cal value for ν_{01} can be calculated to an accuracy of about 1 ppm and hence, together with the use of our reported experimental value for ν_{01}, should determine the fine-structure constant α to an accuracy of better than 1 ppm.

A more detailed report of our experiment will be submitted for publication later.

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Anomalous Microwave Absorption Near the Plasma Frequency*

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Measurements in a highly ionized plasma of anomalously large absorption and the threshold field required for its onset are presented as a function of plasma density for conditions free of electron inelastic effects. In the weak-field limit the measured absorption is in good agreement with the classical theory for the high-frequency resistivity.

Absorption of intense electromagnetic waves near the critical density on a plasma profile, where the electron plasma frequency ω_p approximately equals the wave frequency ω, has recently become important in connection with laser and rf heating of plasmas. Theory indicates the possibility of enhanced absorption[1] when the intense fields excite high-frequency instabilities.[2,3] This Letter reports such absorption measurements made on the highly ionized plasma column of the single-ended Los Alamos Q machine. By operating over a large range of electric fields and plasma densities, our measurements yield (1) a quantitative value for the classical resistivity, (2) the threshold electric field for the onset of instability and anomalous dissipation, and (3) an experimental estimate for the anomalous dissipation under conditions shown to be free of electron inelastic effects.

In our experiment, potassium ions, produced by contact ionization on a grounded 2500°K tungsten hot plate (HP), drift along the applied magnetic field B_0 until they are collected on a cold copper collector biased negatively to the potential V_c. The plasma is limited by a 2.45-cm-diam aperture near the HP. The electrostatic sheath at the HP reduces the current of thermionic electrons by the Boltzmann factor $\exp(e\varphi/kT)$, where

$\varphi<0$ is the potential of the plasma and T is the HP and electron temperature. The negative bias applied to the collector reflects all electrons whose axial energy is less than $U=e(\varphi-V_c)$. For our operating conditions, $U\gg kT$, the relative drift velocity between the electrons and ions is about 10^5 cm/sec.[4] The electron density (or ω_p^2), varied by changing T or the flux of K atoms incident on the HP, is determined, when possible, from the resonant-frequency shift Δf of a microwave cavity, and from the density profile determined with a Langmuir probe. At the higher densities the probe was inoperative and ω_p^2 information was obtained from Δf alone using the assumed density profile indicated on the figures.

The experiment determines the absorption by plasma of microwaves stored in a 11.7-cm-diam. TM_{010} microwave cavity. The cavity resonates near 2 GHz with a Q of 21 000 without plasma, and is coaxial with the plasma column for 8.6 cm. The largest of the microwave electric-field components is axial and parallel to B_0. A small radial fringe field exists primarily in the 3.8-cm-diam, 7.5-cm-long cylinders (waveguides below cutoff) which guard the plasma access holes against radiation loss. The distribution of both vacuum electric-field components in the cavity, known for the actual geometry from numerical computations,[5] is used to relate Δf to ω_p^2. The measured increase in inverse Q, $\Delta(1/Q)$, is equal to the additional energy absorption per radian caused by the plasma, divided by stored cavity energy. In analyzing our data we use the perturbation-theory results[6] for Δf and $\Delta(1/Q)$ which are based on the assumption that the vacuum fields are not disturbed much by the plasma. In the stable regime this assumption is valid[7] because the collisionless-plasma skin depth, $(c/\omega)[(\omega_p/\omega)^2-1]^{-1/2}$, is never smaller than the plasma diameter in our experiment. Under unstable conditions its validity remains to be established because anomalous absorption may shorten the skin depth.

At each plasma density we measure Q and the peak microwave field E_0 for a range of incident microwave powers. Such measurements must be made in times short compared to the period of any shifts in the resonant frequency caused by ambient density fluctuations. To achieve this observation speed we modulate the frequency of the microwave source with a linear sweep. This results in constant power incident on the cavity with a constant positive or negative rate of change of frequency with time, $\beta=d\omega/dt$. The power transmitted through the cavity, P_T, as a function of ω or t is monitored by a well-matched, calibrated crystal detector. When displayed on an oscilloscope it represents the response of the resonator. Our theoretical analysis of P_T includes the distortion and beats produced when $\beta\gtrsim(\omega/Q)^2$, and yields an unambiguous Q value for all sweep rates. [For $\beta\ll(\omega/Q)^2$, Q is, of course, equal to the resonant frequency divided by the bandwidth of the resonator.] Although E_0 can be determined from the power absorbed in the resonator, P_A, and Q, it is easier to monitor P_T rather than P_A. Therefore, we determine the constant α in $P_T=\alpha E_0^2$ by measurements made at low power levels for each density, and assume it to be independent of incident power. This is valid provided the cavity field pattern does not change with power.

Figures 1(a)-1(f) show oscilloscope traces of the crystal response to P_T for various experimental conditions. For small ω_p^2 [Figs. 1(a)-1(d)] P_T depends upon v_E/v_T and upon the algebraic sign of β although no evidence of instability or anomalous heating is observed. Here $v_T=(2kT/m)^{1/2}$, $v_E=eE_0/m\omega$, and m is the electron mass. For $\beta>0$ [Figs. 1(a) and 1(b)] the increase of v_E/v_T results in an apparent increase in Q (as witnessed by the appearance of beats on the high-frequency side of the resonance), while for $\beta<0$ [Figs. 1(c) and 1(d)] the opposite is true. We believe this is a high-frequency electromagnetic pressure effect. According to single-particle orbit computations, a substantial number

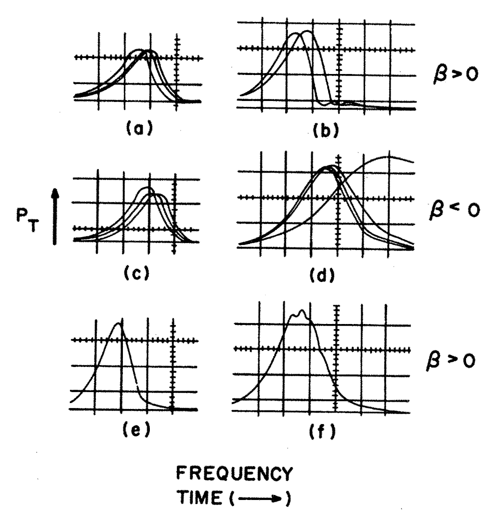

FIG. 1. P_T vs t (5 μsec per division) or frequency f. The following parameter values apply to (a)-(f), respectively: $\omega_{po}^2/\omega^2=0.5$, 0.5, 0.5, 0.5, 1.0, 1.0; $v_E/v_T=0.051$, 0.300, 0.051, 0.300, 0.072, 0.080; $f=70$, 70, 70, 70, 150, 150 kHz per division.

of electrons can be reflected by microwave fields and prevented from entering the cavity when $v_E/v_T \approx 1$. The presence of ions and the requirements of quasineutrality may reduce the magnitude of this reflection effect, but can not eliminate it. Thus, as strong fields ($v_E/v_T = 0.3$) build up in the resonator, ω_p^2 is decreased, shifting the resonant frequency to lower values. In this case $\beta > 0$ sharpens the response curve while $\beta < 0$ broadens it as shown. For $v_E/v_T = 0.05$, the field is too weak to the field is too weak to affect the electron orbits significantly, and P_T becomes independent of the sign of β. Figure 1(d) includes one response curve which is excessively broadened by a rapid ambient density fluctuation.

For large ω_p^2, instabilities are excited when E_0 exceeds a threshold. This effect is shown in Figs. 1(e) and 1(f) for values of v_E/v_T just below and above threshold. In these two traces P_T shows a similar increase with time, but for $v_E/v_T = 0.080$ it begins to exhibit fluctuations and broadening near the maximum while for the slightly smaller value 0.072 these effects are absent. This observation is independent of the sign of β and occurs at a value of v_E/v_T for which electron reflection is not important. Significantly larger threshold values for v_E/v_T are found to be required only for densities causing a Δf of 80-85 MHz. This is shown in Fig. 2, which is a compilation of measured v_E/v_T thresholds as a function of Δf. For lower ω_p^2 ($\Delta f < 80$ MHz) no instabilities at all are observed, while for higher ω_p^2 the threshold v_E/v_T saturates at about 0.075 (i.e., for $E_0 = 15$ V/cm).

Figure 2 also compares our measured values to the theoretical threshold predicted by the ac

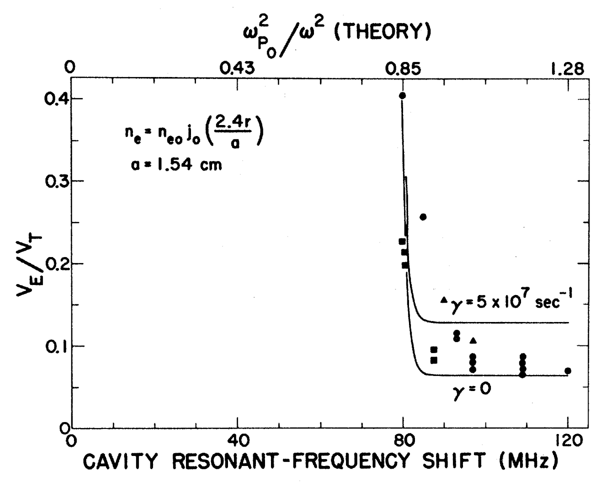

FIG. 2. Threshold v_E/v_T values versus Δf or ω_{p0}/ω^2. Points are measured data. Solid lines are from theory.

two-stream instability theory[3] for two values of the growth rate γ. Although this theory describes a uniform infinite plasma, we apply it locally to the density profile. In this we assume that the threshold for the finite plasma is the minimum threshold for the density range encountered on the profile. For large ω_p^2, $\Delta f > 85$ MHz, the theoretical minimum threshold is due to those electrons on the wings of the density profile for which $1.0 > (\omega_p/\omega)^2 > 0.90$, rather than due to those near the center of the plasma column where $\omega_p = \omega_{p0}$ and $(\omega_{p0}/\omega)^2 > 1$. The electron collision rate used in the calculation includes both electron-ion collisions and an effective collision rate due to Landau damping. The profile radius is adjusted so that experiment and theory agree at $\Delta f = 80$ MHz. This comparison is not intended to imply that we have identified the instability as the ac two-stream instability. Moreover, it is noteworthy that the parametric excitation of the ion acoustic waves[2] (for $T_e = T_i$) would require only half the threshold field computed for Fig. 2. The parametric excitation of other plasma modes is also not ruled out by the data presented here.

Examination of the current I_{ce} drawn by the cold collector, discloses the appearance of a pulse (see insert in Fig. 3) whenever unstable conditions exist. This pulse is believed to be due to electrons because its peak amplitude I_{ce0} decreases as the collector voltage V_c is biased more negatively. The onset of I_{ce0}, as shown in Fig. 3, agrees with the threshold found from the response curves. Although these pulses have U/kT as large as 30, their axial speed deduced from the 1-μsec delay between the application of microwave power and appearance of the electron pulse at the collector is characteristic of electrons with a drift energy of about $7kT$. This indicates a mechanism more complicated than the free flight of heated electrons.

That enhanced absorption begins at the onset of the instability is clear from the measurements of $\Delta(1/Q)$ which are summarized in Fig. 4 for four densities. At high densities, $\Delta(1/Q)$ increases sharply at the threshold described in Fig. 2. Gekker and Sizukhin[8] arrive at a similar conclusion as a result of observing anomalously small microwave reflection, but for plasma conditions which are not as well understood.

Figure 5 shows $\Delta(1/Q)$ as a function of $(\Delta f/f)^2$ at low power. We believe that this is the first report of an observation which can be compared quantitatively with the weak-field theory for the high-frequency resistivity.[9] This comparison,

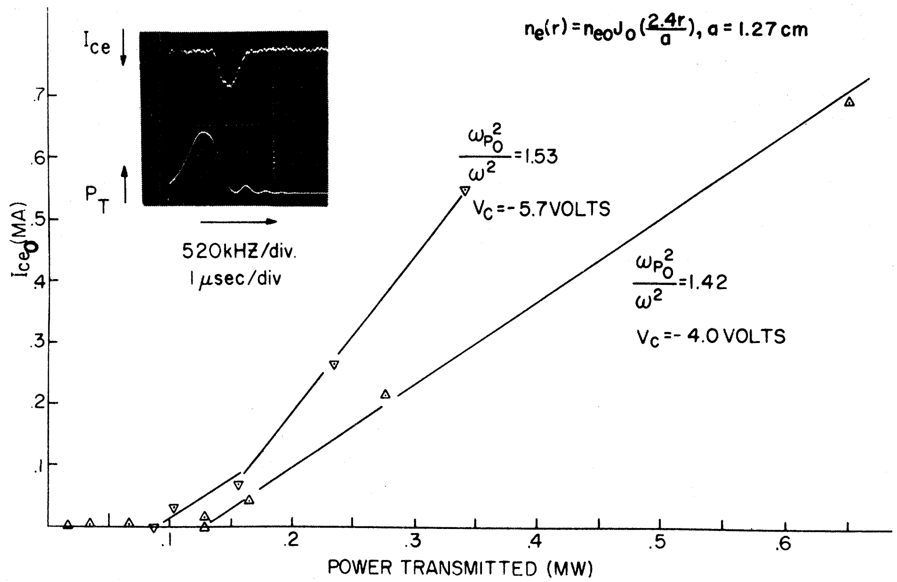

FIG. 3. I_{ce0} vs P_T (in milliwatts) for two plasma densities. Insert shows electron current collected and P_T vs t.

carried out for three different density profiles, takes into account the radial variation of density in the volume integration of the resistive part of the plasma dielectric coefficient.

We have found no influence upon the threshold due to electron inelastic collisions with K^+ ions or background K atoms. First, by varying T, with potassium flux F to the hot plate held constant, it is possible vary ω_p^2 although the K-atom density n_g remains constant. In this case the threshold near $\Delta f = 80\text{-}85$ MHz (Fig. 2) was found to be identical with the value observed when ω_p^2 is varied by changing F. The sudden increase in $\Delta(1/Q)$ observed with increasing v_E/v_T is found to be strictly a function of ω_p^2 rather than n_g. Second, Q values for $\beta > 0$ and $\beta < 0$ showed no measurable difference. This rules out ionization as a sudden dissipation source since electron production during the sweep through resonance would broaden the resonance for $\beta > 0$ and narrow it for $\beta < 0$.

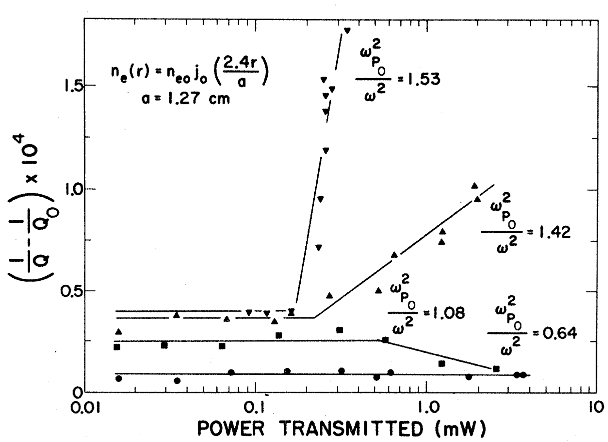

FIG. 4. $\Delta(1/Q)$ vs P_T (in milliwatts) for four densities.

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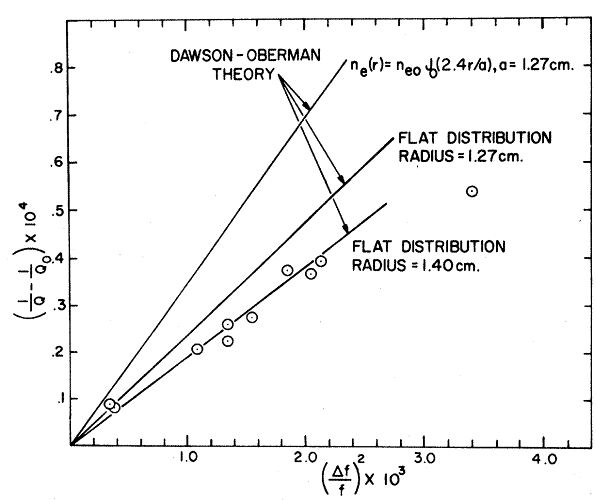

FIG. 5. Low-power $\Delta(1/Q)$ vs $(\Delta f/f)^2$. Points are measured data. Solid lines are from theory for three different assumed profiles.

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Axially Dependent Equilibria for a Relativistic Electron Beam*

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Experimental measurements and numerical simulation show that relativistic electron beams propagating in a drift tube with a net current less than the Alfvén critical current assume an axially dependent equilibrium. The equilibrium of a 40-kA, 3-MeV electron beam propagating in air at 22.5 Torr has been studied analytically, numerically, and experimentally. Reasonable agreement among the results of the three approaches has been found.

Yoshikawa[1] has recently proposed an axially uniform, force-free equilibrium for high-intensity electron beams. Although it is conceivable that one could produce such a beam in the laboratory, experiments with the propagation of high-current electron beams at Sandia Laboratories indicate that the (quasi) equilibria assumed by relativistic beams are in general z dependent. Thus, as an alternative to the axially uniform equilibria of Yoshikawa, or that of Hammer and Rostoker,[2] we propose an axially varying equilibrium.

Figure 1 shows experimental current-density profiles for a 3-MeV electron beam propagating in air at a pressure of 22.5 Torr. The profiles were obtained with an array of Faraday cups which were located radially at 0, 1, 2, and 3.25 in. and azimuthally at 0°, 90°, 180°, and 270°.[3] Measurements taken at a given time and axial position were averaged azimuthally. The primary current in the diode was measured to be ~45 kA maximum and the net current in the drift tube was ~22.5 kA. Two pinches (regions of radial constriction) are visible.

We consider the problem of a beam of electrons injected through a conducting plane. The beam is assumed to be instantaneously charge neutralized, and all electric fields are neglected. Hence, we are considering just the Alfvén problem,[4] except that we have done it self-consistently. We have approached this problem in two ways: the relativistic, temperature-dependent fluid equations of Toepfer,[5] and a computer simulation technique.

The fluid approach is complicated by difficulties with the higher-moment equations, and we will present only the zero-temperature case (as done by Yoshikawa).

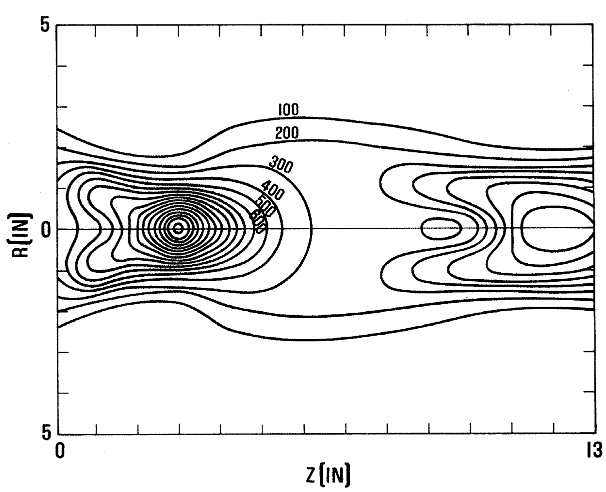

FIG. 1. Experimental profiles of constant current density (A/cm^2) during the time of maximum pinching for a 3-MeV beam propagating in a drift tube filled with air at 22.5 Torr. The primary current in the drift tube was ~40 kA and the net current was ~22.5 kA.

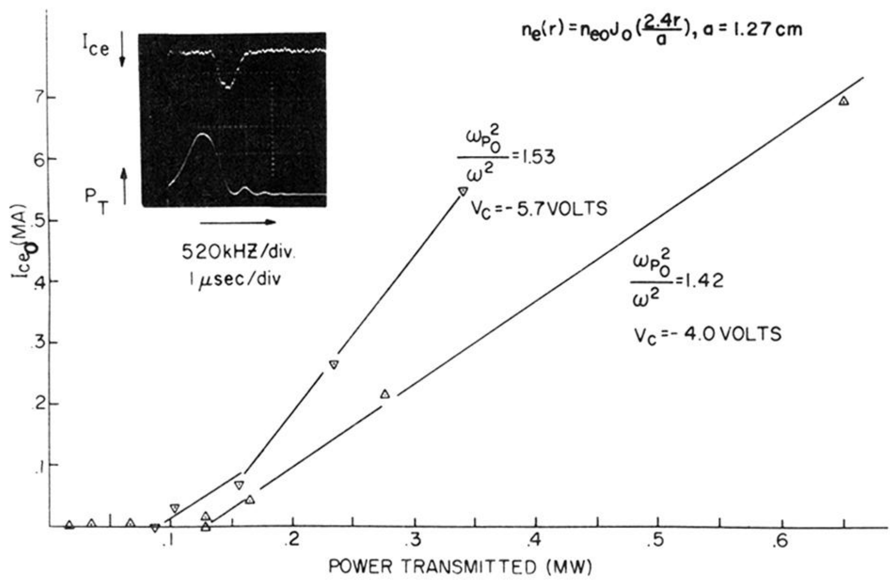

FIG. 3. I_{ce0} vs P_T (in milliwatts) for two plasma densities. Insert shows electron current collected and P_T vs t.