According to (4), in contrast, the constant b defined in (3) is positive, and hence χ increases definitely as H increases as long as H is less than $H^*/\sqrt{e}$. For real solids, such a simplified expression as (4) may not necessarily hold because of complicated band structure and the most general expression (1) should be applied instead. However, it seems to be the case, without detailed analysis, that the appearance of an $H^2 \ln H$ term may alter whether χ increases or decreases with H from what is expected on the simple band model.

Taking, in particular, nearly ferromagnetic Fermi liquids for which the enhancement factor $(1+2\varphi g)^{-1}$ is very large, one finds that there exists a certain similarity between the temperature variation and the field variation of the susceptibility. If the density-of-states function φ_ϵ is assumed, near the Fermi level, to be relatively smooth and if its higher-order derivatives may be ignored, then the logarithmic temperature dependence of χ is determined dominantly by the term $-\varphi^2 g''(1+2\varphi g)^{-2}$, where $g'' \equiv \partial^2 g_{\epsilon\mu}/\partial\epsilon^2$ [cf. Eq. (2) in Ref. 6], while the magnitude of the logarithmic field variation is given by $-\varphi^2 g''(1+2\varphi g)^{-4}$ as is seen from (1). From this correspondence and from the negative definiteness of the sign of the $T^2 \ln T$ term,[6] it follows that the sign of b in (3) is positive for these systems. This may resolve the contradiction between the band theory and the experiment for the H dependence of χ in Pd.[9] Finally it should be mentioned that such a prevailing view,[10] in which the sign of ν and the magnitude of D can be determined on the basis of (2) from experiment, is losing ground.

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Self-Focusing of Electromagnetic Radiation in Semiconductors

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We propose a new mechanism for the self-focusing of electromagnetic radiation in degenerate semiconductors. The effect is due to the velocity-dependent mass of the conduction electrons. Calculations for InSb and a typical laser intensity give a theoretical focal length of 2.06 mm.

Important nonlinear optical properties of degenerate semiconductors stem from the nonparabolicity of the electronic conduction bands. A striking manifestation of this effect—the mixing of light waves—has been demonstrated experimentally[1] and calculated theoretically.[2] In this Letter we calculate the essential limitation on this and similar nonlinear optical experiments imposed by a well-known basic process of recent interest, namely, self-focusing.[3] Specifically we find that in InSb a light beam of intensity 2.8×10^7 W/cm^2 will self-focus in 190 vacuum wavelengths. This demonstrates that at these high intensities one cannot treat this and similar classes of problems without properly taking into account the effects of the nonlinear conductivity of the medium on the propagation of the primary waves.

Since the original work in self-focusing, this topic has received much attention in the litera-

ture, both theoretically and experimentally.[3] While initially it was studied in liquids, recent investigations have expanded to solids, vapors, and plasmas.[4] Previously considered effects leading to self-focusing include electrostriction, the Kerr effect, thermal perturbation of the medium, nonlinear electronic polarization, and forward stimulated Brillouin scattering.

We propose here a new mechanism in which the conduction electrons in semiconductors play a strategic role. We show that the nonparabolicity of the energy-momentum relation can give rise to strong nonlinearities in the current, conductivity, and dielectric constant. Indeed, for InSb, our calculations[5] yield a nonlinear dielectric constant ϵ_2 whose magnitude is among the largest known for any system. In the most general case, both relativistic and band-structure effects are responsible for the nonparabolicity. Even for the free-electron gas, nonlinear effects arise in the interaction of light with plasmas when $v/c \sim 1$. This is usually an unimportant effect in solid-state situations since there $v/c \ll 1$. However, because of band-structure interactions, in degenerate semiconductors the Hamiltonian in the effective-mass approximation near the bottom of the conduction band formally resembles a relativistic Hamiltonian. This similarity may be exploited to develop a pseudo-relativistic dynamics for the conduction electrons. We show here that this process can lead to the self-focusing of a single intense light beam. This mechanism may well dominate the interaction of laser light with semiconductors at sufficiently high intensities. Possible application of the self-focusing effect can be found in the study of the electronic properties under the influence of powerful radiation.

In the absence of external fields, the electronic Hamiltonian may be expressed as

$$H_0 = [(E_g/2)^2 + E_g p^2/2m^*]^{1/2} = [(m^*c^*)^2 + (c^*p)^2]^{1/2}, \quad (1)$$

where E_g is the gap energy, and m^* and p are the effective mass and momentum of the electron, respectively. We have defined a speed c^* by $c^* = (E_g/2m^*)^{1/2}$. As far as the dynamics of the electrons are concerned, c^* plays the same role as the conventional speed of light, c. Its magnitude is roughly two orders of magnitude smaller than the speed of light in the medium, however.

The external field may be introduced via the minimal coupling $\vec{p} \to \vec{p} + e\vec{A}/c$, the equations of motion of the electrons being

$$\frac{d}{dt}\frac{\vec{v}}{[1-(v/c^*)^2]^{1/2}} = \frac{e}{m^*}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right). \quad (2)$$

Since we are interested in the long-wavelength response, quantum dynamics is unnecessary. If we use $v < c^* \ll c$, it follows that

$$\frac{d}{dt}\frac{\vec{v}}{[1-(v/c^*)^2]^{1/2}} \approx \frac{e}{m^*c}\frac{d\vec{A}}{dt}. \quad (3)$$

Thus, unlike the situation in a true relativistic plasma, magnetic effects are negligible. Assuming the vector potential to be switched on adiabatically, we may integrate Eq. (3) to give

$$m^*\vec{v}[1-(v/c^*)^2]^{-1/2} - e\vec{A}/c = \vec{q}, \quad (4)$$

where $\vec{q}$ is the initial momentum. The initial momenta are assumed to be distributed according to the Fermi-Dirac distribution function $\{\exp[\beta(H_0 - E_F)] + 1\}^{-1}$, where $H_0 = [(m^*c^{*2})^2 + (qc^*)^2]^{1/2}$.

The current $\vec{J}$ is $-ne\vec{v}$, where n is the conduction-electron number density. After some algebra we derive the following expression for the current density:

$$\vec{J} = -\frac{ne^2\vec{A}}{m^*c} \times\left[\left\langle\frac{1+\frac{2}{3}\Delta}{(1+\Delta)^{3/2}}\right\rangle - \frac{3}{8}\left(\frac{eA}{m^*c^*c}\right)^2\left\langle\frac{1}{(1+\Delta)^{7/2}}\right\rangle\right], \quad (5)$$

where $\Delta = (q/m^*c^*)^2$. Here we have made a power-series expansion in A and have retained terms through order A^3. Only the fundamental harmonic term has been kept. From Eq. (5) it is a simple matter to derive expressions for the conductivity or the dielectric constant $\epsilon = \epsilon_0 + \epsilon_2 E^2$:

$$\epsilon_0 = \epsilon_L - (4\pi ne^2/m^*\omega_0^2)A_0, \quad (6a)$$

and

$$\epsilon_2 = \tfrac{3}{8}(4\pi ne^2/m^*)(e/m^*c^*\omega_0^2)^2 A_2, \quad (6b)$$

where ω_0 is the angular frequency of the light and ϵ_L is the lattice dielectric constant. The dielectric constant of Eq. (6b) is numerically identical to the nonlinear mixing coefficient of Wolff and Pearson.[2] It should be pointed out, however, that these quantities represent inherently different physical entities, and indeed the higher ϵ_n terms will not be numerically equal. A_0 and A_2 respectively refer to the first and second averages in Eq. (5), and for the range of n of interest in this paper they may be replaced by unity. This is equivalent to making the cold-plasma approximation. Actually, in this case,

an expression valid for arbitrary intensity exists and is given by[6]:

$$\epsilon = \epsilon_L - \frac{4\pi ne^2}{m^*\omega_0^2} \frac{F(\frac{1}{2},\frac{1}{2};2;\gamma/(1+\gamma))}{(1+\gamma)^{1/2}}, \quad (6c)$$

where $\gamma = (eE_0/m^*c^*\omega_0)^2$ and F is the Gauss hypergeometric function. Here E_0 is the electric field strength in the medium.

Having derived an expression for the dielectric constant, the remainder of the problem is straightforward. From Akhmanov, Sukhorukov, and Khokhlov[3] we obtain the following expression for the focal length:

$$R = \tfrac{1}{2}ka^2(P_i/P_i^{cr} - 1)^{-1/2}, \quad (7a)$$

where the critical incident power is[7]

$$P_i^{cr} = (c\epsilon_0/32\epsilon_2 k^2)(1+\sqrt{\epsilon_0})^2. \quad (7b)$$

Here P_i is the incident power in the beam, a is the beam radius, and k is the wave-propagation vector in the crystal. In Eqs. (7) we have taken account of the reflection of the beam at the surface.

Naively interpreted, the model predicts the focusing of the beam to a point in a distance R. In actuality, because of the omission of higher-order terms in Eq. (5), neglect of losses, and neglect of coupling to the higher harmonics, R represents the distance over which a dramatic shrinkage of the beam size has occured.

An estimate of the focal length for InSb and a typical Q-switched-laser intensity is made using the above formulas. The following values were employed[6]:

$\omega_0 = 1.742\times10^{14}$ rad/sec (10.81 μm), $\epsilon_L = 16$,

$n = 2.0\times10^{16}$ cm^{-3}, $P_i = 2557$ W,

$m^* = m_e/60$, $E_g = 0.234$ eV,

$a = 0.0054$ cm, $T = 77$°K;

and this gave $R = 1.79$ mm for the theoretical focal length.

From our formula (6c) it is clear that the dielectric function will saturate at high field strengths to the value ϵ_L. We note that, in contrast with other calculations, in the present work the analytic expression for the dielectric constant valid for all field strengths is obtained. This feature is essential to the proper description of the self-focusing profile since the contraction of the beam produces a considerably amplified field intensity. Consequently, formulas (7a) and (7b) are not strictly valid near the focal point. A detailed numerical calculation[5]

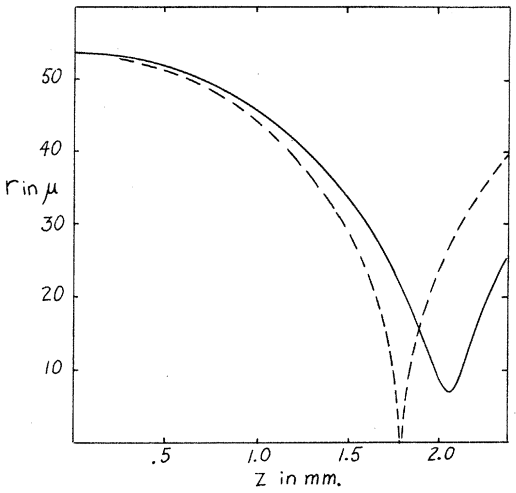

FIG. 1. Radius of the beam, r, as a function of depth in the crystal, z.

changes the focal length from 1.79 to 2.06 mm. Unlike the previous case the minimum beam size is finite and equals 6.66 μm. Beyond the focal length the radius of the beam is found to bounce back. The radius of the beam as a function of distance is given in Fig. 1 by the solid curve. Notice that the focusing occurs in a fairly gradual manner, in contradiction to other calculations which predict a precipitous contraction to the focal point. The difference is traced to the use of Eq. (6c) rather than the quadratic form. The saturation tendency manifests itself in strong deviations from the quadratic formula at even moderate values of E^2. Thus, as the beam contracts, the effective ϵ_2 decreases. This in turn causes the contraction to proceed at a slower rate. For comparison's sake the beam profile for the quadratic approximation is indicated by a broken line in Fig. 1. Thus, as the focal point is approached, the two curves depart from each other.

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Thermal Boundary Resistance

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A simple model of heat flow across a metal-insulator interface is used to calculate the spatial variation of the temperature. The characteristic distance for the temperature gradient to reach its bulk value is shown to be of the order of the phonon mean free path in both the metal and the insulator. Explicit expressions are presented for the electron and phonon contributions to the interfacial thermal resistivity.

In this Letter we present a semiclassical description of heat transport across a metal-insulator interface. We shall be particularly interested in the temperature profile near the interface, where the usual local relation between the heat flux and the temperature gradient is no longer satisfied.

A knowledge of the temperature distribution is important in determining the thermal boundary resistance between two solids, or a solid and a liquid, since the temperature at the interface is usually obtained by a linear extrapolation[1] of the temperature measured relatively far from the boundary. Most theoretical studies[2] of these phenomena seek to determine the temperature difference across the interface, and do not consider the details of its spatial variation.

In addition, the electron contribution to the heat flux across the interface has received relatively little attention since it is commonly assumed that they are almost uncoupled to the phonons in the insulator. Little,[3] and subsequently Andreev,[4] presented the first models of just how such a coupling might arise.

In the present work we use a simple Boltzmann-equation approach to provide a phenomenological description of thermal transport across a metal-insulator interface. This allows an explicit calculation of the temperature profile and a qualitative assessment of the effects of different surface conditions as well as the *nonlocality* of the transport near the interface. The latter leads to some rather interesting results concerning the role of electrons, even in the limiting case of zero electronic heat flux exactly at the surface.

We take the metal to occupy the right half-space $x > 0$, and the insulator the left half-space. A steady heat flux J is imposed, and we seek the steady-state temperature distribution.

The electronic contribution to the heat flux J_e is obtained by solving the usual Boltzmann equation[5] for the electron distribution function $f = f_0(T(x)) + \varphi$:

$$V_x \frac{\partial \varphi}{\partial x} + \frac{\varphi}{\tau} = V_x \frac{\partial f_0}{\partial \epsilon} \left(\frac{\epsilon - u}{T} \frac{\partial T}{\partial x} \right), \tag{1}$$

where f_0 is the Fermi-Dirac distribution at the local temperature $T(x)$ and Fermi energy $u(x)$, and τ is the relaxation time. The thermoelectric force, which is necessary to assure that the electrical current is zero, will be neglected here since it leads to a correction to ∇T of the order of $(kT/u)^2$.

The relaxation-time approximation is a crude representation of the effect of collisions and is subject to the usual criticism in that it does not accurately describe inelastic scattering. It furthermore fails to take account of the different relaxation rates of the various anisotropies in the spherical harmonic