, J. G. Beery, and A. G. Blair, Phys. Rev. 182, 1113 (1969).

[5]See, for example, L. S. August, P. Shapiro, and L. R. Cooper, Phys. Rev. Lett. 23, 537 (1969).

[6]P. D. Kunz, private communication.

[7]F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).

[8]P. M. Endt and C. vander Leun, Nucl. Phys. A105, 1 (1967).

[9]C. E. Moss, R. V. Poore, N. R. Roberson, and D. R. Tilley, Nucl. Phys. A144, 577 (1970).

[10]See further R. W. West, Phys. Rev. 141, 1033 (1966).

Axisymmetric Black Hole Has Only Two Degrees of Freedom

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A theorem is described which establishes the claim that in a certain canonical sense the Kerr metrics represent "the" (rather than merely "some possible") exterior fields of black holes with the corresponding mass and angular-momentum values.

The purpose of this Letter is to give the statement (with a very brief outline of the proof) of a theorem which shows the crucial significance of the Kerr metrics[1] for the study of the ultimate state of a collapsing star, from the point of view of an external observer, in terms of a model based on classical (i.e., unquantized) matter fields obeying Einstein's equations of general relativity in an asymptotically flat space-time manifold. Penrose's theorem[2,3] shows that in such a problem one cannot expect that the manifold will be able to be extended so as to be complete in the strong sense. It is, however, reasonable to suppose that, at least in some cases, <u>the manifold can be extended sufficiently for it to contain the Schmidt completion[4] of its domain of outer communications,</u> i.e., of the set of points lying on timelike curves coming from and returning to asymptotically large distances. This is a mathematically precise formulation of what is usually meant by the statement that collapse takes place in such a way that "no naked singularities occur" or, equivalently, that "all singularities are hidden in black holes" (the "black holes" being regions of space-time beyond the domain of outer communications) since it automatically ensures that the domain of outer communications is nonsingular in the sense of Schmidt.[4] (The extent to which singularities really would be hidden has been much debated[5,6] and the balance of informed opinion seems inclined towards the idea that, at least in a wide class of cases—including those which do not differ too greatly from spherical symmetry—and conceivably in all cases, naked singularities will not occur.) When the singularities are hidden, it seems reasonable to suppose that the observable regions of space-time will (under physically realistic conditions) tend asymptotically with time towards a <u>pseudostationary</u> final state (i.e., one in which space-time is invariant under an isometry group generated by a Killing vector field which is timelike at least at sufficiently large asymptotic distances), since one would expect that nonstationary motions will in general be damped out (by gravitational radiation, viscosity, etc.), and since there is only a finite time during which the energy of such motions can be replenished from the interior parts of the star before they disappear through the horizon bounding the domain of outer communications. It also seems reasonable to suppose that such a final state will be a vacuum under most natural conditions, i.e., to suppose that in the long run all matter will have either fallen through the horizon or been ejected to asymptotically large distances. The theorem presented here shows that, subject provisionally to certain qualitative simplifying con-