To obtain (7) one takes the solution

$$\rho = M^{\dagger} W^{\dagger} \qquad (14)$$

of (13), inserts it in (11), and multiplies the result by W. The parameter ν has to be purely imaginary:

$$\nu = i\mu. \qquad (15)$$

A particular case of this theorem has been obtained by Dashen when c_0 and c_8 only are nonzero to start with.

We can now draw the main conclusions from this theorem:

(1) If the CP-invariance–violating term which is present in the weak interaction is due to a semistrong or/and electromagnetic breaking term belonging to a $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ representation of SU(3) ⊗ SU(3), then $\mu \neq 0$. This CP-invariance–violating term is a pure *singlet* of SU(3). As a consequence, if the weak-interaction Hamiltonian satisfies the rule $|\Delta \vec{I}| = \frac{1}{2}$, so should the CP-invariance violating part. If octet dominance is correct, then the CP-invariance violating part should be dominated by the octet. We stress again that this is true both for CP-invariance violation originating from a semistrong interaction as well as for a CP-invariance violation originating from an electromagnetic interaction. The crucial restriction is that the breaking term belongs to a $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ representation. This would predict the equality of the η's characterizing the decay $K_L^0 \to 2\pi$. In particular[3]

$$|\eta_{00}|^2 = |\eta_{+-}|^2. \qquad (16)$$

(2) This violation can be made as small as one wants, as one can easily convince oneself on specific examples.

(3) Using the freedom which is left of an arbitrary SU(3) rotation $U = V$ which does not change the trace in (6b), and thus the maximum, one can write the most general H' as

$$H_B' = c_0 u_0 + c_8 u_8 + c_3 u_3 + d_0 v_0. \qquad (17)$$

The physical interpretation of this form is as follows: u_0 picks out that SU(3) representation which is responsible for the existence of multiplets; u_8 is the semistrong breaking term; u_3, the electromagnetic breaking; and d_0 [a *singlet* of SU(3)], the CP-invariance violating part.

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[1]R. Dashen, Phys. Rev. D 3, 1879 (1971).

[2]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[3]Recent experiments seem to favor this equality, as preliminary results of CERN-Orsay and CERN-Aachen-Torino collaborations show.

Dynamics of Broken SU(3)⊗ SU(3)

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We present a method of using mass dispersion relations for relating symmetry-breaking parameters in Lagrangians to breaking effects in states and deducing precise relations between physical S-matrix elements from operator algebras. We apply the method with the canonical equal-time and light-cone operator structures of the gluon model to deduce some SU(3) ⊗ SU(3) symmetry-breaking effects. We find $f_K/f_\pi \simeq 1.20$, $\xi(0) = -0.7$, a 50% correction to the Callan-Treiman relation, and $\alpha_8 \equiv$ [SU(3) breaking parameter] $\simeq 0.14$ GeV, and explain why these values are completely consistent with $Z_K/Z_\pi \gg 1$.

The Gell-Mann chiral SU(3) ⊗ SU(3) current algebra[1]

$$\delta(x_0)[V_0^a(x), V_0^b(0)] = if^{abc} V_0^c(0)\delta^4(x), \text{ etc.}, \qquad (1)$$

implies many interesting low-energy theorems, a number of which [usually via partial conservation of axial-vector current (PCAC)] can be compared with experiment. The agreement with experiment is quite good and supports the validity of Eqs. (1). The status of the conjectured corresponding broken SU(3) ⊗ SU(3) symmetry of the hadrons is much less clear.[3] The axial charges certainly do not give rise to a good symmetry of the states but seemingly to a Goldstone-Nambu-type symmetry. SU(3) symmetry, on the other hand, is clearly reflected in the hadron-mass spectrum. The precise nature of the symmetry-breaking interactions, and the precise connection between the magnitudes of the symmetry-breaking parameters in the Lagrangian and the magnitudes of the symmetry-breaking effects in the states, have remained, however, very obscure. In this paper we will assume the validity of the simplest possible chiral symmetry-breaking scheme and address ourselves to the problem of effectively determining the consequent departures from symmetry for the states and the magnitudes of the symmetry-breaking parameters.

We thus assume $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ symmetry breaking[3] (S^a = scalar nonet, p^a = pseudoscalar nonet). We shall, in fact, be more specific, and abstract some additional algebraic relations from the gluon Lagrangian model, in which the quark fields ψ interact via a neutral vector meson B_μ. The currents are then simply Dirac bilinears and we have the additional relations[4]

$$\delta(x_0)[S^a(x),P^b(0)]=id^{abc}A_0{}^c(0)\delta^4(x),\ \text{etc.} \qquad (2)$$

Since it is, of course, presently out of the question to solve the above model, we cannot directly deduce consequences for observable quantities like physical matrix elements of the currents. Our approach will therefore be indirect. We will abstract further formal properties from the model: specifically, the light-cone (LC) operator product expansions.[5,6] For example,

$$P^a(\tfrac{1}{2}x)S^b(-\tfrac{1}{2}x)\underset{x^2\to 0}{\longrightarrow}\partial_\mu(x^{-2})\sum_n d^{abc}O_{\alpha_1\cdots\alpha_n}{}^{\mu,c}(0)x^{\alpha_1}\cdots x^{\alpha_n}+\partial_\mu\partial_\nu(\ln x^2)$$
$$\times\sum_n f^{abc}O_{\alpha_1\cdots\alpha_n}{}^{\mu\nu,c}(0)x^{\alpha_1}\cdots x^{\alpha_n}+(x^{-2})d^{abc}\sum_n O_{\alpha_1\cdots\alpha_n}{}^{c}(0)x^{\alpha_1}\cdots x^{\alpha_n}. \qquad (3)$$

There is, by now, considerable support for the validity of such expansions and, equally important, the following properties which we assume that they possess: (A) The asymptotic behaviors (6), etc., set in quite fast, completely dominating for $x^{-2}\sim q^2\gtrsim 2.5\ \text{GeV}^2$. (B) They display a smooth threshold behavior in that the Fourier transforms of the coefficients of the LC singularities vanish rapidly near the boundaries of the physical region. A review of the experimental evidence for these properties is given in Ref. 6.

Before using this formalism, let us recall the Fubini-Furlan[7] dispersive approach to the calculation of corrections to SU(3). We have previously shown, for example, how, in the context of the above gluon-model relations, it leads to the result[8,9]

$$c/\sqrt{2}\equiv\alpha_8/\sqrt{2}\alpha_0\simeq -0.2. \qquad (4)$$

Our method involves the use of mass dispersion relations[10] to provide algebraic relations between the values of amplitudes at zero mass (given by equal-time commutation relations), at physical points (given by experiment), and at mass $\simeq 2.5\ \text{GeV}^2$ (given by the LC expansions). Since use of the smooth-threshold assumption relates the LC behavior back to the equal-time behavior, we end up with coupled equations for various physical parameters which can be solved simultaneously. This is how our algebraic equations for operators lead to algebraic equations for physical parameters and how the (exact) operator symmetry embodied in Eqs. (1)-(6) leads to (broken) symmetry for the physical states.[11]

In this paper we will apply our methods to discuss the vacuum–one-pseudoscalar-meson matrix elements of $A_\mu{}^a$ and P^a. We will ignore terms of order $\alpha_8{}^2$, but will *not* assume that the order α_8 terms are small compared to the order 1 terms. We keep all orders of α_0. We define the usual pion- and kaon-decay constants f_π and f_K and "renormalization" constants

$$\langle 0|P^{\pi^-}|\pi^+\rangle=Z_\pi,\quad \langle 0|P^{K^-}|K^+\rangle=Z_K. \qquad (5)$$

We always label on- or off-shell pions (kaons) with momentum p_μ (k_μ). On shell, $p^2=m_\pi{}^2=\mu^2$, $k^2=m_K{}^2=m^2$. The divergence equations

$$D^{\pi^-}\equiv\partial^\mu A_\mu{}^{\pi^-}=(2/\sqrt{3})(\sqrt{2}\alpha_0+\alpha_8)P^{\pi^-}\equiv(2/\sqrt{3})\epsilon_2P^{\pi^-}, \qquad (6a)$$

$$D^{K^-}\equiv\partial^\mu A_\mu{}^{K^-}=(2/\sqrt{3})(\sqrt{2}\alpha_0-\tfrac{1}{2}\alpha_8)P^{K^-}\equiv(2/\sqrt{3})\epsilon_KP^{K^-}, \qquad (6b)$$

immediately give the relations

$$\mu^2 f_\pi = (2/\sqrt{3})(\sqrt{2}\alpha_0 + \alpha_8)Z_\pi, \quad m^2 f_K = (2/\sqrt{3})(\sqrt{2}\alpha_0 - \tfrac{1}{2}\alpha_8)Z_K. \tag{7}$$

Thus, with (4), we have $(\mu^2/m^2)f_\pi/f_K = 0.77 Z_\pi/Z_K$. We will see that our formalism provides an elegant understanding of this large difference between f_π/f_K and Z_π/Z_K.

For use below, we parametrize matrix elements of some other operators occurring in (3), etc.:

$$\langle 0|i\bar{\psi}\gamma_\alpha\gamma_5(\tfrac{1}{2}\lambda^K)(i\Delta_\beta)\psi|K\rangle = A_K(k_\alpha k_\beta - \tfrac{1}{4}m^2 g_{\alpha\beta}), \tag{8}$$

$$\langle 0|\bar{\psi}\gamma_5(\tfrac{1}{2}\lambda^K)\Delta_\beta\psi|K\rangle = B_K k_\beta, \quad \Delta_\beta \equiv \overleftrightarrow{\partial}_\beta + 2igB_\beta. \tag{9}$$

We have taken (8) to be traceless because, according to the field equations, the trace is higher order in α_8. Likewise, to the order of interest, we can take $A_\pi = A_K = A$ and $B_\pi = B_K = B$, where A_π and B_π are defined as above with $\pi \to K$.

We consider first the vertex function $(k = p + q)$

$$A(p^2, q^2) = -i\int dx\, e^{i(p-q)\cdot x/2}\langle 0|TP^{\pi^-}(\tfrac{1}{2}x)\mathfrak{D}^{\bar{K}^0}(-\tfrac{1}{2}x)|K\rangle, \tag{10}$$

where $\mathfrak{D}^{\bar{K}^0} \equiv \partial^\nu V_\nu^{\bar{K}^0} = -i\sqrt{3}\alpha_8 S^{\bar{K}^0} \equiv -i\epsilon_3 S^{\bar{K}^0}$. The zero-mass theorem $A(m^2, 0) = iZ_K$ follows immediately. The residue of the pion pole in $A(p^2, q^2)$ can be simply expressed in terms of the K_{l3} form factors defined by

$$\langle\pi|V_\mu^K|K\rangle = 2^{-1/2}[(k_\mu + p_\mu)f_+(q^2) + (k_\mu - p_\mu)f_-(q^2)]. \tag{11}$$

Writing the matrix element of (3) as

$$\langle 0|P^{\pi^-}(\tfrac{1}{2}x)\mathfrak{D}^{\bar{K}^0}(-\tfrac{1}{2}x)|K\rangle \to (k\cdot\partial)(x^{-2})f_1(x\cdot k) + (k\cdot\partial)^2(\ln x^2)f_2(x\cdot k) + (x^{-2})f_3(x\cdot k), \tag{12}$$

it follows as usual[6] that

$$A \underset{q^2\text{ fixed}}{\xrightarrow{p^2\to\infty}} \int_{-1}^{1} d\eta\, F_1(\eta)\frac{q^2-p^2}{(q-p+\eta k)^2} + \int_{-1}^{1} d\eta\, F_2(\eta)\left[\frac{q^2-p^2}{(q-p+\eta k)^2}\right]^2 + \int_{-1}^{1} d\eta\, F_3(\eta)\frac{1}{(q-p+\eta k)^2}, \tag{13}$$

where the $F_i(\eta)$ are proportional to the Fourier transforms of the $f_i(\lambda)$. The normalizations of the $F_i(\eta)$ are conveniently fixed by considering the Bjorken[12] limit $p_0 \to \infty$, $\vec{p}$ fixed.

We thus learn from (13) that

$$A(p^2, 0) \xrightarrow{p^2\to\infty} -i\epsilon_3(f_K + A) - \frac{i\epsilon_3}{p^2}\left[\frac{m^2}{2}(f_K + A) + (2/\sqrt{3})\epsilon_2 Z_K\right], \tag{14}$$

where we have invoked the smooth-threshold assumption[6] in order to neglect, e.g., $\int d\eta F_i(\eta)\eta$ compared with $\int d\eta F_i(\eta)$.

We have now derived enough results to use the mass dispersion relation

$$A(p^2, q^2) = \frac{1}{\pi}\int_0^\Lambda dz \frac{\mathrm{Im}A(z, q^2)}{z - p^2} + \frac{1}{2\pi i}\oint_{c_\Lambda} dz \frac{A(z, q^2)}{z - p^2} \tag{15}$$

and the relation

$$0 = \pi^{-1}\int_0^\Lambda dz\, \mathrm{Im}A(z, q^2) + (2\pi i)^{-1}\oint_{c_\Lambda} dz\, A(z, q^2) \tag{16}$$

effectively, where $\Lambda = 2.5$ GeV^2 and c_Λ is the circular contour $|z| = \Lambda$. We evaluate (15) at $p^2 = m^2$, $q^2 = 0$. The result (14) [and Assumption (A)] can be used to evaluate the contour integrals, and (11) can be used to evaluate the pion-pole contributions to the $(0 \to \Lambda)$ integrals. The difference $\mathrm{Im}\bar{A}$ of $\mathrm{Im}A$ and its pion-pole contribution should not oscillate in the short integration range $9\mu^2 < z \leq \Lambda$, and so the mean-value theorem can be used to conclude that

$$\int_0^\Lambda dz\, \mathrm{Im}\bar{A}(z, 0)/z = M^{-2}\int_0^\Lambda dz\, \mathrm{Im}\bar{A}(z, 0), \quad 0 \leq M^2 \leq \Lambda. \tag{17}$$

From previous experience,[6] we expect that $M^2 \simeq 1.5$. Putting all this into (15) and (16), we obtain the relation

$$Z_K = Z_\pi f_+(0)\frac{M^2 - \mu^2}{M^2 - m^2} - \epsilon_3\left[(f_K + A)\left(1 - \frac{m^2}{2(M^2 - m^2)}\right) - \frac{2\epsilon_2 Z_K}{\sqrt{3}(M^2 - m^2)}\right]. \tag{18a}$$

In exactly the same way, we can obtain a second relation by taking the K, rather than the π, off shell and considering $\langle 0|TP^K\mathfrak{D}^{\bar{K}}|\pi\rangle$ instead of (10). We get, in this way,

$$Z_\pi = Z_K f_+(0)\frac{M^2-m^2}{M^2-\mu^2} + \epsilon_3\left[(f_\pi - A) - \frac{2\epsilon_K Z_\pi}{\sqrt{3}M^2}\right]. \tag{18b}$$

In deriving Eqs. (18), we used zero-mass theorems ($q=0$, $p^2=m^2$ or $k^2=\mu^2$) obtained from $\langle 0|TP\partial^\mu V_\mu \times|\pi \text{ or } K\rangle$. If, instead, we consider $\langle 0|T\partial^\mu A_\mu \mathfrak{D}|\pi \text{ or } K\rangle$, we get zero-mass theorems at $p=0$ ($q^2=m^2$) or $k=0$ ($q^2=\mu^2$). Using these theorems, our method leads to the new relations

$$-\epsilon_3 Z_K = f_\pi(m^2-\mu^2)\left[f_+(m^2) + \frac{m^2}{m^2-\mu^2}f_-(m^2)\right] + \frac{2}{\sqrt{3}}\epsilon_2\epsilon_3\left[f_K\left(1+\frac{3m^2}{2M^2}\right) + A\left(1+\frac{7m^2}{2M^2}\right) - \frac{2\epsilon_2 Z_K}{\sqrt{3}M^2}\right], \tag{19a}$$

$$-\epsilon_3 Z_\pi = f_K(m^2-\mu^2)\left[f_+(\mu^2) + \frac{\mu^2}{m^2-\mu^2}f_-(\mu^2)\right] + \frac{2}{\sqrt{3}}\epsilon_3\epsilon_K\left[f_K\left(1+\frac{3\mu^2}{2M^2}\right) - A\left(1+\frac{7}{2}\frac{\mu^2}{M^2}\right) - \frac{2\epsilon_2 Z_K}{\sqrt{3}M^2}\right]. \tag{19b}$$

We consider next the vertex function

$$-i\int dx\, e^{i(p-q)\cdot x/2}\langle 0|TD^\pi(\tfrac{1}{2}x)V_\mu{}^K(-\tfrac{1}{2}x)|K\rangle = (k+p)_\mu f_+(p^2,q^2) + (k-p)_\mu f_-(p^2,q^2). \tag{20}$$

The zero-mass theorem is $f_+(0,m^2)+f_-(0,m^2)=f_K$, and the asymptotic behavior, determined as above, is easily found to be

$$f_+(p^2,m^2)+f_-(p^2,m^2) \underset{p^2\to\infty}{\longrightarrow} p^{-2}(2\epsilon_2/\sqrt{3})[2Z_K - (2/\sqrt{3})f_K(\epsilon_2+\epsilon_K) - 3B]. \tag{21}$$

The mass-dispersion relations, used as above, now give

$$f_K = f_\pi[f_+(m^2)+f_-(m^2)] - (2\epsilon_2/\sqrt{3}M^2)[2Z_K - (2/\sqrt{3})f_K(\epsilon_2+\epsilon_K) - 3B]. \tag{22a}$$

The relation obtained by taking the π, rather than the K, off shell is

$$f_\pi = f_K[f_+(\mu^2) - f_-(\mu^2)] - (2\epsilon_K/\sqrt{3}M^2)[2Z_\pi - (2/\sqrt{3})f_\pi(\epsilon_2+\epsilon_K) - 3B]. \tag{22b}$$

Using the relations (4) and (7) and the experimental values $f_K/f_\pi f_+(0)=1.28$ and $f_\pi = 0.96\mu$, our results (18), (19), and (22) constitute six equations in the six unknowns $f_+(0)$, $\epsilon(0)\equiv f_-(0)/f_+(0)$, $f_+(m^2)+f_-(m^2)$, α_8, A, and B. The solution is

$$f_+(0) \cong 0.94, \tag{23a}$$

$$\xi(0) \cong -0.7, \tag{23b}$$

$$f_+(m^2)+f_-(m^2) \cong 1.45 f_K/f_\pi, \tag{23c}$$

$$\alpha_8 \cong -140 \text{ MeV}. \tag{23d}$$

These values were obtained using the usual value $M^2=1.5$ but they are quite insensitive to M^2 in the range $1.5 \leqslant M^2 \leqslant 2$.

Let us comment on our results.[13] Eq. (23a) is in good agreement with the Ademollo-Gatto theorem. Eq. (23b) is in good agreement with the recent K_{l3} data. Note that it was obtained without making any assumptions about the q^2 dependence of the form factors. Eq. (23c) represents a 50% violation of the Callan-Treiman relation, but in the opposite direction from what we previously expected.[14] Eq. (23d) is, as expected, a small SU(3)-nonconserving parameter. The chiral SU(2) ⊗ SU(2) symmetry breaking is thus $\epsilon_2 = \sqrt{2}\alpha_0 + \alpha_8 \cong 580$ MeV. We see that our formalism leads to a completely consistent picture of SU(3) ⊗ SU(3) symmetry breaking.

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[1]For convenience we exhibit the local commutation relations although we only use them in their integrated form.

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algebra.

[5]R. A. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1971).

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[8]R. A. Brandt and G. Preparata, Ann. Phys. (New York) 61, 119 (1970).

[9]M. Gell-Mann, R. J. Oakes, and B. Renner [Phys. Rev. 175, 2195 (1968)] have used a different set of assumptions in connection with the $(3,3^*)\oplus(3^*,3)$ model and have found $c/\sqrt{2}\simeq -1$. The recent accurate numerical evaluation of $T^{\pi N(+)}(\nu=0,\ \nu_B=0)$ by T. P. Cheng and R. F. Dashen [Phys. Rev. Lett. 26, 594 (1971)], however, makes it highly unlikely that both these different assumptions and the $(3,3^*)\oplus(3^*,3)$ model are correct.

[10]R. A. Brandt and G. Preparata, Phys. Rev. Lett. 25, 1530 (1970).

[11]Although our language is different, we are adopting here the philosophy of Weinberg (Ref. 2) in making a sharp distinction between the operator algebras and the symmetry properties of the states.

[12]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[13]See Ref. 2 for a discussion of some previous attempts to calculate these quantities.

[14]For most of the cases considered in Ref. 8, we used Regge theory to determine which amplitudes extrapolated smoothly off-shell. For the K_{l3} amplitudes, Regge theory is inapplicable; but we argued that the off-shell extrapolation defined by $\mathfrak{D}$ was smooth and that the one defined by V_μ was not. This led to $f_+(m^2)+f_-(m^2)\simeq 0.3f_K/f_\pi$. Now, however, we need not make such assumptions since we can reliably compute off-shell corrections. Our result is that neither extrapolation is smooth.

Description of High-Energy Scattering

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A model of high-energy scattering is developed which incorporates assumed analytic properties. The approximations involved in the phenomenological fits of Orear and Krisch are discussed. An improved formula is suggested which has correct analytic properties and reproduces the Orear and Krisch fits in appropriate limits.

At present there is no general theory available for explaining high-energy hadron collision cross sections over the entire angular range. For proton-proton scattering there exist at least two excellent phenomenological fits,[1] the Orear[2] formula

$$S d\sigma/d\Omega = A e^{-(p\sin\theta)/b} \tag{1}$$

and the Krisch[3] formula

$$d\sigma/dt = \sum_i A_i \exp(-\alpha_i \beta^2 p^2) = \sum_i A_i \exp(-\alpha_i ut/s), \tag{2}$$

with $A_1=90$, $A_2=0.74$, $A_3=0.0029$, $\alpha_1=10$, $\alpha_2=3.45$, and $\alpha_3=1.45$. Here s, t, and u are the usual Mandelstam variables, θ is the c.m. scattering angle, and p is the c.m. momentum. The Orear fit is good for large angles around $\theta\simeq 90°$, but deviates substantially for small angles. A single exponential[4] in t fits the data for small t, i.e., for θ close to 0°, and is quite bad for larger angles. It is worth mentioning that a fit which combines both the Krisch and Orear type of exponentials,

$$d\sigma/d\Omega = A \exp(\alpha p_\perp + \beta p_\perp^2), \tag{3}$$

had been proposed by Narayan and Sarma[5] as early as 1964.

So far, no attempts have been made to propose phenomenological formulas which have the correct analytic properties of scattering cross sections. Since the analytic properties are due to the nature of forces responsible for scattering of hadrons, a formula satisfying the requirements of analyticity could provide better representations for scattering data. This was first pointed out by Cutkosky and Deo[6] and Cutkosky[7] and also explicitly demonstrated by them[8] and Chou.[9] They have been able to construct polynomial expansions where each term has the assumed analytic property and more importantly, the series converges very rapidly. We propose to extend some of these ideas to the high-energy scattering for all angles in the diffraction region. At these energies, resonances have ceased to