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Upper Cutoff in the Spectrum of Solar Particles

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Arguments are advanced in favor of the existence of an upper cutoff in the spectrum of solar protons. In the case of the event of 28 January 1967, on the basis of the recordings made by the worldwide network of neutron monitors, this cutoff is found to be 4.3 ±0.5 GeV.

The purpose of this Letter is to show that an upper cutoff exists in the spectrum of solar protons. By upper cutoff is meant an energy level beyond which there are no accelerated particles. The extensive work done in the last fifteen years on solar proton events has seldom mentioned the existence of this cutoff notwithstanding the fact that this quantity is a useful parameter for studying the mechanisms of particle acceleration.

The method resorted to consists in using the worldwide network of neutron monitors as an energy spectrometer while adding the presence of a maximum energy in the proton spectrum. Mountain stations were ignored. The percentage increases were related to a pressure of 760 mm Hg by a double correction of the barometric effect. The method of Palmeira, Bukata, and Gronstal[1] was applied with the following attenuation lengths: $\lambda g = 140$ g/cm^2 for galactic particles; $\lambda f = 103$ g/cm^2 for solar particles.[2] With this correction made, the percentage increase F_2 for a neutron monitor may be formulated as follows[1]:

$$F_2 = \frac{A_1}{N_g}\int_{P_c}^{\infty}\left(\frac{dj}{dP}\right)_f S(P)dP, \tag{1}$$

where A_1 is a constant, N_g the counting rate due to galactic cosmic rays with a standard neutron monitor located in a place of magnetic rigidity P_c, P the magnetic rigidity of the protons, $(dj/dP)_f$ the differential spectrum of the solar protons, and $S(P)$ the proton specific-yield function. Here we use for N_g the values obtained by Carmichael et al.[3] during their latitude survey in North America in 1965, and for P_c the values calculated by Shea et al.[4] Use is made of Lockwood and Webber's specific-yield function.[5] In order to simplify numerical calculations, this function is represented by power laws in different rigidity bands. Lastly, when considering the existence of a cutoff P_m in the differential spectrum under the power law, we may write

$$F_2 = (A_2/N_g)\int_{P_c}^{P_m} P^{-\mu} S(P)dP. \tag{2}$$

By using for each time interval the percentage increases at the various neutron monitor stations, and by applying the method of least squares, it is possible to determine A_2, μ, and P_m.

The proton differential spectrum can be written as a function of energy in the form of a power law with an upper cutoff E_m, whence

$$F_2=(A_3/N_g)\int_{E_c}^{E_m}E^{-\gamma}S(E)dE, \qquad (3)$$

where E_c corresponds to P_c for protons and $S(E)$ is the proton specific-yield function expressed as a function of the energy. Proceeding as before, it is possible to determine A_3, γ, and E_m.

The events of 28 January 1967 and 30 March 1969 were analyzed by this method with hourly counting rates of the worldwide network of neutron monitors and the two presumed forms of the differential spectrum [Eqs. (2) and (3)]. Figure 1 shows the values obtained for γ, E_m, μ, and P_m as a function of time for the event of 28 January 1967. An examination of these curves leads to the following observations:

(1) Allowing for statistical errors, E_m and P_m remain substantially constant for a lengthy part of the event and do not follow its profile. The mean value found for P_m is approximately 5.3 ± 0.7 GV. Allowing for the energy-rigidity relationship in the case of the protons, this agrees well with $E_m=4.5\pm0.5$ GeV.

(2) The solid curves [Figs. 1(b) and 1(d)] show the pattern of variation of γ and μ with time and agree well with experimental results. These curves were plotted for an energy band of 0.4 to 3 GeV and a rigidity band of 1 to 4 GV on the basis of the Krimigis model[6] for diffusion in interplanetary space. It is assumed for the purpose that the particle source spectra are, respectively, of the form $E^{-2.8}dE$ and $P^{-3.9}dP$, that propagation of the particles is velocity dependent,[7] and that the diffusion coefficient takes the form $D=(0.08/3)vr$,[6,8] where v is the velocity of the particles in A.U. per hour and r is the distance from the sun in A.U. The time of ejection of the particles in the case of this event was assumed to be 0730 UT.[9-11]

The differential spectrum can be studied in detail. In Fig. 2 the percentage increase in the counts of the neutron monitors at 1200 UT is plotted against P_c, and the figure shows the curves predicted for various forms of the differential spectrum. It is found that best agreement with the experimental points is obtained for $\exp(-P/0.6)$[10] and $P^{-4.1}$ with an upper limit of 5.3 GV, which in this domain merges with the E^{-3} law accounting for the $E_m=4.3$-GeV limit. The $P^{-5.0}$ law[1,10,12] differs markedly from experimental observations.

Figure 3 shows the integral spectrum determined at 1600 UT by satellite and balloon[11] plot-

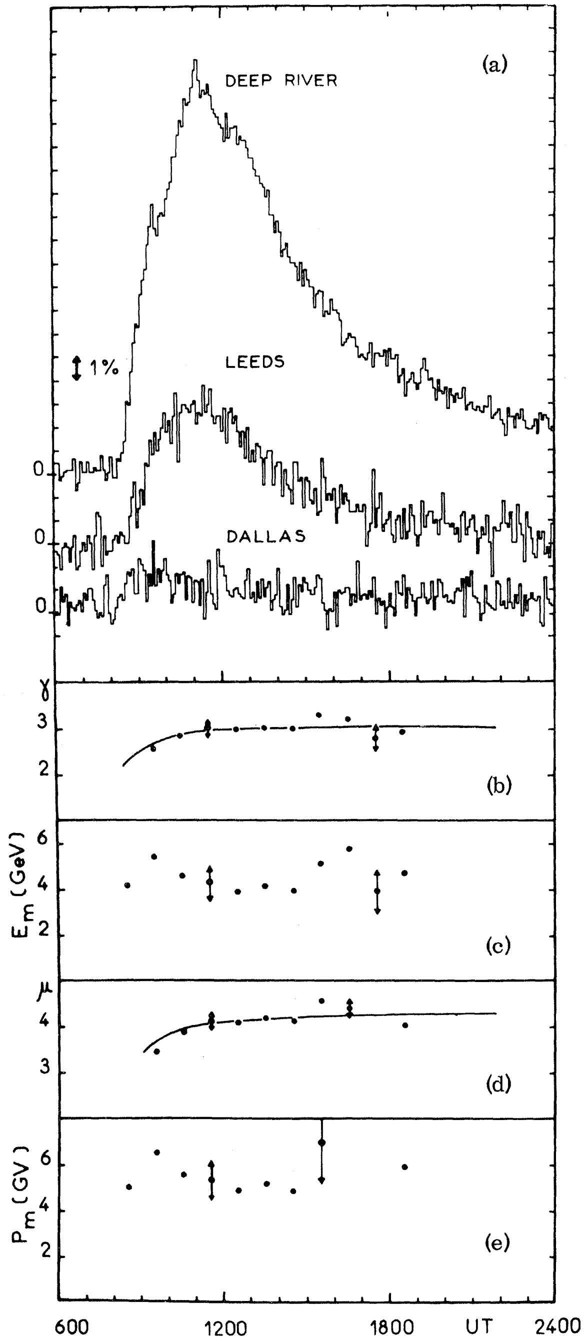

FIG. 1. Time behavior and spectral parameters of the solar proton event of 28 January 1967. (a) Recordings by three typical stations. (b) Exponent of the differential energy spectrum. The solid curve is theoretical. (c) Upper cutoff in the differential energy spectrum. (d), (e) Same as (b) and (c) but for rigidity instead of energy.

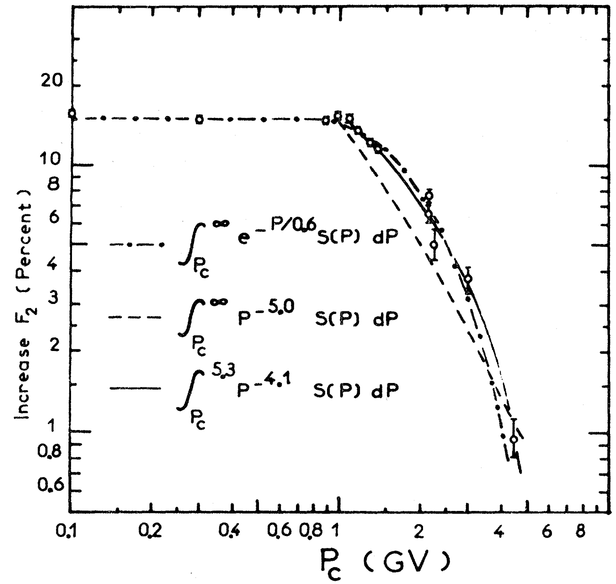

FIG. 2. Percentage increase F_2 in the sea-level neutron monitors at 1200 UT on 28 January 1967, compared with different theoretical curves.

ted against energy, and shows the different spectrum forms noted. It will be seen that outside the neutron monitor domain, the exponential law does not agree at all with the measurements made. It should be noted that, beyond a few hundred MeV, the rigidity and energy power laws with upper limits can be merged.

It can be seen therefore that it is the energy power law with an E_m that best represents the proton spectrum throughout the energy domain explored. The agreement between the experimental and theoretical γ and μ and between the spectrum determined and the measurements in the low-energy domain demonstrates the validity of the calculated parameters. Moreover the timewise constancy of E_m and P_m, as well as their equivalency, are arguments in favor of the existence of an upper cutoff in the spectrum.

The errors given for the above discussed parameters are the statistical ones. Additional errors may be introduced by the uncertainties in the specific-yield function (SYF). The influence of this latter parameter on the results has been studied by using Webber's SYF.[13] We obtained

$$E_m = 4.4 \pm 0.5 \text{ GeV}, \quad \gamma = 3.2 \pm 0.3 \text{ at 1200 UT}.$$

We conclude that with both forms of SYF the results remain almost similar.

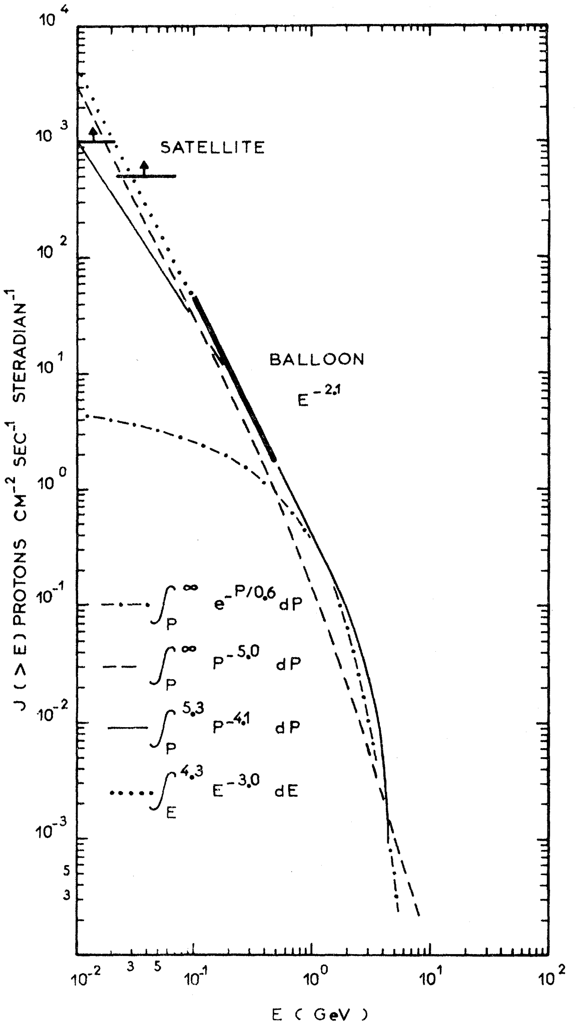

FIG. 3. Integral spectrum of the event of 28 January 1967.

An analysis of the event of 30 March 1969 gave $E_m = 3.7 \pm 0.9$ GeV, but because this event was of a small amplitude the errors affecting the parameters are greater than in the case of the event of 28 January 1967. It is to be noted that the usually steep integral spectrum of the solar particles remains unaffected in the lower energy bands by the presence of this cutoff. Direct measurements of the latter from balloons or satellites are therefore tricky to make. The tendency towards growth of the proton SYF of the neutron monitor favors the share of the high energies in the counting rate, relatively speaking. This explains why the influence of this cutoff can be seen over a substantial part of the additional counting rate curve of the neutron monitors as a function of P_c. However, the cutoff may not be quite as sharp as has been assumed, but the method used prevents a more detailed spectrum form from being obtained. It should be noted moreover that this method is applicable only for that part of an event where isotropy is assured, as was the case for the event of January 1967 from 0900 UT onwards.[10] It is worth noting that Frêon, Berry, and Folques,[14] by extrapolating the balloon recorded spectrum[15] and using the percentage counts of the neutron monitors, find maximum energy for the event of 7 July 1966. While this method may furnish a means of making measurements, it does not bring out the existence of this cutoff.

It is important to ascertain whether this cutoff is a consequence of the particles' propagation or acceleration. Since galactic cosmic rays of a few GeV are diffused by the interplanetary magnetic field, the same must be true of solar particles. The authors therefore feel that this cutoff exists in the particles' acceleration spectrum and that it is one of its important parameters.

In models proposed for the acceleration of solar particles, the existence of a limit energy is sometimes taken into account. Friedman and Hamberger,[16] in developing Petschek's "wave model,"[17] predict a maximum energy of the order of 10^{11} to 10^{12} eV, which is two to three orders of magnitude greater than that found in this paper.

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