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Single-Particle Distributions of π Mesons Produced in π^+p, K^+p, pp, and π^-p Collisions at High Energies*

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A comparison is made of single-particle momentum distributions of pions produced in 7.0-GeV/c π^+p, 12.7-GeV/c K^+p, 28.5-GeV/c pp, and 24.8-GeV/c π^-p interactions. We observe that the distributions for negative pions which have low laboratory momentum (target fragments) in π^+p, K^+p, and pp interactions agree with each other when normalized by their respective asymptotic total cross sections. However, the corresponding distributions from the π^-p reaction are significantly different from the rest. This difference is particularly interesting in view of recent theoretical predictions for inclusive reactions.

In anticipation of studies to be made of very high-energy collisions at the National Accelerator Laboratory in this country and at the Intersecting Storage Rings (ISR) in Europe, much attention has been focused recently on both theoretical and experimental aspects of multiparticle production processes. The simplest distributions to study are of the inclusive variety, i.e., $a+b \to c+\text{anything}$, where the spectrum of particle c is studied irrespective of other produced particles.

We report results on a comparison study of the single-particle spectra in the following inclusive reactions:

$$\pi^+ + p \to \pi^- + \text{anything at 7 GeV}/c, \quad (1)$$

$$K^+ + p \to \pi^- + \text{anything at 12.7 GeV}/c, \quad (2)$$

$$p + p \to \pi^- + \text{anything at 28.5 GeV}/c, \quad (3)$$

$$\pi^- + p \to \pi^- + \text{anything at 24.8 GeV}/c, \quad (4)$$

$$\pi^- + p \to \pi^+ + \text{anything at 24.8 GeV}/c. \quad (5)$$

The data are from four alternating-gradient synchrotron experiments utilizing the Brookhaven National Laboratory 80-in. bubble chamber.[1] The π^+p, K^+p, pp, and π^-p data are from Rochester-Yale,[2] Rochester,[3] Brookhaven,[4] and Wisconsin[5] experiments, respectively.

Feynman[6] and, independently, Benecke, Chou, Yang, and Yen[7] have predicted that the spectra of produced particles in high-energy collisions approach finite, energy-independent limits with increasing incident energy. Furthermore, since total and elastic cross sections in many reactions appear to approach limiting values at energies of a few GeV (e.g., in K^+p and pp collisions), one might expect limiting behavior in the spectra of produced secondaries at equally low energies.

To test limiting behavior one needs data from a range of incident energies; however, if limiting behavior sets in at relatively low energies, it may be possible to discover relationships among distributions derived from reactions involving different colliding particles. Specifically, it is of interest to test for factorization, i.e., to determine whether the limiting distributions of target fragments are independent of the nature of the projectile, and projectile fragments of the nature of the target.

We now describe a theoretical framework which suggests how the single-particle spectra should be normalized to test for factorization, and which predicts for which reactions factorization is expected to work best. Based on the arguments of Mueller,[8] the inclusive reaction $a+b\rightarrow c+$anything can be related, through a generalized optical theorem, to the imaginary part of the three-particle $a\bar{c}b \rightarrow a\bar{c}b$ scattering amplitude. When vacuum exchange becomes the dominant mechanism in the reaction $a\bar{c}b \rightarrow a\bar{c}b$, particle c is predicted to exhibit limiting behavior.[9] Using the duality hypothesis, Chan *et al.*[10] point out that, when the quantum numbers of the $a\bar{c}b$ system are exotic, the dominance of vacuum exchange, and consequently the onset of limiting behavior, may occur at relatively low energies.

On the other hand, Ellis *et al.*[11] assert that this condition is not sufficient for early onset of limiting behavior, and suggest that both $a\bar{c}b$ and ab must be exotic for limiting behavior to prevail. If only $a\bar{c}b$ needs to be exotic, Reactions (1), (2), and (3) will approach limiting behavior more rapidly than the rest, while if both $a\bar{c}b$ and ab must be exotic, then only Reactions (2) and (3) approach limiting behavior at low energies. In either case, the $\pi^- p$ reactions (4) and (5) are considered nonexotic, and their spectra should approach limits more slowly. The question as to which criteria are most meaningful must be answered by examination of the data.

We study the single-particle spectra, Reactions (1) through (5), in either the target or the projectile rest frames, depending on whether we are considering the low-momentum "fragments" of the target or projectile particle, respectively. We wish to compare the pion spectra among the various reactions (1) through (5). In order to do so, we normalize each distribution by dividing by the asymptotic total cross section[12] of each reaction, as suggested by Chan *et al.*[10]

We consider target fragmentation first. Figure 1 displays the normalized distributions for the longitudinal momentum P in the laboratory system L for Reactions (1) through (4).[13] The data points have been fitted with polynomials in P_L, and we show these fitted curves along with some representative experimental points. It is apparent that the normalized spectra for Reactions (1), (2), and (3) are in agreement with each other at very low laboratory momenta, where proton fragmentation is expected to dominate.

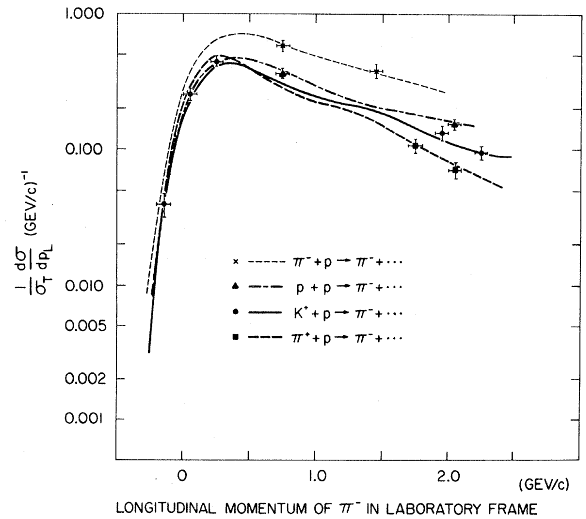

FIG. 1. Longitudinal-momentum distributions in the laboratory system for Reactions (1)-(4). Curves are polynomial fits to the data; for clarity of presentation we show only representative data points. Each reaction is normalized by its asymptotic total cross section (see text and Ref. 12).

This agreement can be expressed quantitatively by dividing the cross sections for the region $P_L<0.5$ GeV/c by the total asymptotic cross section for each reaction. These ratios are 0.23 ± 0.02, 0.20 ± 0.02, 0.23 ± 0.02, and 0.32 ± 0.02 for Reactions (1) through (4), respectively. The errors include systematic as well as statistical uncertainties.

Figure 2 shows the normalized distributions in the square of the transverse momentum, $P_\perp^2$, for these same reactions, but with the restriction $P_L<0.5$ GeV/c. The distributions are again in good agreement for reactions (1) through (3), except for large $P_\perp^2$ where the cross sections are small.

The agreement among Reactions (1) through (3) is evidence for factorization, and hence also for limiting behavior for those reactions which have $a\bar{c}b$ exotic. The lack of agreement of Reaction (4) may be interpreted as evidence that limiting behavior has not yet been reached for this nonexotic reaction.[14]

Now turning to projectile fragmentation, Fig. 3 displays the normalized distributions for the longitudinal momentum P in the projectile rest frames b for Reactions (1), (2), (3), and (5).[15] The normalized transverse-momentum distributions for $P_b<0.5$ GeV/c are shown in Fig. 4. The spectra for Reactions (1) and (5) are in much closer agreement with each other than with the

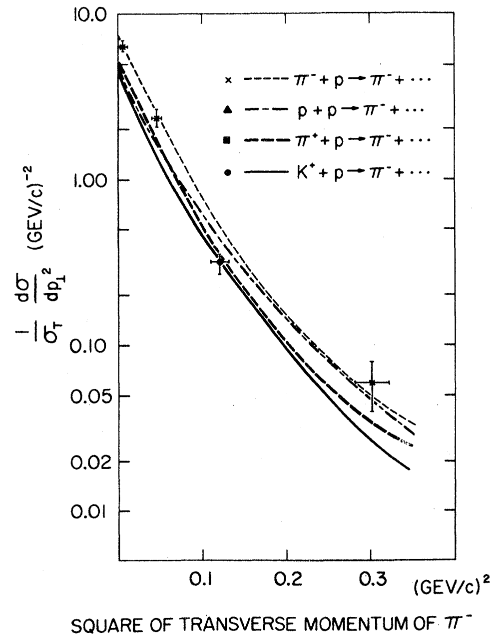

FIG. 2. Transverse-momentum distributions for Reactions (1)-(4) for particles whose longitudinal momentum in the laboratory is less than 0.5 GeV/c.

spectra from the remaining reactions. The normalized production ratios for $P_b < 0.5$ GeV/c are within two standard deviations of each other: 0.036 ± 0.004 for Reaction (1) and 0.028 ± 0.003 for Reaction (5); for $P_b < 1.0$ GeV/c these ratios are 0.090 ± 0.008 and 0.077 ± 0.007, respectively. In the projectile rest frame it is difficult to separate kinematic from dynamic effects since Reactions (2) and (3) involve different projectiles than Reactions (1) and (5). However, the relative agreement between Reactions (1) and (5) in the projectile fragmentation region is also an example of factorization, since in this case the $\pi^\pm$ fragmentation is independent of the s-channel quantum numbers. Furthermore, it is interesting that this agreement occurs in spite of the nonexotic nature of π^-p (and of π^+p according to Ellis *et al.*[11]).

In conclusion, we summarize the results of our investigation of the inclusive reactions (1) through (5). We observe that the cross sections for small longitudinal laboratory momenta (target fragmentation region) in Reactions (1), (2), and (3) are in excellent agreement (within 10%) with each other when they are normalized by the respective total cross sections, while the cross section for Reaction (4) in the same momentum range is ≈ 50% higher. In the projectile-fragmentation region, the normalized cross sections

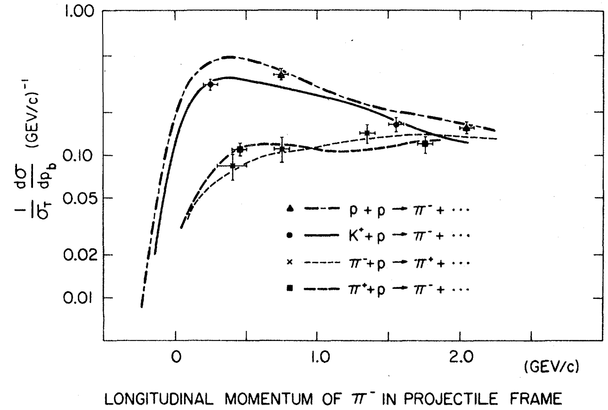

FIG. 3. Longitudinal-momentum distributions in the projectile frame for Reactions (1), (2), (3), and (5).

for Reactions (1) and (5) are also found to be in close agreement. Moreover, if we regard the disparity observed in the target-fragmentation region between Reaction (4) and Reactions (1), (2), and (3) as being due to the nonexotic nature of Reaction (4), then our results may be interpreted as supporting the arguments of Chan and collaborators[10] concerning the relationship between exotic channels and limiting behavior. The requirement that ab must also be exotic for limiting behavior to occur, as suggested by Ellis *et al.*,[11] is not required by our data. Further studies of these and other reactions at sev-

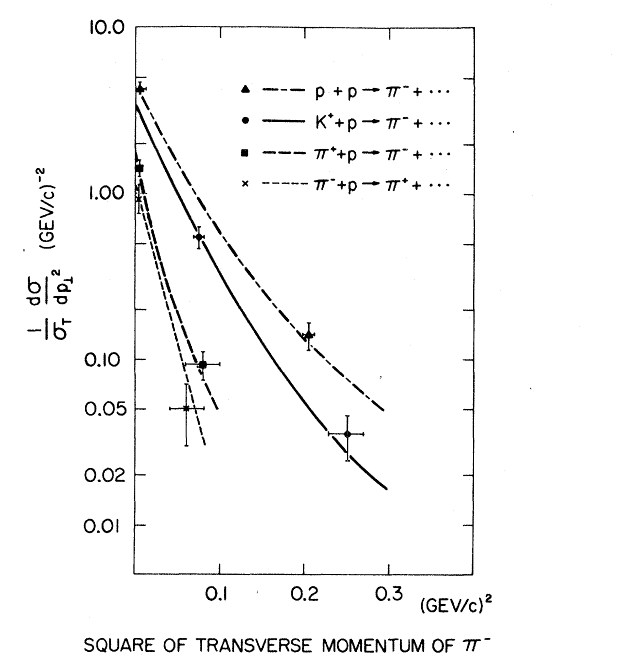

FIG. 4. Transverse-momentum distributions for Reactions (1), (2), (3), and (5) for particles with longitudinal momentum in the projectile frame <0.5 GeV/c.

eral energies must be carried out to establish firmly the role of factorization in inclusive processes, although the agreement among the target fragmentation distributions in Reactions (1), (2), and (3) is already an intriguing result.

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†Alfred P. Sloan Foundation Fellow.

[1]In the reactions studied, nearly all topologies have been measured. To account for the unmeasured topologies, scaling factors have been introduced so that the cross sections represent the inclusive reactions within the quoted errors.

[2]P. Slattery *et al.*, Nuovo Cimento 50A, 377 (1967); S. Stone *et al.*, University of Rochester Report No. UR-875-335 (to be published).

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[5]J. W. Elbert *et al.*, Nucl. Phys. B19, 85 (1970); J. W. Elbert *et al.*, Phys. Rev. D (to be published).

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[7]J. Benecke *et al.*, Phys. Rev. 188, 2159 (1969); see also N. F. Bali *et al.*, Phys. Rev. Lett. 25, 557 (1970); M.-S. Chen, L.-L. Wang, and T. F. Wong, Phys. Rev. Lett. 26, 280 (1971); C. E. DeTar, Phys. Rev. D 3, 128 (1971); H. Abarbanel, Phys. Rev. D (to be published).

[8]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[9]It has been suggested previously that the onset of limiting behavior may be particularly rapid in the target-fragmentation or projectile-fragmentation regions (see Ref. 7).

[10]Chan H.-M. *et al.*, Phys. Rev. Lett. 26, 672 (1971).

[11]J. Ellis *et al.*, CERN Report No. TM-1316 (unpublished).

[12]We use 23.4, 17.4, 39.8, and 24.9 mb as the asymptotic values of the total cross sections for the π^+p, K^+p, pp, and π^-p incident channels, respectively. These values are obtained from a smooth extrapolation from current energies [G. Giacomelli, CERN Report No. CERN/HERA 69-3, 1969 (unpublished)].

[13]The data for Reaction (5) are not given in the laboratory frame because of the experimental uncertainties in the identification of the slow π^+ meson (see Ref. 4).

[14]We note, however, that Reaction (4) is the only reaction in which the selected secondary particle has the same charge as the beam. This means that, for a given multiplicity, Reaction (4) has one additional π^- compared with Reactions (1)-(3); consequently the normalized *total* π^- cross section is expected to be higher. Since the extra π^- might be expected to be a "leading" particle, its contribution should be in the high-momentum part of the spectrum; however, we also observe a disparity in the low-momentum region.

[15]In the projectile frame, the positive direction is defined by the target proton direction. We do not display the data from Reaction (4) in the projectile rest frame, since this inclusive process contains the elastic channel and therefore cannot be compared directly with any of the other reactions. For Reaction (3) we use the laboratory spectra displayed previously instead of the spectra as calculated in the projectile frame. Because of the indistinguishability of the two incident protons, the laboratory spectrum must be identical with the projectile spectrum; we choose to display the laboratory distribution since the experimental resolution is superior for the slower fragments of the target than for the fast fragments of the beam.