above, this model has no applicability to alloys for Z even, and only extremely limited applicability to alloys where Z is ± 1. The self-consistency requirement on the potential requires that the perturbation of an impurity be more extended than allowed by the strongly localized model. Although calculations in the CPA have been closely tied in with the strongly localized perturbation model, the inapplicability of this model does not mean that the CPA is not applicable nor does it necessarily mean that all conclusions on the properties of alloys obtained from this model are incorrect. One would expect that the model cannot be trusted to give quantitative results, and qualitative results should be treated with some caution. To conform better with reality, the model used will have to employ a more extended perturbation, requiring a reformulated form[17,18] of the CPA or a new approach.[19] In addition, the self-consistency of the perturbation must also be solved simultaneously with the rest of the calculation. It is not possible to treat the perturbation as an independent variable to be varied at will.

*Research supported in part by the U. S. Air Force Office of Scientific Research, Office of Aerospace Research.

†On sabbatical leave from the University of Washington, Seattle, Wash. 98105. Work supported in part by a National Science Foundation Senior Postdoctoral Fellowship.

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Logarithmic Field Dependence of the Susceptibility of a Paramagnetic Fermi Liquid—the Pd Problem

Setsuo Misawa
Department of Physics, College of Science and Engineering, Nihon University, Kanda-Surugadai, Tokyo, Japan
(Received 2 February 1971)

The susceptibility $\chi(H)$ of a paramagnetic Fermi liquid in a magnetic field is shown to have the logarithmic form $\chi(H)=\chi(0)(1-bH^2\ln H)$, and hence the free energy of the system cannot be expanded in powers of H or in powers of the magnetization. Contrary to the result of the band model, this prediction is consistent with the experimental field dependence observed in Pd.

Among various metals, palladium is one of the most intractable and hence the most fascinating from the standpoint of studying magnetism. Recently, for Pd, the magnetic field dependence of the paramagnetic susceptibility $\chi(H)$ (or nonlinear magnetization) has been measured,[1] the result being expressed approximately in the form $\chi(H)=\chi(0)(1+\beta H^2)$ with $\beta=6.0\times10^{-14}$ Oe^{-2}. Prior to this experiment Wohlfarth[2] gave an expression for β on the basis of the band theory with consideration for the effect of exchange enhancement. Most recent data[3] on the band structure of Pd give us the theoretical value of β as negative: $\beta=-2.5\times10^{-13}$ or -1.1×10^{-12} Oe^{-2}, which apparently contradicts experiment.

Here we shall give a clue as to how this dis-

crepancy can be resolved. The above fact, that the observed field dependence is not compatible with the band theory, is related closely to the fact that the rigid band model[4] cannot account for the anomalous peak of χ observed in Pd about 80°K.[5] It was shown previously[6] that the peak may in fact be attributed to the $T^2 \ln T$ temperature variation of χ. By essentially the same reasoning, we shall derive the conclusion that the field dependence of χ is also given by a logarithmic formula. It is to be noticed that, owing their origin to the sharpness of the Fermi surface, these logarithmic terms appear in any normal Fermi liquid.

On the basis of the Landau Fermi-liquid theory in the Usui form,[7] the formal expansion of the susceptibility in powers of H can be obtained as

$$\frac{\chi(H)}{\mu_B^2}=\frac{2\varphi_\mu}{1+2\varphi_\mu g_{\mu\mu}}+\frac{1}{3}(\mu_B H)^2\left\{\frac{\partial^2}{\partial\epsilon^2}\left[\varphi_\epsilon\left(1-\frac{2\varphi_\mu g_{\epsilon\mu}}{1+2\varphi_\mu g_{\mu\mu}}\right)^4\right]-3\frac{1+2\varphi_\mu h_{\mu\mu}}{\varphi_\mu(1+2_\mu g_{\mu\mu})^4}\left(\frac{\partial\varphi_\epsilon}{\partial\epsilon}-2\varphi_\mu{}^2\frac{\partial}{\partial\epsilon}g_{\epsilon\epsilon}\right)^2\right.$$
$$-4\frac{\partial^2}{\partial\epsilon\partial\epsilon'}\left[\varphi_\epsilon\varphi_{\epsilon'}h_{\epsilon\epsilon'}\left(1-\frac{2\varphi_\mu g_{\epsilon\mu}}{1+2\varphi_\mu g_{\mu\mu}}\right)\left(1-\frac{2\varphi_\mu g_{\epsilon'\mu}}{1+2\varphi_\mu g_{\mu\mu}}\right)\right]$$
$$\left.+\frac{8\varphi_\mu}{1+2\varphi_\mu h_{\mu\mu}}\left\{\frac{\partial}{\partial\epsilon}\left[\varphi_\epsilon h_{\epsilon\mu}\left(1-\frac{2\varphi_\mu g_{\epsilon\mu}}{1+2\varphi_\mu g_{\mu\mu}}\right)\right]\right\}\right\}_{\epsilon=\epsilon'=\mu}+\cdots, \tag{1}$$

where μ_B is the magnetic moment of one particle, φ_ϵ denotes the density-of-states function with the quasiparticle energy ϵ as a variable, μ is the chemical potential, and $h_{\epsilon\epsilon'}$ and $g_{\epsilon\epsilon'}$ are, respectively, the spin symmetric and antisymmetric parts of the quasiparticle interaction (Landau f function). The explicit form for the functional dependence of φ_ϵ, $h_{\epsilon\epsilon'}$, and $g_{\epsilon\epsilon'}$ on the field will not be mentioned here for reasons of brevity. If one sets $h\equiv 0$ and $\partial g_{\epsilon\mu}/\partial\epsilon\equiv\partial g_{\epsilon\epsilon}/\partial\epsilon\equiv 0$ in the above, the expression (1) is reduced to the Wohlfarth formula,[2]

$$\chi(H)=\chi(0)[1+\tfrac{1}{6}\mu_B{}^2\nu D^3H^2], \tag{2}$$

where $\chi(0)=2\mu_B{}^2\varphi D$, $D=(1+2\varphi g)^{-1}$ is the exchange-enhancement factor, and $\nu=\varphi''/\varphi-3(\varphi'/\varphi)^2$.

Now the analytical properties of the functions φ, h, and g are our main concern. It was shown previously[6] that such derivatives as $\partial^2\varphi/\partial\epsilon^2$, $\partial^2 h_{\epsilon\epsilon'}/\partial\epsilon\partial\epsilon'$, and $\partial^2 g_{\epsilon\mu}/\partial\epsilon^2$, when considered as functions of temperature, diverge logarithmically as $T\to 0$. If one regards them as functions of a magnetic field at the absolute zero, these derivatives also diverge logarithmically as $H\to 0$. The H dependence of the quasiparticle energy or of the f function gives a factor $\ln H$ which prevents the divergence. Thus from Eq. (1) we are led to the conclusion that the H dependence of χ is given by the logarithmic formula

$$\chi(H)=\chi(0)[1-bH^2\ln(H/H^*)], \tag{3}$$

b and H^* being constants independent of H. From this field dependence it follows directly that the free energy of the system, F, is given by a logarithmic formula: $F\sim H^2+H^4\ln H$, or, in terms of the magnetization M, $F\sim M^2+M^4\ln M$.

In contrast to the case of the temperature variation,[6] the thermodynamic argument may not be used here for determining the sign of b in (3). Alternatively, we may glean some information concerning this sign from the result of perturbation theory. We now consider a system of Fermi particles interacting through a short-range potential. By evaluating the energy of the spin-polarized system up to the second order in the gas parameter λ (or s-wave scattering amplitude),[8] we can obtain the following result:

$$\frac{\chi(H)}{\chi(0)}=1-\frac{2}{3\Gamma^3}\left(\frac{\mu_B H}{\epsilon_F}\right)^2\left[\tfrac{16}{5}\lambda^2\ln\left(\frac{\mu_B H}{\Gamma\epsilon_F}\right)+1+\Lambda\right]+O(H^4\ln H), \tag{4}$$

where $\chi(0)=3N\mu_B{}^2/2\Gamma\epsilon_F$, N is the number of particles, ϵ_F is the Fermi energy, $\Lambda=\frac{8}{15}(3-2\ln 2)\lambda^2$, and $\Gamma=1-4\lambda-\frac{64}{15}(2+\ln 2)\lambda^2$ is the inverse of the exchange-enhancement factor in this approximation. If we observe (1) and (4), the mutual consistency between the results of the general theory, Fermi-liquid theory, and that of perturbation theory is seen to be satisfactory.

In deriving Eq. (4), a parabolic form for the single-particle spectrum has been assumed. Since $\varphi_\epsilon\propto\sqrt{\epsilon}$ for this spectrum, the Wohlfarth formula based on the simple band theory, (2), predicts that χ should decrease as H increases.

According to (4), in contrast, the constant b defined in (3) is positive, and hence χ increases definitely as H increases as long as H is less than $H^*/\sqrt{e}$. For real solids, such a simplified expression as (4) may not necessarily hold because of complicated band structure and the most general expression (1) should be applied instead. However, it seems to be the case, without detailed analysis, that the appearance of an $H^2 \ln H$ term may alter whether χ increases or decreases with H from what is expected on the simple band model.

Taking, in particular, nearly ferromagnetic Fermi liquids for which the enhancement factor $(1+2\varphi g)^{-1}$ is very large, one finds that there exists a certain similarity between the temperature variation and the field variation of the susceptibility. If the density-of-states function φ_ϵ is assumed, near the Fermi level, to be relatively smooth and if its higher-order derivatives may be ignored, then the logarithmic temperature dependence of χ is determined dominantly by the term $-\varphi^2 g''(1+2\varphi g)^{-2}$, where $g'' \equiv \partial^2 g_{\epsilon\mu}/\partial\epsilon^2$ [cf. Eq. (2) in Ref. 6], while the magnitude of the logarithmic field variation is given by $-\varphi^2 g''(1+2\varphi g)^{-4}$ as is seen from (1). From this correspondence and from the negative definiteness of the sign of the $T^2 \ln T$ term,[6] it follows that the sign of b in (3) is positive for these systems. This may resolve the contradiction between the band theory and the experiment for the H dependence of χ in Pd.[9] Finally it should be mentioned that such a prevailing view,[10] in which the sign of ν and the magnitude of D can be determined on the basis of (2) from experiment, is losing ground.

The author wishes to thank Professor M. Shimizu of Nagoya University for bringing the author's attention to the present problem.

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Self-Focusing of Electromagnetic Radiation in Semiconductors

N. Tzoar* and J. I. Gersten
Department of Physics, City College of the City University of New York, New York, New York 10031
(Received 22 February 1971)

We propose a new mechanism for the self-focusing of electromagnetic radiation in degenerate semiconductors. The effect is due to the velocity-dependent mass of the conduction electrons. Calculations for InSb and a typical laser intensity give a theoretical focal length of 2.06 mm.

Important nonlinear optical properties of degenerate semiconductors stem from the nonparabolicity of the electronic conduction bands. A striking manifestation of this effect—the mixing of light waves—has been demonstrated experimentally[1] and calculated theoretically.[2] In this Letter we calculate the essential limitation on this and similar nonlinear optical experiments imposed by a well-known basic process of recent interest, namely, self-focusing.[3] Specifically we find that in InSb a light beam of intensity 2.8×10^7 W/cm^2 will self-focus in 190 vacuum wavelengths. This demonstrates that at these high intensities one cannot treat this and similar classes of problems without properly taking into account the effects of the nonlinear conductivity of the medium on the propagation of the primary waves.

Since the original work in self-focusing, this topic has received much attention in the litera-