Limiting Distribution and the Average $\pi^\pm$ Multiplicities in $\pi^- + p \to \pi^\pm$ + Anything Reactions*

Min-Shih Chen, Ling-lie Wang, and T. F. Wong
Brookhaven National Laboratory, Upton, New York, 11973
(Received 30 November 1970)

The experimental data for the single-pion distribution in the reaction $\pi^- + p \to \pi^\pm$ + anything at incident energies between 2.5 and 6 GeV and at 25 GeV show evidence for a limiting distribution and scaling behavior. The slopes of the logarithmic increase of the average $\pi^\pm$ multiplicities calculated from the 25-GeV data are found to be comparable with those obtained from accelerator pp reactions and cosmic-ray experiments. We make some comments on possible future experiments and on an apparent discrepancy between experiments at 10, 20, and 30 GeV and a limiting distribution.

The single-particle distribution in inclusive reactions of the type $A+B\to C$ + anything provides the simplest test of various high-energy models.[1-3] For a single outgoing hadron of mass μ and momentum $\vec{p}$, the invariant differential cross section,

$$f(s,p_\parallel,p_\perp) \equiv (p_\parallel^2 + p_\perp^2 + \mu^2)^{1/2} d^2\sigma/\pi dp_\parallel d(p_\perp^2), \qquad (1)$$

where s is the square of the total invariant mass, is in general a function of three variables. Different models give a common prediction that as $s\to\infty$, this function approaches a limiting distribution of only two variables. One of these two variables is the transverse momentum $p_\perp$.[4] The other one is either the longitudinal momentum $p_\parallel$ in the laboratory (projectile) frame[2] or the scaling variable[1] $x \equiv 2p_\parallel^*/s^{1/2}$, where $p_\parallel^*$ is the longitudinal momentum in the center-of-mass frame. At infinite incident energy, one can easily show that the variables $p_\parallel$ and x are equivalent.

For finite energies, it is generally believed that the limiting distribution in $p_\perp$ and $p_\parallel$ already gives a good description.[2] Recently, this has been further confirmed by the measurement of the backward $\pi^\pm$ distribution in $\pi^- p$ reactions with the incident π^- energy varying between 2.5 and 6 GeV.[5] However, we would like to point out that the limits in $p_\parallel$ and x are already equivalent at finite energies. In Fig. 1, we plot the single-$\pi^\pm$ invariant cross section of Ref. 5 as a function of s for various values of x. As can be seen, it becomes a function of the scaling variable x only and is quite independent of s for $s \geq 8$ (GeV)^2. It also seems that the x dependence gets weaker for smaller values of $|x|$.

Notice also that if the invariant cross section reaches a limit at quite low energies, the essential energy dependence of $d^2\sigma/\pi dx d(p_\perp^2) = f(x,p_\perp)/[x^2+4(p_\perp^2+\mu^2)/s]^{1/2}$ is contained in $[x^2+4(p_\perp^2+\mu^2)/s]^{-1/2}$. This energy dependence is most dramatic at $x\approx 0$. Unfortunately there are no data points at $|x| \leq 0.4$ in Ref. 5. Thus the crucial behavior

$$\frac{d^2\sigma}{dx d(p_\perp^2)} = \frac{\pi f(0,p_\perp)}{[x^2+4(p_\perp^2+\mu^2)/s]^{1/2}}, \quad x=0, \qquad (2)$$

cannot be tested. On the other hand, a measurement of the $\pi^\pm$ distribution near $x=0$ has recently been carried out at 25 GeV/c by Elbert, Erwin, and Walker.[6,7] For more reliable statistics,[7] the differential cross section $d\sigma/dp_\parallel^*$ is given integrated over $p_\perp$. Since it is generally recognized that the differential cross section is a rapidly decreasing function of $p_\perp$, we can approximate the scaling limit of Eq. (1) by[8]

$$\frac{d\sigma^\pm}{dx} = \int \frac{f^\pm(x,p_\perp)}{[x^2+4(p_\perp^2+\mu^2)/s]^{1/2}}\pi d(p_\perp^2) = \frac{1}{[x^2+4(\langle p_\perp^2\rangle^\pm+\mu^2)/s]^{1/2}}\int f^\pm(x,p_\perp)\pi d(p_\perp^2), \qquad (3)$$

where $\langle p_\perp^2\rangle^\pm$ are certain mean values of $p_\perp^2$, usually of the order of a few hundred MeV^2. If $f(x,p_\perp)$ is smooth enough near $x=0$, the measured $d\sigma^\pm/dx$ should be able to be fitted by the form $a/[x^2+4(\langle p_\perp^2\rangle+\mu^2)/s]^{1/2}$ near $x=0$. Since we have small-x data only at one energy, to magnify the effect we have fitted

$$d\sigma^\pm/dp_\parallel^* = a^\pm/(p_\parallel^{*2}+b^{\pm 2})^{1/2} \qquad (4)$$

for small values of $p_\parallel^*$, where $a^\pm$ and $b^\pm$ are constants.

In our fit,[9] the range of $p_\parallel^*$ chosen is such that the confidence level $P(\chi^2)$ is maximized. In Fig. 2 we present the result of such a fit. The largest value of $p_\parallel^*$ fitted is indicated by an arrow. For the

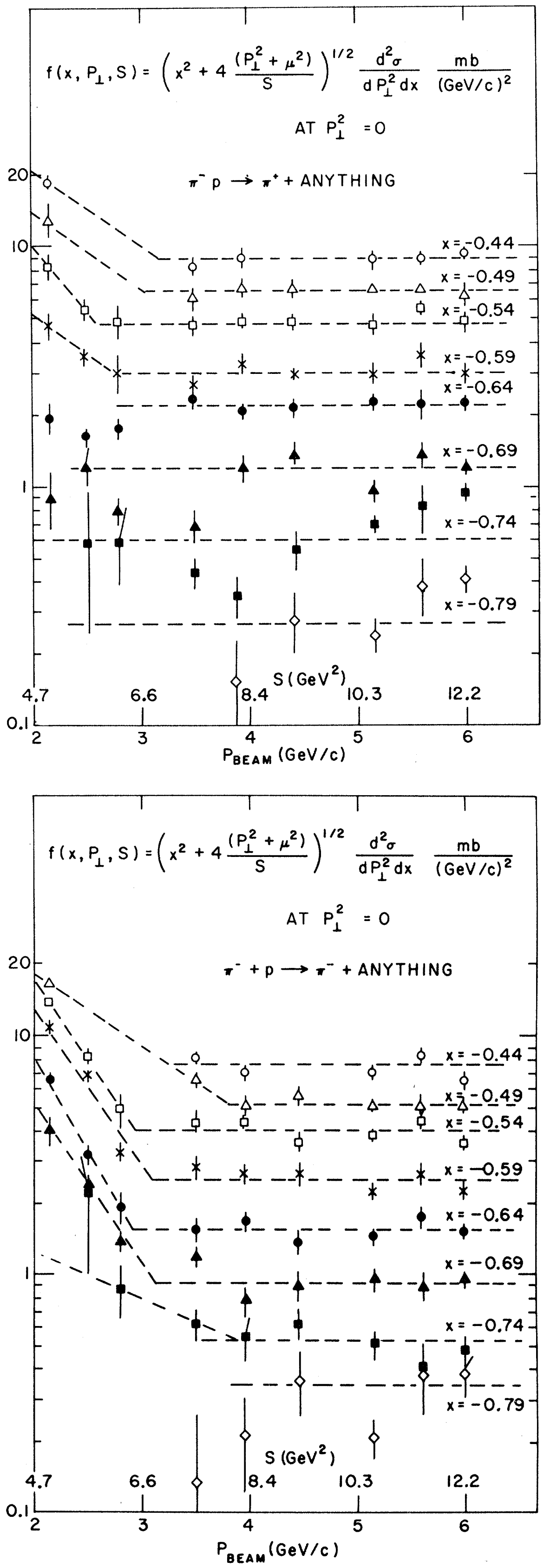

FIG. 1. The pion distribution of Ref. 5 plotted as a function of incident energy and x. The dashed lines are hand drawn through the points of equal x.

π^+ distribution, the data can be fitted by Eq. (4) up to $p_\parallel^* = 0.45$ GeV/c ($x = 0.13$) with $P(\chi^2) = 0.91$, while for π^-, the data are fitted up to $p_\parallel^* = -0.37$ GeV/c ($x = -0.11$) with $P(\chi^2) = 0.74$. Acutally the π^+ data can be fitted up to $p_\parallel^* = 1$ GeV ($x = 0.3$) with $P(\chi^2) \geq 0.07$. The parameters obtained from the fit are $a^+ = 8.48 \pm 0.87$, $b^+ = 0.21 \pm 0.02$, $a^- = 6.16 \pm 0.63$, and $b^- = 0.15 \pm 0.02$.[10] From this satisfactory fit, we can see that the behavior given by Eq. (2) is quite well satisfied. In Fig. 3, we present a plot of

$$F^\pm(x) = (x^2 + 4b^{\pm 2}/s)^{1/2}) d\sigma^\pm/dx, \tag{5}$$

which is consistent with $F^\pm(x)$ being constant in x for $0 \leq |x| \lesssim 0.1$.

As pointed out by Bali et al.[11] in an earlier Letter, if the single-$\pi^\pm$ distribution function $f^\pm(x, p_\perp)$ is already asymptotic and constant in x for $x \approx 0$, the $\pi^\pm$ multiplicities can be calculated as follows:

$$\bar{n}^\pm \equiv \frac{1}{\sigma_{\text{tot inel}}} \int \frac{d^2\sigma^\pm}{dp_\parallel^* d(p_\perp^2)} d(p_\perp^2) dp_\parallel^* \tag{6}$$

$$= \frac{2}{\sigma_{\text{tot inel}}} \int_0^1 \frac{F^\pm(x)dx}{(x^2 + 4b^{\pm 2}/s)^{1/2}},$$

$$\simeq \frac{1}{\sigma_{\text{tot inel}}} \left[a^\pm \ln s + \text{const} + O\left(\frac{1}{s}\right) \right]$$

$$\equiv c^\pm \ln(E/m) + d^\pm + O(1/E). \tag{7}$$

With $\sigma_{\text{tot inel}} = 20.8 \pm 0.2$ mb,[12] and with $a^\pm$ obtained before, we have

$$c^+ = 0.41 \pm 0.05, \quad c^- = 0.30 \pm 0.03.$$

For $d^\pm$, we can numerically integrate $d\sigma/dp_\parallel^*$ to obtain $\bar{n}^\pm$ according to Eq. (6) and then obtain $d^\pm \simeq \bar{n}^\pm - c^\pm \ln(E/m)$. The results are $d^+ = -0.54 \pm 0.36$ and $d^- = -0.33 \pm 0.23$. The large errors are results from the subtraction of two comparable numbers. In addition, there is also a contribution from the $(E/m)^{-1}$ term in Eq. (7). Thus the values of $d^\pm$ are not very reliable.

Comparing with the cosmic-ray results[11,13] $c^+ = c^- = 0.36$ and with the accelerator pp reaction results[11] $c^+ = 0.46 - 0.54$, $c^- = 0.24 - 0.37$, we find

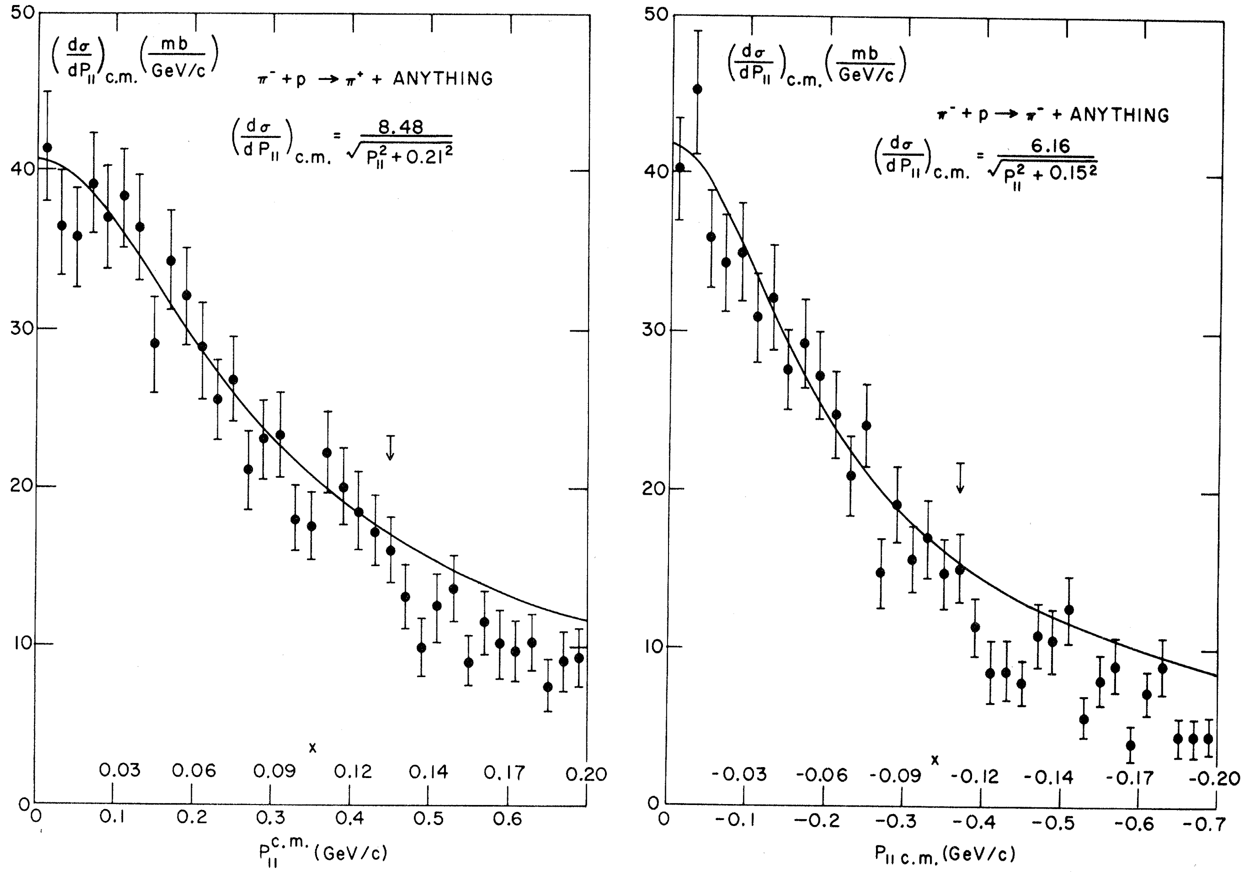

FIG. 2. Fit of the 25-GeV data of Refs. 6 and 7 with Eq. (4).

quite surprisingly that the $c^{\pm}$ of πp reactions are comparable with those of pp reactions. This is qualitative support of factorization as suggested by multi-Regge models.[3] An ultimate test of the factorization would be the comparison of the $\pi^{\pm}$ distribution in different reactions.[14]

Future experiments measuring the following quantities will give a more detailed test of the prediction of limiting distribution and factorization:

(1) Single-$\pi^{\pm}$ distribution from pp, Kp, γp, and $\bar{p}p$ reactions: If measured, we can first see if the prediction of a limiting distribution is true. If true, we can compare the limiting distributions and the multiplicity constants of all these different reactions to check factorization.

(2) Proton production distribution from $pp \to p$ + anything at higher energies: It was observed that the double differential cross section $d^2\sigma/dp_{\parallel}{}^*d(p_{\perp}{}^2)$ is independent of $p_{\parallel}{}^*$ and s, i.e.,

$$d^2\sigma/dp_{\parallel}{}^*d(p_{\perp}{}^2) \simeq g(p_{\perp}) \tag{8}$$

at incident proton energies of 10, 20, and 30 GeV.[15] It is very easy to see that if such behavior persists at high energies, it will apparently contradict the prediction of a limiting distribution. If Eq. (8) were always true, then we would have from Eq. (1) that

$$f(s,p_{\parallel}{}^*,p_{\perp}) \simeq (s^{1/2}/2\pi)[x^2+4(p_{\perp}{}^2+m_p{}^2)/s]^{1/2}g(p_{\perp}),$$

or, expressed in terms of the laboratory momenta $p_{\parallel}$ and $p_{\perp}$, for large s,

$$f(s,p_{\parallel},p_{\perp}) \simeq (s^{1/2}/\pi m_p)[(p_{\parallel}{}^2+p_{\perp}{}^2+m_p{}^2)^{1/2}-p_{\parallel}]g(p_{\perp}).$$

Thus the invariant cross section, as well as the average proton multiplicity, would increase linearly with $s^{1/2}$. Therefore, it is very interesting to see whether the proton distribution approximated by Eq. (8) persists at higher energies. If not, it is also interesting to see when and how it changes.

We would like to thank Dr. A. R. Erwin and

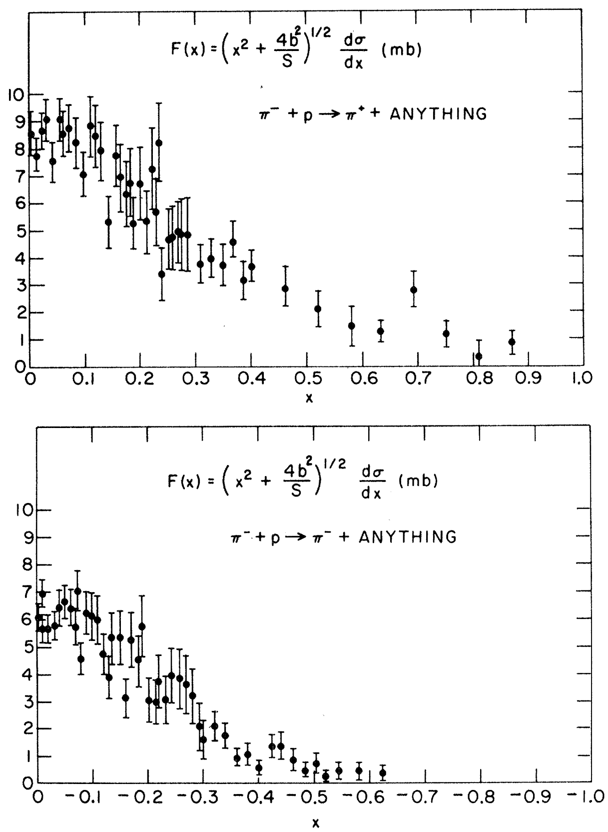

FIG. 3. Plot of $F^{\pm}(x)$, defined by Eq. (5), by using the 25-GeV data and the $b^{\pm}$ from our fit.

Mr. J. W. Elbert for invaluable discussions and providing us their data. We would also like to thank Dr. A. H. Mueller for helpful discussions.

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*Work performed under the auspices of U. S. Atomic Energy Commission.

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[3]D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962); K. G. Wilson, Acta Phys. Austr. 17, 37 (1963); L. Caneschi and A. Pignotti, Phys. Rev. Lett. 22, 1219 (1969); A. H. Mueller, to be published; N. F. Bali, A. Pignotti and D. Steele, to be published; D. Silverman and C. I. Tan, to be published; C. E. DeTar, Lawrence Radiation Laboratory, University of California Report No. UCRL-20056 (to be published); H. Cheng and T. T. Wu, Phys. Rev. Lett. 23, 1311 (1969).

[4]The longitudinal direction is always chosen to be the initial direction of the interaction in any Lorentz frame in which the two initial particles are aligned.

[5]R. W. Anthony, C. T. Coffin, E. S. Meanley, J. E. Rice, K. M. Terwilliger, and N. R. Stanton, to be published.

[6]J. W. Elbert, A. R. Erwin, and W. D. Walker, to be published.

[7]J. W. Elbert and A. R. Erwin, private communication.

[8]At finite energies, it is important to use the expression $[x^2+4(p^2+\mu^2)/s]^{-1/2}$; not $1/x$, as often used.

[9]In our analysis, we took the forward π^+ data of Ref. 6 to minimize the proton contamination and the backward π^- data to minimize the leading π^- contribution. However, we have also fitted the forward π^- distribution with no appreciable change of a^- and b^-.

[10]In general there is no reason that $a^+=a^-$, $b^+=b^-$. However from multi-Regge model $f^+(x,p_\perp)\approx f^-(x,p_\perp)$ for small values of x, therefore a^+ and b^+ should be equal to a^- and b^-, respectively. The fact that they are not equal from our analysis may be due to the theoretical approximation involved or may be due to the systematic errors in the experiments. Notice also the difference of $f^{\pm}(x,p_\perp)$ in Fig. 1. To answer this question needs further study.

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[12]J. W. Waters, W. D. Walker, A. R. Erwin, and J. W. Elbert, Nucl. Phys. B17, 445 (1970).

[13]L. W. Jones, in *Proceedings of the International Conference on Expectations for Particle Reactions at New Accelerators, Madison, Wisconsin, April 1970* (Physics Dept., Univ. of Wisconsin, Madison, Wis., 1970); K. N. Erickson, thesis, University of Michigan, 1970 (unpublished).

[14]Such a distribution for $pp\to\pi^-+$anything has been compiled from the existing data with an extrapolation in $p_\perp$; see S. D. Drell, in *Proceedings of the International Conference on Expectations for Particle Reactions at the New Accelerators, Madison, Wisconsin, April 1970* (Physics Dept., Univ. of Wisconsin, Madison, Wis., 1970). The shape is quite similar to ours in Fig. 3(b) for small values of $|x|$. See also Ref. 5.

[15]E. W. Anderson *et al.*, Phys. Rev. Lett. 19, 198 (1967); E. W. Anderson and G. B. Collins, Phys. Rev. Lett. 19, 201 (1967).