14, 544 (1965); V. P. Silin, Zh. Eksp. Teor. Fiz. 48, 1679 (1965) [Sov. Phys. JETP 21, 1127 (1965)].

[3]K. Nishikawa, J. Phys. Soc. Jap. 24, 916, 1152 (1968); P. K. Kaw and J. M. Dawson, Phys. Fluids 12, 2586 (1969); J. R. Sanmartin, Phys. Fluids 13, 1533 (1970).

[4]H. Dreicer, D. B. Henderson, and D. Mosher, to be published.

[5]H. Dreicer and W. F. Rich, in *Proceedings of the International Conference on Physics of Quiescent Plasmas, Paris, 1968* (Ecole Polytechnique, Paris, France, 1969), Part III, p. 135.

[6]V. E. Golant, Zh. Tekh. Fiz. 30, 1265 (1960) [Sov. Phys. Tech. Phys. 5, 1197 (1961)].

[7]S. J. Buchsbaum, L. Mower, and S. C. Brown, Phys. Fluids 3, 806 (1960).

[8]I. R. Gekker and O. V. Sizukhin, Pis'ma Zh. Eksp. Teor. Fiz. 9, 408 (1969) [JETP Lett. 9, 243 (1969)].

[9]J. M. Dawson and C. R. Oberman, Phys. Fluids 5, 517 (1962), and 6, 394 (1963).

Axially Dependent Equilibria for a Relativistic Electron Beam*

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(Received 8 April 1971)

Experimental measurements and numerical simulation show that relativistic electron beams propagating in a drift tube with a net current less than the Alfvén critical current assume an axially dependent equilibrium. The equilibrium of a 40-kA, 3-MeV electron beam propagating in air at 22.5 Torr has been studied analytically, numerically, and experimentally. Reasonable agreement among the results of the three approaches has been found.

Yoshikawa[1] has recently proposed an axially uniform, force-free equilibrium for high-intensity electron beams. Although it is conceivable that one could produce such a beam in the laboratory, experiments with the propagation of high-current electron beams at Sandia Laboratories indicate that the (quasi) equilibria assumed by relativistic beams are in general z dependent. Thus, as an alternative to the axially uniform equilibria of Yoshikawa, or that of Hammer and Rostoker,[2] we propose an axially varying equilibrium.

Figure 1 shows experimental current-density profiles for a 3-MeV electron beam propagating in air at a pressure of 22.5 Torr. The profiles were obtained with an array of Faraday cups which were located radially at 0, 1, 2, and 3.25 in. and azimuthally at 0°, 90°, 180°, and 270°.[3] Measurements taken at a given time and axial position were averaged azimuthally. The primary current in the diode was measured to be ~45 kA maximum and the net current in the drift tube was ~22.5 kA. Two pinches (regions of radial constriction) are visible.

We consider the problem of a beam of electrons injected through a conducting plane. The beam is assumed to be instantaneously charge neutralized, and all electric fields are neglected. Hence, we are considering just the Alfvén problem,[4] except that we have done it self-consistently. We have approached this problem in two ways: the relativistic, temperature-dependent fluid equations of Toepfer,[5] and a computer simulation technique.

The fluid approach is complicated by difficulties with the higher-moment equations, and we will present only the zero-temperature case (as done by Yoshikawa).

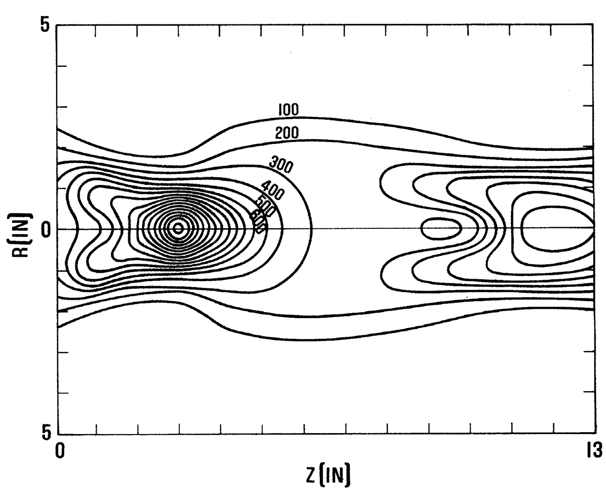

FIG. 1. Experimental profiles of constant current density (A/cm^2) during the time of maximum pinching for a 3-MeV beam propagating in a drift tube filled with air at 22.5 Torr. The primary current in the drift tube was ~40 kA and the net current was ~22.5 kA.

With the use of mks units and cylindrical coordinates r, θ, and z (where we assume all quantities independent of θ), the fluid equations take the form

$$\frac{\partial}{\partial z}(nv_z)+\frac{1}{r}\frac{\partial}{\partial r}(rnv_r)=0, \tag{1}$$

$$v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}=\frac{e}{m\gamma}v_zB_\theta, \tag{2}$$

$$v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}=-\frac{e}{m\gamma}v_rB_\theta, \tag{3}$$

$$B_\theta=-(e\mu_0/r)\int_0^r nv_z r dr. \tag{4}$$

Here e and m are, respectively, the electron charge and mass, $\vec{v}=(v_r,0,v_z)$ is the velocity, n is the density, and B_θ is the magnetic field. Since the velocity at the injection plane ($z=0$) is $(0,0,v_0)$, we must have $B_r=B_z=v_\theta=0$. Also $\gamma=(1-v_0^2/c^2)^{-1/2}$ will be a constant.

If we consider the streamline $R(z)$, it is easily shown that the total current I inside this streamline is independent of z. Thus along $R(z)$ we have $B_\theta=\mu_0 I/2\pi R$; and from (3),

$$dv_z/dR=e\mu_0 I/2\pi m\gamma R. \tag{5}$$

Since $v_z=v_0[1+R'(z)^2]^{-1/2}$, where $R'(z)=dR/dz$, the equation for the streamline is found to be

$$R'(z)=-\{[2(\nu/\gamma)\ln(R/R_0)+1]^{-2}-1\}^{1/2}, \tag{6}$$

where $R_0=R(z=0)$ and ν is the Budker parameter,

$$\nu=(e^2/4\pi\epsilon_0 mc^2)\int_0^{R_0}n(z=0)2\pi r dr.$$

Equation (6) applies only to streamlines for which $R(z)$ is single valued; roughly, these are streamlines for which $\nu/\gamma\lesssim 1$.

Physically, the assumption of zero temperature made in deriving Eq. (6) cannot be correct since even if all particles are injected with velocity $(0,0,v_0)$, they will acquire a spread in v_r and v_z as the beam pinches (although $v_r^2+v_z^2$ will remain constant for each particle). We cannot expect Eq. (6) to describe details of a pinch region with any accuracy, but we find that Eq. (6) does give a reasonable approximation to the location of the first pinch. To understand the role of temperature in the pinch region, one refers to the work of Bennett[6,7] or to the more recent work of Toepfer.[5]

The computer simulation technique consisted of injecting a stream of simulation electrons at $z=0$ and following their motion using the relativistic single-particle equations, the field B_θ being determined by Eq. (4). (Temperature effects are of course automatically included.) Details of the method will be given elsewhere.[8] The basic result is that for $\nu/\gamma\lesssim 1$, the system soon settles into a z-dependent steady state. For $\nu/\gamma\gtrsim 1$, i.e., the current exceeding the Alfvén critical current, part of the beam is reflected back out of the system at $z=0$. (See Ref. 4, Fig. 2.)

A picture of the simulation results for $\nu/\gamma=0.2$ is shown in Fig. 2. The $z=0$ density profile was taken to be uniform out to the beam edge, a rough approximation to the profile of Fig. 1. For comparison with experiment, we define a parameter α = pinch-to-pinch distance divided by the maximum beam diameter. The results for $\nu/\gamma=0.2$ are as follows:

Method of determination	α
Experiment	1.5
Analytic theory, Eq. (6)	1.7
Computer simulation ($T_0=0$)	1.8
Computer simulation ($T_0=T_{\rm exp}$)	1.4

The experimental value here is based on the spatial current profiles of Fig. 1. The analytic value was obtained by solving Eq. (6) numerically. The simulation values for α were obtained by dividing the z value of the first pinch by the maximum beam radius. In the simulation results, T_0

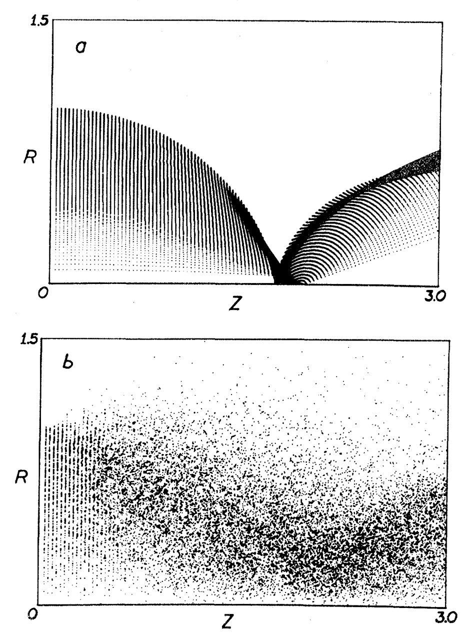

FIG. 2. Computer-obtained z-dependent equilibrium for the case where $\nu/\gamma=0.2$. Each dot represents a simulation electron; the density of dots at a given position (r,z) is proportional to the product of r and the actual electron density for (a) $T_0=0$, (b) $T_0=T_{\rm exp}$.

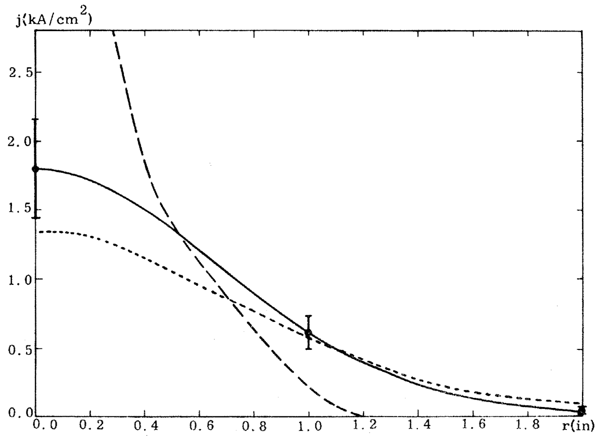

FIG. 3. Radial distribution of primary current in the region of maximum pinching for a 3-MeV electron beam with net $\nu/\gamma = 0.2$. The three data points indicate the results obtained experimentally. The solid curve was calculated from the finite-temperature fluid model with $T = 420$ keV and was fitted to the data at $r = 0$. The dashed and dotted lines were obtained from the numerical simulation for beams with $T_0 = 0$ and $T_0 = T_{\text{exp}}$, respectively. The value of j injected at $z = 0$ was 0.2 kA/cm^2.

refers to the spread in v_r and v_θ given to the particles at the $z = 0$ injection plane. The $T_0 = T_{\text{exp}}$ case was obtained by giving the particles a Gaussian distribution in v_r and a v_θ of about 0, such that the width of the Gaussian corresponded to the estimated scattering of a 3-MeV electron in the 0.005-in.-thick titanium anode foil.[9]

Figure 3 compares the experimental (3-MeV, 22.5-Torr), computer simulation, and the analytic results for the radial profile of the current density in a region of maximum pinching. The computer-code results for $T_0 = 0$ (a monoenergetic beam injected normally at $z = 0$) is too peaked at the center, but the $T_0 = T_{\text{exp}}$ case is in reasonable agreement with experiment. The "finite-temperature–model" curve is based on the axially uniform, analytic equilibria of Ref. 5, assuming a net ν/γ of 0.2 and a beam temperature of 420 keV. The justification for applying an axially uniform theory in a region of maximum pinching is that all fluid quantities in such a region are slowly varying in z. The fact that the analytic curve passes almost exactly through the three experimental points indicates that some aspects of beam behavior can be described by axially uniform theories. The theory of Bennett might also be expected to apply in a pinch region; however, the Bennett profile did not fit the data at large radii.

In summary, experimental observations on relativistic electron beams force one to look for axially dependent theoretical equilibria. We have found such equilibria and obtained reasonable quantitative agreement with experiment.

The authors thank Professor N. Rostoker for several helpful discussions, and F. O. Lane for programming assistance.

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*Work supported by the U. S. Atomic Energy Commission.

[1]S. Yoshikawa, Phys. Rev. Lett. 26, 295 (1971).

[2]D. A. Hammer and N. Rostoker, Phys. Fluids 13, 1831 (1970).

[3]J. G. Kelly, Bull. Amer. Phys. Soc. 15, 1308 (1970).

[4]H. Alfvén, Phys. Rev. 55, 425 (1939).

[5]A. J. Toepfer, Phys. Rev. A 3, 1444 (1971).

[6]W. H. Bennett, Phys. Rev. 45, 890 (1934).

[7]W. H. Bennett, Phys. Rev. 98, 1984 (1955).

[8]J. W. Poukey, to be published.

[9]G. Knop and W. Paul, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland, Amsterdam, 1966), Vol. 1, pp. 4-7.