Photon Mass and the Galactic Magnetic Field

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If the photon has a finite rest mass m, a filament of magnetic flux sustained by a partially ionized gas decays exponentially at a rate proportional to m^2. Arguing from assumptions regarding the galactic magnetic field that are plausible though not yet rigorously established, we find an upper limit $m \leqslant 3\times10^{-56}$ gm, corresponding to a Compton wavelength of 6 lt yr.

It is known that our galaxy (and presumably it is not unique) possesses magnetic fields largely directed along the spiral arms and supported by currents flowing in the partially ionized interstellar medium. If the medium is perfectly conducting and there are no outside influences, the magnetic field is permanently frozen in. If the medium has a finite conductivity, the usual theory of plasmas shows that the field decreases at a rate determined by the dimensions of the supporting plasma and by its conductivity. In this note we wish to show that if the photon were to have a finite mass the situation would be different: The field now diminishes at a rate determined by the plasma dimensions and the photon's Compton wavelength, the smaller number predominating. Numerical estimates based on the persistence of the field then enable us to set a new upper limit on the photon's mass.

We imagine a galactic arm straightened out so as to form a long filament of plasma with a magnetic field running down it. To simplify the analysis we suppose that the plasma is electrically neutral and that there is no electric field along the filament, so that the currents flow perpendicular to its length. We assume that the spatial distribution of the galactic plasma does not change significantly during the million years considered here; thus we need not deal with the hydrodynamic equations for the plasma.

We first calculate the electric field in a partially ionized plasma[1,2] carrying a current density

$$\vec{j} = n_e e(\vec{v}_i - \vec{v}_e),$$

where the v's are mean velocities and the densities of electrons and ions are assumed equal. Ignoring inertial forces, we equate the electrodynamic forces on the electrons to the damping forces exerted by their interaction with the ions and the atoms,

$$-e\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{H}\right) + \frac{m}{\tau_{ie}}(\vec{v}_i - \vec{v}_e) + \frac{m}{\tau_{ae}}(\vec{v}_a - \vec{v}_e) = 0.$$

Similarly, the ions are damped by collisions with neutral atoms,

$$e\left(\vec{E} + \frac{1}{c}\vec{v}_i\times\vec{H}\right) + \frac{M_i}{\tau_{ia}}(\vec{v}_a - \vec{v}_i) = 0.$$

Also, let f be the fractional mass of the neutral atoms,

$$n_a M_a = f(n_a M_a + n_i M_i);$$

in "cold" clouds, $f \approx 0.999$.

Finally, we define a coordinate system with respect to which the center of mass of ions and atoms is unaccelerated,

$$n_a M_a \vec{v}_a + n_i M_i \vec{v}_i = 0.$$

Eliminating the particle velocities from the last five equations gives

$$\vec{E} = \frac{m}{n_e e^2}\left(\frac{1}{\tau_{ie}} + \frac{1}{\tau_{ae}}\right)\vec{j} + \frac{1}{n_e e c}\vec{j}\times\vec{H} - \frac{\tau_{ia}}{n_e M_i c^2}(\vec{j}\times\vec{H})\times\vec{H}, \qquad (1)$$

where f has been replaced by 1 and terms in m/M have been neglected. The first term is Ohmic, the second corresponds to the plasma Hall effect, and the third (which is of principal interest here) reflects the dissipation of energy in ion-atom collisions.

The coefficient of $\vec{j}$ in (1) will be called σ_e^{-1}, and for order-of-magnitude calculations we can linearize the last term by writing

$$\tau_{ia} H^2 / n_e M_i c^2 = \sigma_i^{-1}.$$

Then in the case considered, we find

$$E_x = (\sigma_e^{-1} + \sigma_i^{-1}) j_x + H j_y / n_e e c,$$
$$E_y = (\sigma_e^{-1} + \sigma_i^{-1}) j_y - H j_x / n_e e c. \qquad (2)$$

The known properties[3] of "cool" plasmas (Strömgren's H I zones[4,1]) give

$$\sigma_e \approx 10^{10}\ \mathrm{sec}^{-1}, \quad \sigma_i \approx 10^{-3}\ \mathrm{sec}^{-1},$$

so that the dissipation of energy proceeds mostly from ions to atoms and we can ignore the electronic contribution.

We assume that a massive photon can be described by the Proca equations

$$\partial\vec{H}/\partial t = -c\nabla\times\vec{E} \tag{3}$$

and

$$\nabla\times\vec{H} - c^{-1}\partial\vec{E}/\partial t + \mu^2\vec{A} = 4\pi c^{-1}\vec{j}, \tag{4}$$

where μ^{-1} is the photon's reduced Compton wavelength. If $\vec{j}$ as given by (4) is put into (2) and the result put into (3), we find that

$$\partial H/\partial t = \nu(\Box - \mu^2)H + \gamma\nabla\cdot(\vec{j}H), \tag{5}$$

$$\nu = c^2/4\pi\sigma_i \approx 10^{23}\ \mathrm{cm^2/sec}, \quad \gamma = (n_e e)^{-1}. \tag{6}$$

In order to estimate the dissipation of magnetic field, we multiply (5) by H and integrate over a cylindrical volume of unit length along the filament, and of radius r which will eventually become infinite. Neglecting the second time derivative, which is found to be incredibly small, we have

$$\tfrac{1}{2}dW/dt = \nu\left(\int H\nabla^2 H dv - \mu^2 W\right) + \gamma\int H\nabla\cdot(\vec{j}H)dv, \tag{7}$$

where $W = \int H^2 dv$. The first term on the right can be expressed on integration by parts as $-l^{-2}W$, where l is a length of the order of the radius of the spiral arm; in the last term we write

$$H\nabla\cdot(\vec{j}H) = \tfrac{1}{2}[\nabla\cdot(\vec{j}H^2) + H^2\nabla\cdot\vec{j}],$$

and this gives no contribution for a neutral plasma in which, on the average, $\nabla\cdot\vec{j} = 0$. The solution of (7) is then

$$W \sim \exp\left[-\frac{2\nu}{l^2}(1+\mu^2 l^2)t\right].$$

The result without the term in μ is well known.[2,5] With the numerical estimate (6), the decay time in this case is effectively infinite.

It is difficult to estimate the stability of the galactic magnetic field, but fortunately we can afford to be conservative. The evidence of nuclear particle tracks in lunar samples shows[6] that the flux of primary cosmic rays has remained roughly constant, on the average, over the last 10^6 yr, which implies that the galactic field has at least a comparable duration. Further, the known velocities of galactic matter make it difficult to see how the field could be maintained[7] magnetohydrodynamically if it had a tendency to collapse in so short a period. Assuming the field constant over 10^6 yr leads to the estimate

$$\mu < 10^{-18}\ \mathrm{cm}^{-1},$$

This corresponds to a Compton wavelength of about 6 lt yr (much smaller than the extent of the observed fields) and a rest mass less than 3.4×10^{-56} gm. This is eight orders of magnitude smaller than the estimates derived from geomagnetic data[8] and nine orders below the most sensitive laboratory determination.[9] Admittedly, our conclusion is only as certain as the underlying astrophysical hypotheses. Those that seem to us least sure are the minimum decay time of 10^6 yr and the assumption that the currents flow at least roughly perpendicular to the magnetic field in a spiral arm. The decay time could be considerably smaller without hurting our result very much, but if it should turn out that the currents and fields are very nearly parallel, then the plasma resistance that in our calculation leads to a rapid decay rate is diminished by several orders of magnitude, and our conclusion is correspondingly weakened. We hope that within a few years astrophysicists will be able to clear up these points.

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[3] Ref. 1, p. 228.

[4] B. Strömgren, Astrophys. J. 89, 526 (1939).

[5] As seems to be usual, the departure from the zero-mass result is given by the ratio of the size of the system considered to the photon's Compton wavelength. [A. S. Goldhaber and M. M. Nieto, Phys. Rev. Lett. 26, 1390 (1971); D. Park and E. R. Williams, Phys. Rev. Lett. 26, 1393 (1971); N. M. Kroll, Phys. Rev. Lett. 26, 1395 (1971)].

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[9] E. R. Williams, J. E. Faller, and H. A. Hill, Phys. Rev. Lett. 26, 721 (1971).

[10] In a forthcoming review article on the photon rest mass [Rev. Mod. Phys. 43, 277 (1971)], A. S. Goldhaber and M. M. Nieto comment upon our proposed test and discuss some of the difficulties.