tic planes in the smectic A phase orient normal to a surface.

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Superfluidity in Quantum Crystals

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The speculations of Chester, Andreev and Lifshitz, and Leggett concerning the possibility of Bose-Einstein condensation and/or superfluidity in quantum crystals are examined. The existing body of experimental data on these systems places extremely strong constraints on the experimental realization of these speculations.

Recently Chester,[1] Andreev and Lifshitz,[2] and Leggett[3] have speculated about the possibility of the occurence of Bose-Einstein condensation and/ or superfluidity in solids. Each of these speculations has contained the suggestion that the most likely candidates for the observation of this superfluidity is the quantum crystal, solid ^{4}He. The purpose of this paper is (1) to describe what we believe to be the essential physical content of each of these speculations and (2) to scruntinize them in light of the large body of data on quantum crystals.

It is useful at the outset to introduce a model Hamiltonian which gives an adequate description of the important particle motions in quantum crystals and which provides a concrete framework on which to base our discussion. We take the particles in solid ^{4}He to be described by

$$\mathcal{H}_{PM}=\sum_R \epsilon_0 b_R{}^\dagger b_R+\sum_{RR'} t(RR')b_R{}^\dagger b_{R'}+\varphi_0\sum_R b_R{}^\dagger b_R{}^\dagger b_R b_R, \qquad (1)$$

where $b_R{}^\dagger$ creates a ^{4}He atom in the ground state at lattice site R.[4,5] The energies ϵ_0, $t(RR')$, and φ_0 are ϵ_0, the energy of a particle in the ground state at lattice site R; $t(RR')$, the energy associated with tunneling from lattice site R to lattice site R' (approximately the off-diagonal kinetic energy); and φ_0, the hard-core energy associated with double occupation of a lattice site.[6] The particles may move through the solid because of $t(RR')$, the tunneling term, but they are hindered in their motion by the strong repulsive interaction, the hard-core term. For the purposes of this model the numbers ϵ_0, t, and φ_0 can be taken to be about 20, 0.1, and 10 K, respectively.

The particle motions that are necessary to have a superfluid component in the solid are difficult to achieve because of the hard-core repulsion between pairs of particles. We may partially avoid this difficulty by taking the $T=0$ ground state of the solid to contain a finite number of vacancies.

Case I.—At $T=0$ the ground state of the solid is N particles described by $\mathcal{H}_{PM}$ on $N+n_v$ lattice sites. Here n_v is the number of vacancies in the ground state. These vacancies are called ground-state or "frozen-in" vacancies. The presence of n_v vacancies in the ground state does not mean that there are n_v vacant lattice sites. It means that there is a single-particle density in the vicinity of each lattice site which is less than 1, i.e., $N/(N+n_v)$. In his discussion of Bose-Einstein condensation in solids, Chester suggests that the usual proofs that this condensation cannot occur in solids are invalidated by the presence of "frozen-in" vacancies in the $T=0$ ground state.[1] The vacancies of Andreev and Lifshitz, which have a condensate and lead to the superfluid properties they discuss, are vacancies present in the $T=0$ ground state.[7] Thus we take case I to represent the physical content of the speculations of Chester and of Andreev and Lifshitz.

Table I. Vacancy-wave parameters.

V (cm^3/mole)	Phase	Bandwidth (K)	t (K)	Excitation temperature (K)
		^{3}He		
24.7	bcc	~10	0.6	5-6
22.0	bcc	9	0.5	9
20.0	bcc	6	0.4	14.5
19.0	hcp	?	?	>20
		^{4}He		
21.0	bcc	~8	0.5	~5-6
21.0	hcp	?	?	~15

Because of the tunneling term in Eq. (1), the vacancies in solid ^{4}He exist as vacancy waves.[8] The motion of a vacancy wave through the solids yields motions of the single-particle density that could lead to superfluidity. The thermally activated vacancy waves in solid ^{4}He have the dispersion relation

$$\hbar\omega(k) = \varphi_0 - 2t(\cos k_x\Delta + \cos k_y\Delta + \cos k_z\Delta), \quad (2)$$

where Δ is the near-neighbor distance and we have used simple cubic geometry. The vacancy-wave bandwidth $2zt$ and the vacancy-wave excitation temperature $\varphi_0 - 6t$ are known from NMR experiments[9] (see Table I). If we assume that the ground-state vacancies have the same dynamics as the thermally activated vacancies, then the ground-state vacancy waves have the dispersion relation[10]

$$\hbar\omega(k) = -2t(\cos k_x\Delta + \cos k_y\Delta + \cos k_z\Delta). \quad (3)$$

The system of n_v ground-state vacancies will undergo a transition to the superfluid state at temperatures such that the thermal de Broglie wavelength is comparable with the interparticle spacing, $\lambda_T^3 = [\hbar/(2m^*k_BT_c)^{1/2}]^3 \approx V/n_v \approx \Delta^3 N/n_v$, where m^* is the effective mass deduced from Eq. (2). For $n_v \approx N$ we have $T_c \ll zt/10 \approx 1$ K, using $t \approx 1$ K from Table I.

How many ground-state vacancies are present in solid ^{4}He? There are two tests in the NMR data on ^{3}He and ^{3}He-^{4}He mixtures that bear on this point.

(1) There is direct evidence about the presence of vacancy waves in solid ^{4}He in the experiments of Richardson and co-workers.[11] These experiments see the motion of vacancy waves in ^{4}He through their effect on small concentrations of ^{3}He which serve as probe particles. Richardson and co-workers find that (a) the vacancy-wave bandwidth is about the same in ^{4}He as in ^{3}He, (b) the thermally activated vacancies in bcc ^{4}He have an activation energy of about 6 K, and (c) the thermally activated vacancies in hcp ^{4}He have an activation energy of about 15 K. The data of Richardson and co-workers at $V = 20.0$ cm^3/mole and $T = 1.5$ K can be understood qualitatively and quantitatively by the presence of a concentration of 10^{-4} of thermally activated vacancies.[12] Thus we conclude that the number of ground-state vacancies in ^{4}He is certainly less than $10^{-4}N$. For this limit, $n_v = 10^{-4}N$, the transition to the superfluid would occur at $T \ll 50$ mK.

(2) If we argue that the energetics of vacancy formation are the same in ^{3}He as in ^{4}He, then if there are ground-state vacancies in ^{4}He they are also very likely to be present in ^{3}He and vice versa. Are there ground-state vacancies in ^{3}He? The NMR data of Giffard and Hatton[13] at $V = 20.0$ cm^3/mole and $T \approx 0.4$ K can be understood by the presence in the system of a concentration of 10^{-14} of thermally activated vacancies. In the absence of an argument for why the ground-state vacancy waves obey a different dynamics and have a different effect on an NMR experiment than the thermally activated vacancy waves, we must take this result to mean $n_v(^3\mathrm{He}) < 10^{-14}N$. This is equivalent to $n_v(^3\mathrm{He}) \equiv 0$. Further, in the absence of an argument for why the energetics for vacancy formation in solid ^{3}He should be greatly different from that in solid ^{4}He, we can transfer this result to solid ^{4}He and conclude that $n_v(^4\mathrm{He}) \equiv 0$.[14]

We believe that there are no ground-state vacancies in quantum crystals.

Case II.—At $T = 0$ the ground state of the solid has N particles described by $\mathcal{H}_{PM}$ on N lattice sites. For this case the possible motions available to the particles in the solid are highly constrained. The primary mechanism of particle motion is the cooperative tunneling process which permits particles that are near neighbors of one another to change place. When the particles are tagged by spins this process is described by a pseudospin Hamiltonian and leads to the Heisenberg antiferromagnet description of solid ^{3}He.[5,9] Let us extract from $\mathcal{H}_{PM}$ the part which describes the cooperative tunneling process. We write

$$\mathcal{H}_{PM} \to \mathcal{H}_T = \sum_R \epsilon_0 b_R{}^\dagger b_R + \sum_{RR'} \frac{t(RR')^2}{-\varphi_0} n_R n_{R'}, \quad (4)$$

where $n_R = b_R{}^\dagger b_R$. The second term in this Hamiltonian couples the single-particle density at lattice site R to the single-particle density at lattice site R'. In solid ^{3}He the coupling constant $t(RR')^2/\varphi_0 = M(RR')$ is the exchange interaction J. The quantity $t(RR')^2/\varphi_0$ for solid ^{4}He can be determined from experiments on ^{3}He and the use of reliable scaling procedures.[4,15] The single-particle density described by Eq. (4) has a set of normal modes that are found by examining $G(RR') = i\langle T b_R(t) b_{R'}{}^\dagger(t')\rangle$. In the Hartree approximation the normal modes of the single-particle density are (for a cubic crystal)

$$\hbar\omega(k) = -M(k) = -(t^2/\varphi_0)2(\cos k_x\Delta + \cos k_y\Delta + \cos k_z\Delta). \quad (5)$$

We take case II to represent the physical content of the speculation of Leggett. The coherent diffusion process[16] called for by Leggett is the coupled motion of the single-particle density described by the modes in Eq. (5). Bose-Einstein condensation in this solid will occur at temperatures given by $\lambda_T{}^3 \approx \Delta^3$. Using the effective mass m^* from Eq. (5) in the calculation of λ_T leads to the condition $T \ll zt^2/10\varphi_0$. The superfluid properties of the solid will be seen at temperatures about an order of magnitude below the bandwidth $2zt^2/\varphi_0$. For ^{3}He at 20 cm^3/mole the exchange interaction J has strength 20 μK.[9] The highest density ^{4}He solid is at about this molar volume so that a very optimistic estimate of t^2/φ_0 for solid ^{4}He is 20 μK. Thus we estimate that the solid must be carried to temperatures well below 0.02 mK$\approx zJ/10$ for the superfluid transition to occur. Leggett suggests that the superfluid fraction of the total density, ρ_s/ρ, is on the order of $(t^2/\varphi_0)(\hbar^2/m\Delta^2)^{-1}$. This ratio is reasonably reliably calculated using the theory of tunneling due to Guyer and Zane.[4,15] For hcp ^{4}He at $V \approx 20$ cm^3/mole we have $t^2/\varphi_0 \approx 3$ μK and $\rho_s/\rho \approx 1\times10^{-6}$. The superfluid density is very likely about two orders of magnitude smaller than the estimate of Leggett.

There are two possible mechanisms by which the particles in solid ^{4}He can acquire the mobility that is necessary to permit the kinds of motions that would lead to Bose-Einstein condensation and superfluid phenomena. These mechanisms are (1) motion of the single-particle density due to the presence of "frozen-in" or ground-state vacancies, and (2) motion of the single-particle density due to the cooperative tunneling of pairs of particles. We have argued that the NMR data on solid ^{3}He and solid ^{3}He-^{4}He mixtures provide strong evidence for there being no ground-state vacancies in solid ^{4}He. Thus Bose-Einstein condensation due to the presence of ground-state vacancies is ruled out. Bose-Einstein condensation due to cooperative tunneling would occur at temperatures well below 0.1 mK. If it does occur, the density of the superfluid component will be on the order of 10^{-6} of the bulk density. It seems extremely unlikely that the present generation of low-temperature laboratories are capable of observing this superfluidity.

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[14]The energetics of vacancy formation is well understood theoretically; Ref. 8. There are two conclusions that the theory permits: (1) There are no ground-state vacancies; (2) vacancy formation in solid ^{4}He is more difficult than in solid ^{3}He.

[15]In Ref. 4 it is shown that the essential volume dependence of the cooperative tunneling process is in the factor $\exp[-\frac{1}{2}\alpha^2(\Delta^2+\sigma^2)]$, where σ is the hard-core radius and α^2 is related to the Debye temperature by $\hbar^2\alpha^2/m \approx k_B\Theta_D$; then, $\alpha_4{}^2/\alpha_3{}^2 = (\frac{4}{3})^{1/2}$ [H. H. Sample and C. A. Swenson, Phys. Rev. 158, 188 (1967)].

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Vortex Motion in the Presence of Thermodynamic Fluctuations*

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We have observed an exponential temperature dependence of the quantity $d\rho_f/dH$, where ρ_f is the flux-flow resistivity, in thin-film superconductors in the reduced temperature interval $0.7 \lesssim T/T_c \lesssim 1.0$. The results can be explained qualitatively if it is assumed that the dominant contribution to the flux-flow viscosity is power dissipation resulting from the presence of thermodynamic fluctuations.

The study of current-induced vortex motion in superconductors is of particular interest because it allows the examination of time-dependent effects and dissipative processes in the superconducting state. A comprehensive review of the subject has been given by Kim and Stephen.[1] In the presence of a transport-current density J of sufficient magnitude, steady-state vortex motion of velocity v obtains, where v is determined by equating the net force on a vortex, $f=f_L-f_p$ (f_L being the Lorentz force $J\varphi_0/c$ and f_p being the pinning force), to a viscous drag force ηv:

$$\eta v = J\varphi_0/c - f_p, \tag{1}$$

where $\varphi_0 = hc/2e$ is the flux quantum. This leads to a finite resistivity which, in the limit $f_L \gg f_p$, is given (for the usual thin plate geometry, in which case $n\varphi_0 = H$, n being the vortex density and H the applied perpendicular field) by

$$\rho \to \rho_f = \varphi_0 H/\eta c^2. \tag{2}$$

Since the power dissipated through viscous flow is given by $D=\eta v^2$, Eq. (2) may be cast into the form

$$\rho_f = DH/J^2\varphi_0. \tag{3}$$

The essential physics lies in the understanding of the source of dissipation D. In the usual situation the major contribution to D comes from Joule heating in the vortex cores (actually half of D comes from dissipation in the core and half from dissipation arising from the return currents outside the vortex core).[2] The local model of Bardeen and Stephen,[2] which treats this problem, has been successful in explaining the empirical result, $\rho_f/\rho_n = H/H_{c2}(0)$ for $T\sim 0$, of Kim, Hempstead, and Strnad.[3] While the experimental and theoretical situations are less satisfactory for finite temperatures, it is believed that the basic dissipative mechanism is the same as that for $T\sim 0$. We report here flux-flow results in a regime where the major contribution to the dissipation appears to arise not from the vortex cores, but from the interaction of the current fields of the moving vortices with thermodynamic fluctuations.

The experiments were performed on granular Al films, prepared by the method discussed by Masker, Marčelja, and Parks.[4] Such films are particularly attractive for superconducting studies since the grain size (and hence the scale of structural disorder) is small compared with the coherence length $\xi(T)$.[5,6] Thus, such films, from the standpoint of superconductivity, are particularly homogeneous. This is manifested in the present studies in the extremely small depinning-current densities observed (smaller than in highly annealed bulk specimens).[7] In addition,