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Modulated Electron-Beam Excitation of Low-Frequency Plasma-Cavity Modes*

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Ion heating and relatively large radial, rf electric fields have been observed in an experiment in which an electron beam modulated near the ion plasma frequency excites the resonances of a partially filled plasma cavity. Predictions based on a theoretical self-consistent model for a finite-length beam-plasma system are in good agreement with the experimental observations.

Several reports have been published on observations of plasma ion heating by a modulated electron beam when the frequency of modulation is near the ion plasma frequency. In this paper measurements and calculations are presented which show that the modulated electron beam excites resonances of a partially filled plasma cavity formed by the metal vacuum envelope, and that at the cavity resonant frequencies a relatively large radial, rf electric field is excited in the plasma which produces the observed ion heating. This method of electron-beam excitation of ion oscillations is fundamentally different from other methods[1,2] since it does not require the presence of a strong beam-plasma instability in the frequency range in which rf electric fields are desired. The excitation frequency is determined by the plasma density and geometry, and can be at low frequencies where ion oscillations can be excited. The electron-ion lower hybrid resonance frequency $\omega_{lh}^{-2}=(\omega_{ce}\omega_{ci})^{-1}+(\omega_{pi}^2+\omega_{ci}^2)^{-1}$ is of particular interest because at this frequency the ions oscillate with an average kinetic energy equal to or greater than that of the electrons in the presence of an rf electric field.

The main characteristics of modulated-beam experiments with a beam-generated plasma have been reported by Haas and Dandl[3] for modulation frequencies near ω_{ci} and ω_{pi}. Beeth, Haas, and Eisner[4] have reported measurements on ion heating in nonbeam-generated plasma at frequencies above the ion plasma frequeny. Beam-excited oscillations have been observed when the modulation frequency is near the ion-ion hybrid resonance frequency in a three-component plasma.[5] In work reported earlier, the present authors[6] observed ion heating in a hydrogen plasma at one to three different frequencies immediately above ω_{pi}. In all of these experiments the beam current is modulated by applying an rf modulating voltage to a current-controlling electrode in the electron gun.

The plasma is generated by the electron beam in a metal vacuum system 10 cm in diameter and 61 cm in length measured from the electron gun to the grounded beam collector. The electron beam is produced by a triode electron gun with dc accelerating potentials of 400 to 1000 V. The grid is biased negatively to adjust the average beam current and to insure that sinusoidal current modulation is obtained. Either hydrogen, deuterium, neon, or argon is admitted continuously at a rate that produces pressures in the range of 10^{-4} to 10^{-3} Torr. Beam current and gas pressure are chosen to obtain a beam-generated plasma in the "quiescent" mode[7] with densities of 5×10^8 to 5×10^9 cm^{-3} and a typical electron temperature of 6 eV. The ratio of plasma density to beam density is approximately 25, and the plasma diameter is approximately 6 mm. An axial magnetic field of 300 to 400 G is used. There is a weak magnetic mirror at each end of the system.

The main diagnostic methods used are Langmuir probes with coaxial leads and a gridded retarding-potential probe. Two movable Langmuir probes are used to measure the rf electric field as a function of radius and axial position, and a third probe is used to measure plasma density. The rf probe signal is fed to a matched 100×

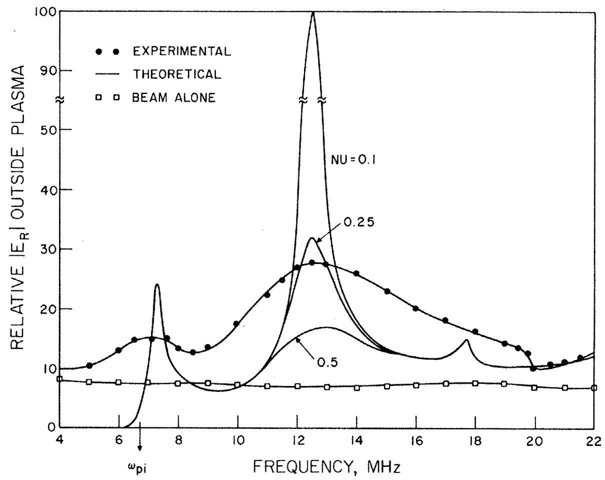

FIG. 1. Radial electric field amplitude at $r=2b$ and $z=0.66L$ as a function of frequency. [Beam voltage $V=600$ V, beam current $I=2.5$ mA, magnetic field $B_0=310$ G, plasma density $n_p=1\times10^9$ cm^{-3}, $L=61$ cm, $b=3$ mm, $NU\equiv\nu/\omega_{pi}(\mathrm{H}^+)$, hydrogen gas.]

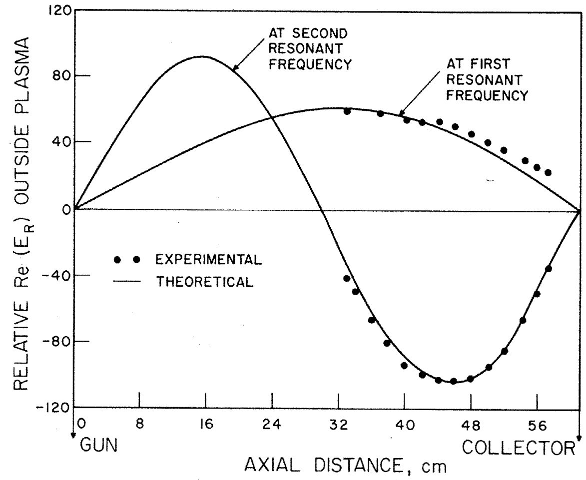

FIG. 2. Real part of the radial electric field at $r=2b$ as a function of axial distance. The parameters are the same as for Fig. 1, with $NU=0.1$.

voltage preamplifier and is either synchronously detected or directly detected with an rf millivoltmeter. The probe frequency response as measured with the modulated electron beam in the absence of a plasma (i.e., at low gas pressure) is shown in Fig. 1.

The excitation of plasma-cavity modes is observed directly by measuring the rf electric field as a function of beam-current modulation frequency, and indirectly by measuring the current carried by energetic ions to the gridded probe as a function of frequency. In these measurements, careful attention is paid to maintaining sinusoidal current modulation with constant rf amplitude at each frequency in the range of 2 to 25 MHz. Two or three peaks are typically observed in the probe response at frequencies that are, in general, not harmonically related. A typical result of these measurements which shows resonances at 7.0, 12.5, and 19.5 MHz is given in Fig. 1.

The gridded probe was located off axis and intercepted mostly energetic ions. Consequently it did not give reliable ion-temperature measurements, but it was useful for detecting the presence of energetic ions at various frequencies or retarding voltages. Measurements of ion current versus retarding potential were made at various modulation frequencies in a hydrogen plasma and indicated that the largest ion-energy spread (up to 35 eV) occurred when the beam was modulated at a resonant frequency.

Interferometric measurements of the radial rf electric field as a function of axial position indicate that at the first two resonant frequencies the cavity is driven in half- and full-wavelength resonances, respectively, as shown in Fig. 2. Comparisons of the phases of the rf fields at two points displaced 180 deg azimuthally showed that the fields were in phase, suggesting that the modes are axisymmetric. It was found that changing the magnetic field strength or mirror ratio had little effect on the resonances.

The beam-plasma system is analyzed as a finite-length, boundary-value problem with a specified driving current. The dispersion equation for the axial propagation constant k_z includes the effect of finite beam and plasma radii, electron-beam space charge, uniform axial magnetic field, and plasma electron-neutral collisions. The plasma is assumed to be cold. The beam and plasma are assumed to have the same radius b, and the metal vacuum-tube radius is assumed to be much larger than b. The quasistatic dispersion equation for perturbations of the form $\exp[i(\omega t-k_z z-m\varphi)]$ in this system is readily available and will not be repeated here.[8] For the axisymmetric ($m=0$) case it is of the form $D(k_z b, \omega)=0$, where D is a transcendental function that admits an infinite number of radial modes, each one of which has six roots of k_z for real ω. Only the lowest-order radial mode is considered. Two of the roots are associated with the electron-beam cyclotron waves and have axial wavelengths given approximately by v_0/f_{ce}. Since the driven electric field was found to have a much larger wavelength, these two roots are neglected. Two of the remaining four roots are associated with

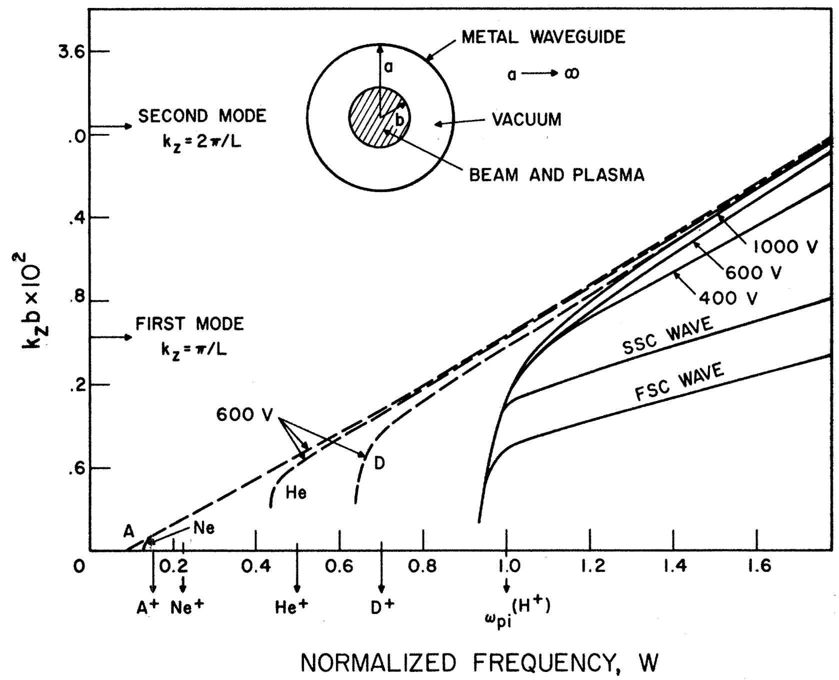

FIG. 3. Effect of ion mass and beam voltage on the dispersion curves of an unfilled beam-plasma waveguide. I, B_0, n_p, L, and b have the same values as in Fig. 1, with $NU=0.1$. $W=\omega/\omega_{pi}(H^+)$, where $\omega_{pi}(H^+)$ is the ion plasma frequency for hydrogen at the given density.

the slow and fast space-charge waves of the electron beam and have a wavelength of approximately v_0/f.[3] The other two roots are approximately the same as the space-charge waves in a plasma column in the absence of the electron beam. A computer program was used to find the roots of k_z for real ω in the frequency range from ω_{ci} to several times ω_{pi}. Figure 3 shows the positive roots as functions of normalized frequency $W = \omega/\omega_{pi}(H^+)$ for five different ion masses, and in the case of H^+ for three beam voltages. Complex roots are present at frequencies above $\omega_{th} \cong \omega_{pi}$, but eventually all four roots become real and separate into distinct beam and plasma waves as shown in Fig. 3 in the normalized frequency range $W>1$. One complex root represents a weak convective instability. When electron-neutral collisions are included, the real roots acquire small imaginary parts. The detailed dispersion in the region $W\cong 1$ in Fig. 3 is complicated and only the real parts of the three positive roots are shown, although the exact roots were used in the normal-mode field calculation. The negative plasma root is essentially a mirror image of the positive root.

The quasistatic potential, the beam-modulation current, and the beam-modulation velocity are expressed as superpositions of the four normal modes of the beam-plasma waveguide, and the following boundary conditions are imposed: (1) zero quasistatic potential at the ends of the system, (2) zero velocity modulation of the electron beam at the electron gun, and (3) a current modulation of the beam given by $J_0 \exp(i\omega t)$ at the gun. The computation proceeds at each frequency by calculating the four values of k_z, the normal-mode complex amplitudes, and the phases and amplitudes of the total electric field, beam current, and beam velocity at an arbitrary position. Typical results of this calculation are shown in Figs. 1 and 2. Figure 1 shows the absolute magnitude of the radial field at a specified axial position as a function of frequency. The chosen axial observation position coincides with that used for the Langmuir probe. Resonances are found at the frequencies 7.0, 12.5, and 17.5 MHz in Fig. 1 for the chosen beam and plasma parameters. The first two resonances have the axial distributions shown in Fig. 2 and are therefore relatively large at the axial position chosen in Fig. 1. The third resonance has an antinode in the radial electric field pattern near the observation position and therefore appears only as a small peak near 17.5 MHz. If the observation point is moved from the antinode position, a large peak occurs in Fig. 1 at this frequency. The quantity $\mathrm{Re}(E_R)$ plotted in Fig. 2 would be the output of a phase-sensitive detector and is in good agreement with the field strength measured by such means as shown in the figure.

The results of the normal-mode analysis are in good agreement with the experiment and can be used to predict the values of the resonant frequencies and their variations with plasma density, beam voltage, and ion mass. The curves for various beam voltages in Fig. 3 for the case of a hydrogen plasma show that the first resonant frequency decreases more slowly than the second as beam voltage is increased, as was observed experimentally. In the experiment ion mass was varied while keeping the beam velocity and dc current constant. Density changed slightly for different gases but was measured for each case. In an experiment with an argon plasma, the first resonant frequency decreased by a factor of 1.13 and the second by a factor of 1.08 compared to the resonant frequencies for a hydrogen plasma with the same density. These changes are much less than the change in ω_{pi} and are in excellent agreement with the changes predicted from Fig. 3 by the intersections of the lines $k_z = \pi/L$ and $k_z = 2\pi/L$ with the 600-V dispersion curves for argon and hydrogen. The independence of the ion mass and the ion heating frequency has been observed in other experiments.[4]

The theoretical variation of radial field strength with frequency as shown in Fig. 1 shows sharper resonances than the experimental curves. Electron-neutral collisions are dominant in the experiment and realistic values of collision frequency give $\nu/\omega_{pi} \cong 0.25$. Curves for $\nu/\omega_{pi} = 0.1$, 0.25, and 0.5 are shown in Fig. 1 at the second resonance. The experimental curve in Fig. 1 has broader resonances probably because of the nonhomogeneous plasma-density profile. The unnormalized field strengths at the first two resonances for the case of $\nu/\omega_{pi} = 0.25$ in Fig. 1 are 35 and 48 V/cm, respectively. Measurements in other experiments have indicated that the electric field strength is of the same order of magnitude.[4]

In summary, a complete field analysis of the beam-plasma, finite-length system is in excellent agreement with the major results of experiments with beam-generated plasmas. The ion heating process suggested by the analysis and experiment is that the electron beam excites resonant modes of the plasma-cavity resonator, and that at these resonances a large radial electric field is created in the plasma which excites ion oscillations. The generation of this large rf field at low frequencies results in the observed ion heating. Evidence of ion heating at the resonant frequencies is obtained with a retarding-potential-energy analyzer. The importance of this result is that the electron beam can transfer energy directly to the plasma ions at frequencies other than those at which there is a strong beam-plasma instability.

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