the energy distribution is at $E = T$. This is clearly not consistent with the results in Fig. 4 for Zr^{90}.

If the results in Fig. 4 are used in conjunction with Eq. (2) and the above stated temperatures to determine σ_i, we obtain the dot-dash curves in Fig. 4. We again see a clear inconsistency between the results for Zr^{90} and Sn^{112}. The σ_i curve for Zr^{90}, which should be reasonably reliable, varies more rapidly with energy than is predicted by optical-model calculations.[4]

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Gravitational Constant: Experimental Bound on Its Time Variation

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Analysis of radar-echo time delays, primarily between Earth and Mercury, yields an upper limit on the fractional variation of the gravitational constant of 4 parts in 10^{10} per year. Continuation of these radar measurements for five more years—even at the present level of accuracy—would allow the uncertainty to be reduced reliably to 3 parts in 10^{11} per year, a limit close to the effect predicted by some theorists.

Many conjectures[1] have been made during the past few decades concerning a possible variation with time of the gravitational "constant" G. The development of planetary radar systems and atomic clocks has made possible[2] the placement of a fairly stringent experimental limit on the magnitude of $\dot{G}$. One can compare gravitational time with atomic time by making, in effect, repeated measurements on an atomic time scale of the orbital period of a planet. We report here the results of such a comparison based primarily on the A.1 time kept by the U. S. Naval Observatory and on a series of atomic-time, interplanetary echo delay measurements made at the Haystack Observatory and the Millstone Hill radar of the Massachusetts Institute of Technology Lincoln Laboratory over the past six years.

We assume that the clocks used for the measurements kept track of atomic time without error since this source of uncertainty was far too small to be of significance for this experiment. The set of intercompared cesium-beam atomic standards used by the U. S. Naval Observatory to determin A.1 time have long-term errors of only about 1 part of 10^{12}, corresponding to less than 200 μsec error in epoch after 6 yr. The effect on the interpretation of a delay measurement between Earth and the fastest moving target, Mercury, is therefore always less than 0.04 μsec—far less than the delay measurement uncertainties themselves which were never less than 3 μsec. Similarly, the clock errors contributed insignificantly to the measurement of delay since the former, over the round-trip times, were always accurate to within 2 parts in 10^{12}, whereas the latter were never more accurate than 2 parts in 10^9. Thus, for both important functions—the determinations of epochs and of intervals—the atomic clocks can be considered errorless.

The somewhat ad hoc model that we used to analyze the data for a possible variation in G can be described briefly: Each planet was assumed to obey the usual equations of motion that follow from the Schwarzschild metric for the sun and from the Newtonian perturbations attributable to the moon and other planets,[3] except that the gravitational constant was replaced by $G_0 + \dot{G}_0(t-t_0)$, where the coordinate time t_0 is some (arbitrary) epoch at which G and $\dot{G}$ are evaluated. This formulation appears adequate to test most cosmological theories, especially since the time span of our data is relatively so short.

Units were selected as follows: The sun's mass M_s was defined to be unity; the A.1 second was used to define the unit of time; and the assumption $(G_0 M_s)^{1/2} \equiv 0.017\,202\,098\,95$ A.U.$^{3/2}$/day (Gauss's value) defined our unit of length—the astronomical unit – after the further stipulation that 1 day = 86 400 sec. For this set of units, conventional in astronomical calculations, one must solve for the speed of light c ($\simeq$173 A.U./day) from the data.

The radar data used in the analysis comprised echo time delay and corresponding, but relatively inconsequential, Doppler observations of Mercury (1966-1969) and Venus (1964-1969). The Mercury data are the more significant in virtue of its five times larger average orbital angular velocity with respect to Earth. Since all of these data are rather insensitive to the small mutual inclinations of the orbital planes, the parameters specifying the spatial orientation of the orbits at epoch – two for each planet – were fixed in accordance with a prior analysis that included U. S. Naval Observatory meridian-circle (optical) observations of the sun and inner planets.[4] The orbit of Mars was also taken from that analysis, whereas the orbits of the outer planets were obtained from a weighted least-squares solution based on U. S. Naval and Greenwich Observatory observations of these bodies from 1840 to 1969.[4] The moon's orbit was taken from a standard source,[5] as were the masses of Venus, Earth, Mars, and the outer planets.[6] The uncertainties in the values for these masses and orbits are far too small to affect significantly our conclusions on $\dot{G}$.

To estimate $\dot{G}$ and its uncertainty from the radar data, we used a computer program to determine the maximum-likelihood values and associated correlation matrix for the remaining 18 significant parameters: $(\dot{G}/G)_0$; c; the mass of Mercury; the Earth-moon mass ratio; the four in-plane osculating elliptic orbital elements at epoch for Earth, Mercury, and Venus; and the mean equatorial radii of the latter two. All of these parameters had to be estimated in this analysis because, with the possible exception of the Earth-moon mass ratio,[6] each is determined more accurately from the present set of data than from other sources.

A potentially important source of error in our theoretical model involves the above-implied assumption that the radius of each target planet is constant over the surface. Venus, at least in the equatorial regions spanned by the subradar point, is remarkably free from surface-height variations; nonetheless those that have been found[7] (up to a few kilometers in magnitude) do not have a serious effect on our results. No surface-height variations have yet been detected reliably on Mercury[7]; any variations present in the equatorial regions covered by the radar observations must be no more than about 1.5 km ($\simeq$10 μsec effect on two-way delay) which is near the present limit of accuracy on the Mercury time-delay measurements.

The use of the usual Schwarzschild metric to determine the gravitational effect on the sun, instead of, say, the corresponding metric for the Brans-Dicke theory, has no substantial effect on our estimate of $\dot{G}$, nor do possible small spatial variations of G within the solar system. A similar conclusion follows for our assumption of a zero solar gravitational quadrupole moment and a zero solar mass loss.

Our result shows no evidence for a time variation of the gravitational constant, the magnitude of the estimate of $\dot{G}/G$ being only a small fraction of the formal standard error, 1.6×10^{-10}/yr. To make a reasonable allowance for unknown but possibly important vitiating effects on our estimate, we take 4×10^{-10}/yr as a more reliable indicator of the actual uncertainty. We are not aware of any other experimental limit on $\dot{G}/G$ of comparable stringency.

The values obtained for the other 17 parameters are of lesser interest for the present paper and will not be discussed in detail. We merely remark that they are consistent with our previous work,[4] though with smaller standard errors. The estimates of these other parameters are, of course, correlated with the estimate of $\dot{G}/G$; all correlations are automatically accounted for in the calculation of the formal standard error.

The post-fit residuals from our 18-parameter solution are acceptably small, the goodness of fit

$$\epsilon \equiv \frac{1}{N-P}\sum_{i=1}^{N}\left(\frac{O_i-C_i}{\sigma_i}\right)^2$$

being 0.8. Here N defines the number of observations, P the number of parameters, O_i the experimental value for the ith observation, C_i the corresponding computed value, and σ_i the estimated standard error for the observation. No trends are apparent in the residuals; however, systematic analyses, including other statistical tests of the significance of the $\dot{G}/G$ estimate, have yet to be carried out.

As another test of the reliability of our bound on $\dot{G}/G$, we relaxed the restriction on the orientation of the orbital planes at epoch and added U. S. Naval Observatory meridian-circle observations of the sun, Mercury, and Venus (1956-1969) to our data set. The effect of errors in A.1 time, which was inaugurated in 1956, on the interpretation of these data is again negligible. In this second analysis we also introduced phase biases through second order of a Fourier-series expansion for the Mercury and Venus optical data, yielding a total of 28 parameters to be estimated. The result for $\dot{G}/G$, as expected, did not differ significantly from that from the first analysis, nor did the standard error of the estimate change appreciably. We omitted optical observations prior to 1956 since these were based on time as kept by the Earth's rotation which would itself be affected in a model-dependent manner by any time dependence of G.

How will future observations improve our bound on $\dot{G}/G$? To answer this question without resorting to extensive computer analysis, we developed a simple model that captures the essence of the dependence of this bound on the distribution of observations. We assumed measurements of time delay to be made between two point planets moving in coplanar, circular orbits. The main effect of a slowly changing gravitational constant was represented by assuming that each planet continues to move around the same circle but with the time dependence of its position L given by $L(t)=L_0+L_1t+\frac{1}{2}L_2t^2$, where L_2, the angular acceleration or rate of change of mean motion, represents the important effect on orbital position caused by $\dot{G}$.[8] (The change in the mean solar distance is not so sensitive a measure of $\dot{G}$ and hence is ignored in this simple model.) With the time-delay data used to estimate simultaneously the three parameters L_j ($j=0$-2) for the inner planet, assuming the outer one to be much further out, we obtain the formal standard error for $\dot{G}/G$:

$$\sigma\left(\frac{\dot{G}}{G}\right)\simeq\frac{3c\sigma_m\{10ha_i[1+(a_i/a_0)^2]\}^{1/2}}{(GM_s)^{1/2}T^{5/2}},$$
$$\simeq 3\times10^{-9}T^{-5/2}/\mathrm{yr},$$

where $\sigma_m=10$ μsec is the standard error for each time-delay measurement, $h\simeq 10$ d is the interval between measurements (assumed equal), $a_i\simeq 0.4$ A.U. is the orbital radius of the inner planet, $a_0\simeq 1$ A.U. is the radius of the outer, and T (in years) is the total time interval spanned by the set of measurements.[9] For $T=3$, the interval encompassing almost all of the Mercury observations, we find $\sigma(\dot{G}/G)\simeq 2\times10^{-10}$/yr. The very good agreement with the standard error obtained from the computer analysis of the actual data is due in part to compensating approximations. If radar time-delay data of the same accuracy were accumulated for 5 more years, the uncertainty in the determination of $\dot{G}/G$ would drop below 3 parts in 10^{11}/yr—nearly the value suggested by Dicke[10] for the fractional yearly rate of change of G. Mercury observations made at Cornell's Arecibo Observatory[11] and at the Jet Propulsion Laboratory's Goldstone site could contribute importantly to the improvement of the $\dot{G}/G$ determination.[12]

We also note that the radar technique is free from the possibly obscuring influences of complicated Earth-moon tidal interactions that affect the lunar laser-ranging experiment,[13] the only other that appears capable of discerning a useful bound on $\dot{G}/G$.

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[12]A further contribution will come from the Mariner Venus-Mercury Flyby Mission scheduled for 1973-1974, which will allow in addition a reduction by about three orders of magnitude in the uncertainty of the estimate of Mercury's mass—the parameter at present most highly correlated (0.5) with the estimate of $\dot{G}/G$. Even for the current radar data set, an independent knowledge of Mercury's mass would reduce the formal standard error in $\dot{G}/G$ by 25%.

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π^0 Photoproduction from Hydrogen with Linearly Polarized Photons*

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The asymmetry in the process $\gamma + p \rightarrow \pi^0 + p$ with polarized photons has been measured at 6 GeV for momentum transfers from $t = -0.4$ (GeV/c)^2 to $t = -1.1$ (GeV/c)^2, using coherent bremsstrahlung from a diamond crystal. A coincidence was made between the recoil proton in a 1.6-GeV/c spectrometer and one of the π^0 decay photons in a Lucite shower counter. The measured asymmetry $(\sigma_\perp - \sigma_\parallel)/(\sigma_\perp + \sigma_\parallel)$ is consistent with strongly dominant natural parity exchange in the t channel.

In a strict Regge-pole model, π^0 photoproduction at small t values and high energies should proceed by Reggized ω, ρ, and B exchange.[1] However, measurements[2,3] of the differential cross section show that if the commonly accepted trajectories for the ω and the ρ are used, cuts or absorption must be included to account for the data. The cross-section data alone cannot differentiate between a wide variety of models[4-7]; in particular, they cannot exclude B exchange. Measurements with linearly polarized photons allow the separation of[8] the natural- and unnatural-parity exchanges to leading orders of t/s. The asymmetry is defined as $A = (\sigma_\perp - \sigma_\parallel)/(\sigma_\perp + \sigma_\parallel)$, where $\sigma_\perp$ ($\sigma_\parallel$) is the cross section with photons polarized normal (parallel) to the reaction plane. Trajectories with a natural-parity sequence (ω, ρ) will contribute only to $\sigma_\perp$, whereas trajectories with unnatural parities (B) will contribute only to $\sigma_\parallel$. Absorption or cuts are expected to make contributions to both.

Previous to this experiment, asymmetry data[9] were available at 3 GeV. The data clearly demonstrated that π^0 photoproduction is dominated by natural-parity exchange even in the region of $t = -0.5$ (GeV/c)^2; however, they still allowed an appreciable amount of B exchange.[4] Furthermore, it was argued that at 3 GeV resonances might still be playing an important role. We report here preliminary results of an experiment at 6 GeV and values of the four-momentum transfer t between -0.4 (GeV/c)^2 and -1.1 (GeV/c)^2.

The layout of the experiment is shown in Fig. 1. A well-prepared electron beam with a phase space $(\Delta x \Delta\theta)^2 = (8 \times 10^{-6})^2$ (cm rad)^2 is focused onto a suitably oriented diamond 0.1 cm thick. After the radiator the electrons are deflected into a beam dump, and the photon beam, as defined by several collimators, is passed through a liquid hydrogen target and stopped in a secondary emission quantameter (SEQ) which was our primary beam monitor. The beam was also monitored by a Cherenkov cell placed just upstream of the target. The process was determined by a coincidence between the recoil proton detected in the 1.6-GeV/c spectrometer, and one of the γ's from the π^0 decay observed by one of two lead-Lucite shower counters.