Analysis of Negative-Pion Photoproduction Near the P_{33} Resonance: Test of the $\Delta I \leq 1$ Rule and Time-Reversal Invariance*

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Angular distributions for the reaction $\pi^-p\to\gamma n$ are analyzed. Existing sets of multipoles do not agree with the data. An acceptable fit is obtained with the Sanda-Shaw model of an electromagnetic current with an isotensor component of $-0.1>x>-0.4$. The model of Christ and Lee is used to estimate the magnitude of a possible T-invariance violation implied by the difference between the data at $\tilde{E}=1245$ MeV and the inverse reaction deduced from γd work. A strong violation in the isovector or isotensor amplitude is excluded, but a maximum violation in the isoscalar amplitude is possible.

In this Letter we present a quantitative analysis of the differential cross section for $\pi^-p\to\gamma n$ for c.m. total energies of 1245, 1337, and 1363 MeV reported in the preceding Letter.[1] The analysis is divided into three parts of increasingly exotic character.

(A) First we attempt to fit the $\pi^-p\to\gamma n$ data with conventional photoproduction multipoles derived under the assumption of the $\Delta I \leq 1$ rule and time-reversal invariance, but we do not consider the data on $\gamma n\to\pi^-p$ deduced from γd work. We are unable to obtain an acceptable fit of our data.

(B) Next, we abandon the $\Delta I\leq 1$ rule. We still assume T invariance and ignore the γd data. We can fit the $\pi^-p\to\gamma n$ data with the Sanda-Shaw[2] model in which the isospin decomposition of the multipoles includes an isotensor term.

(C) Finally, we can fit both the $\pi^-p\to\gamma n$ data and the $\gamma n\to\pi^-p$ cross sections deduced from γd work by introducing a T-invariance violating phase, in addition to the previously found isotensor amplitude.

The conventional multipole amplitudes are obtained from calculations based on fixed momentum-transfer dispersion relations derived by Chew, Goldberger, Low, and Nambu,[3] or based on an isobar model. Among the several calculations using the first approach, we have selected the one by Berends, Donnachie, and Weaver,[4] hereafter labeled BDW, as a starting point for our analysis, because it is fully relativistic and parameter free. Except for the $M_{1-}^{(1/2)}$ multipole, which leads to the inelastic $P_{11}(1460)$ resonance, various published multipole calculations[5] agree on the gross features of the multipoles. The BDW multipoles reasonably describe the many published π^+ and π^0 photoproduction experiments[4] on protons, which include absolute differential cross sections, asymmetry ratios, and recoil-nucleon polarization measurements. The BDW predictions for $\gamma n\to\pi^-p$, which are presumed to be valid to about 10%, are compared with our experimental data at $\tilde{E}=1245$ MeV in Fig. 1(a), and the disagreement is striking. Included in Fig. 1(a) is a datum point at 28° from Ref. 6. Our data at $\tilde{E}=1337$ MeV and $\tilde{E}=1363$ MeV have been averaged to obtain data at $\tilde{E}=1350$, for which extensive $\gamma n\to\pi^-p$ data based on γd work exist. Our results at $\tilde{E}=1350$ MeV are shown in Fig. 1(b), and disagree strongly with the BDW multipole predictions. Berends and Donnachie[7] have suggested a modified version of the BDW multipoles in which the $M_{1-}^{(1/2)}$ is much lower. This leaves the fits of the π^+ and π^0 photoproduction unaffected because of a chance cancelation in the isospin amplitudes. We have included in Fig. 1 the predictions based on the modified multipoles, and it is clear that they do not fit our data either. Next, we have replaced the isospin-$\frac{3}{2}$ multipoles of the modified BDW set by the larger values of Noelle, Pfeil, and Schwela,[10] without obtaining a fit to our data. Finally, we have included in Fig. 1 the predictions by Schwela[8] and by Schmidt.[9] They disagree with our data also. All published theoretical predictions overestimate the experimental π^- photoproduction cross section. We are not aware of an obvious way to adjust the published multipoles and still fit all the pion production data. Of course, one cannot rule out the possibility that a set of conventional multipoles exists that would do this.

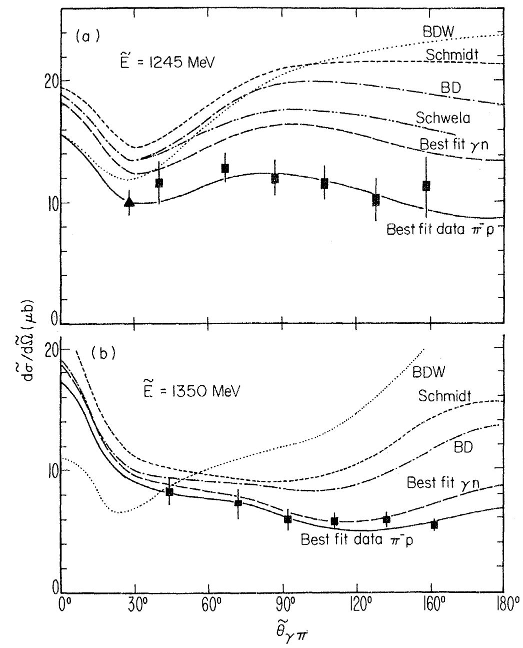

FIG. 1. Comparison of experimental cross sections for $\gamma n \to \pi^- p$ obtained via detailed balance from $\pi^- p \to \gamma n$ (Refs. 1 and 6) with theoretical predictions. Rectangular data points are from Ref. 1 and the triangle point is from Ref. 6. Theoretical predictions are by Berends *et al.*, labeled BDW, Ref. 4; Berends and Donnachie, labeled BD, Ref. 7; Schwela, Ref. 8; and Schmidt, Ref. 9. Also shown are our best fit to $\pi^- p \to \gamma n$ with the parameters of fits 1 and 4, and our best fit to $\gamma n \to \pi^- p$ with the parameters of fits 2 and 5 of Table I.

Sanda and Shaw[2] and others have argued that there is no experimental justification for the validity of the $\Delta I \leq 1$ rule for the electromagnetic current. They have introduced an isotensor component in such a way that the good agreement between the BDW predictions and the proton data is retained but the predictions for γn are lowered. With the Sanda-Shaw model, in which T invariance is assumed, we can obtain a good fit (indicated by the solid line in Fig. 1) to our data for $x = -0.25$, where x is the relative isotensor amplitude defined by Sanda and Shaw. In our fit, we used the BDW multipoles, except for $E_{0+}^{(1/2)}$ and M_{1-} which we allowed to vary. Our value for M_{1-} is consistent with that of Nishikawa et al.[11] The results of our fitting are exhibited in Table I in fits 1-6. The precise value for x and the error involved depend somewhat on the choice of the smaller multipoles. We obtained acceptable fits for x between -0.1 and -0.4. Thus, our $\pi^- p \to \gamma n$ data, analyzed under the hypothesis of T invariance, supports the conclusions of Sanda and Shaw—who analyzed $\sigma_t(\gamma n)$ data deduced from γd experiments—regarding the presence of an isotensor component of the electromagnetic current.

Finally, we complete our analysis by also considering the $\gamma n \to \pi^- p$ data deduced from γd work. As mentioned in the preceding Letter, at $\tilde{E} = 1337$ MeV there is agreement between our data and the inverse, but there is some 30% disagreement at

Table I. Parameters of the best fits to the $\pi^- p \to n\gamma$, $\gamma n \to \pi^- p$, and $\gamma n \rightleftarrows \pi^- p$ data points. Fits 1–6, no T-invariance violation; fits 7–14, T invariance not valid. Reaction ① is $\pi^- p \to \gamma n$, ② is $\gamma n \to \pi^- p$, and ① + ② is $\gamma n \rightleftarrows \pi^- p$. $E_{0+}^{(1/2)}$ and $M_{1-}^{(1/2)}$ are in units $10^{-3}/M_\pi^2$.

Fit No.	Data Pts.	Reaction	$\tilde{E}$ (MeV)	$E_{0+}^{(1/2)}$	$M_{1-}^{(1/2)}$	x	$\pm\phi(M_{1+})$	χ^2 ①	χ^2 ②	$[\chi^2$ ① + χ^2 ②$]$
1	7	①	1245	5.2±3.0	1.5±1.0	−0.25±0.15		2		19
2	24	②	1245	7.5±3.0	0.5±1.0	−0.10±0.10			17	
3	31	① + ②	1245	6.35	1.0	−0.175		11	42	53
4	6	①	1350	3.2±1.3	1.8±0.7	−0.25±0.35		3		23
5	19	②	1350	4.5±1.3	1.1±0.7	−0.10±0.15			20	
6	25	① + ②	1350	3.95	1.45	−0.175		5	27	32
7	31	① + ②	1245	6.35	1.0	−0.175	$\phi(M^{(1/2)})=20°\pm10°$	3	21	24
8	31	① + ②	1245	6.35	1.0	−0.175	$\phi(M^S) = 75°\pm30°$	3	21	24
9	31	① + ②	1245	6.35	1.0	−0.175	$\phi(M^V) = 27°\pm12°$	3	21	24
10	31	① + ②	1245	6.35	1.0	−0.175	$\phi(M^{(3/2)})=10°\pm5°$	11	29	40
11	31	① + ②	1245	6.35	1.0	−0.175	$\phi(M^X)=40°\pm20°$	11	29	40
12	31	① + ②	1245	6.35	1.0	−0.175	$\phi(M^W) = 8.5°\pm4°$	11	29	40
13	25	① + ②	1350	3.95	1.45	−0.175	$\phi(M^{(1/2)})=20°(\pm20°)$	6	18	24
14	25	① + ②	1350	3.95	1.45	−0.175	$\phi(M^{(3/2)})=13°(\pm15°)$	5	19	24

$\tilde{E}=1245$ MeV. We can make a numerical estimate of the possible degree of time-reversal nonivariance that could be involved by taking the cross section for both processes at face value, although the unavoidable uncertainties inherent in extracting $\gamma n \to \pi^- p$ cross sections from γd work preclude making a definite evaluation of T-invariance violation. The existing γd measurements are of two varieties. In one type, a deuterium-filled bubble chamber is exposed to a bremsstrahlung beam and events of the type $\gamma + d \to p + p + \pi^-$ are analyzed with the spectator model. The two published experiments[12, 13] are in agreement. Another type is based on a measurement of the π^-/π^+ ratio and uses counters. Since the $\pi^- pp$ and $\pi^+ nn$ systems in the final state are charge symmetric, the π^-/π^+ ratio is insensitive to possible strong final-state effects. Sufficient π^-/π^+ measurements exist for a reliable interpolation at $\tilde{\theta}_\gamma = 38°$, 90°, and 158° at $\tilde{E}=1245$ MeV. For the data, we used Sands et al. and Neugebauer et al.,[14] Betourné et al. and Fisher et al.,[15] and Beale, Ecklund, and Walker.[16] The resulting $\gamma n \to \pi^- p$ cross sections agree very well with the bubble chamber measurements, except perhaps at 38°. There are several corrections to both types of experiment, which are discussed elsewhere.[12, 17-19] We note that the corrections are thought to be less important for the π^-/π^+ method. The observed agreement of the cross section enhances the confidence in both methods of extracting γn data from γd experiments. The individual data points of the γn experiments are shown in the preceding paper, and we have indicated with a dashed line in Fig. 1 our best fit to the γn data. This fit will be detailed below. For the evaluation of a violation of time-reversal invariance, we use an extension of the model by Christ and Lee[20] in which a phase φ that violates time-reversal invariance is added to the T-conserving phases of the conventional multipoles. The inequality parameter $a(\tilde{E}, \tilde{\theta})$, defined as

$$a(\tilde{E}, \tilde{\theta}) = \frac{d\sigma(\gamma n \to \pi^- p) - d\sigma(\pi^- p \to n\gamma)}{d\sigma(\gamma n \to \pi^- p) + d\sigma(\pi^- p \to n\gamma)},$$

is well suited to determine φ in a particular multipole, since it does not depend critically on the magnitude of the other multipoles. Shown in Fig. 2 is the inequality parameter $a(\tilde{E}, \tilde{\theta})$ as determined by our data and the average of the γn data. Following Christ and Lee, we limit our considerations to the electric and magnetic dipoles. From inspection of Fig. 2(a), we conclude that the E_{0+} multipole does not need to be considered, which leaves only the M_{1+} multipole. Any multipole can be decomposed into an $I=\frac{1}{2}$ and $I=\frac{3}{2}$ component, which in turn can be decomposed as follows: $M^{(1/2)} = M^S - \frac{1}{3}M^V$, where S is the isoscalar and V is the isovector amplitude; while $M^{(3/2)} = M^W + (\frac{3}{5})^{1/2} M^X$, where W is the isovector and X is the possible isotensor amplitude. Christ and Lee have considered the case where φ is added to $M^{(1/2)}$ or $M^{(3/2)}$. We have also considered the case that φ is added to M^S, M^V, M^W or M^X. The numerical results are given in Table I, fits 7-14. In determining the values for φ, we proceeded as follows. We fitted the $\pi^- p \to n\gamma$ and $\gamma n \to \pi^- p$ data points separately, using the BDW values for the multipoles except for the isotensor amplitude x and for $E_{0+}{}^{(1/2)}$ and $M_{1-}{}^{(1/2)}$, which were left as parameters determined by the data. We calculated the average of the two fits and then introduced the time-reversal violating phase φ for the final fit to all the data points. From our fits, given in Table I, we draw the following conclusion regarding time-reversal invariance: Despite the substantial difference in differential cross section at $\tilde{E}=1245$ MeV, the T-invariance violating phase φ of the isovector amplitude appears to be small. When φ is in the isotensor amplitude it is also not maximum. The only case in which φ is close to 90° is when it is associated with the isoscalar amplitude.

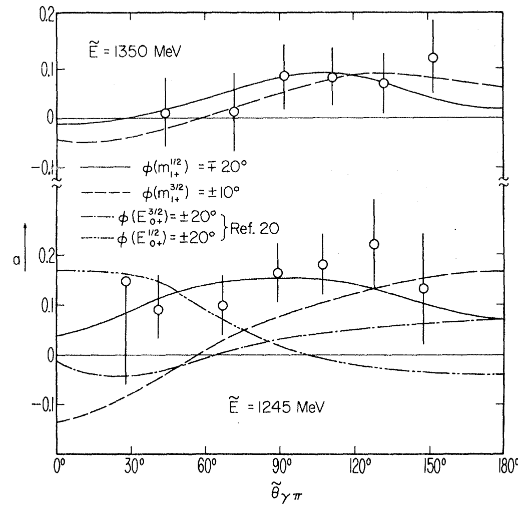

FIG. 2. Comparison of the inequality parameter

$$a = \left[\frac{d\sigma}{d\Omega}(\gamma n \to \pi^- p) - \frac{d\sigma}{d\Omega}(\pi^- p \to n\gamma)\right] \times \left[\frac{d\sigma}{d\Omega}(\gamma n \to \pi^- p) + \frac{d\sigma}{d\Omega}(\pi^- p \to n\gamma)\right]^{-1},$$

calculated for various T-invariance violating phases φ, with experimental data.

We note that a violation of time-reversal invariance in the isoscalar amplitude invalidates the model of Sanda and Shaw regarding the existence of an isotensor because it affects σ_t (see Fig. 2).

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Possible Explanation for the Rapid Approach to "Universality" of the Inelastic Electron-Scattering Structure Functions*

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We have determined the single-variable analyticity of the inelastic electron-scattering structure functions in the complex x plane, with s kept fixed and real, to all orders of Feynman perturbation theory. We find that its Landau singularities, which move as a function of s, rapidly approach their asymptotic s-independent position once s is large. We discuss how this observation offers a possible explanation for a rapid approach to "universality" of the inelastic electron-scattering structure functions and shows that $sW_2(s,q^2/s)$ should "scale" faster than $\nu W_2(s,x)$.

Recent experimental data[1] on inelastic electron scattering indicate that for fixed x the structure functions W_1 and νW_2 become approximately independent[2] of s once s is far above the resonance production region ($s \geqslant 4$ GeV^2). We will call this region the "deep inelastic region." This fact has been referred to as "scaling" of the structure functions.[3,4] We consider two forms of "scaling." The first is the s independence of the magnitude of the structure functions for fixed $x = q^2/(2P\cdot q)$, which we call "universality of magnitude." The second is the s independence of the shape of the curve of the structure functions versus x, which we call the "universality of shape."

We propose that a rapid approach to a universal shape for the νW_2 (or W_1) curve for $s \gg 4$ GeV^2 can be understood as a consequence of the rapid approach of its physical x-sheet singularities to their s-independent asymptotic positions once s is large enough. This is so provided the "strengths" of these