Model for the Production of Particles with Superstrong Interactions: Estimates of Lower Bounds on the Quark and Monopole Masses

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When particles like quarks and magnetic monopoles are produced in pairs, there is a suppression of the production due to the superstrong attractive forces which exist between the constituents of the pair. We present an approximate way of incorporating this effect into a production model, and estimate lower bounds on the masses of the particles from present experimental data using our method in conjunction with the standard statistical model.

If magnetic monopoles exist and interact strongly with hadrons, one should be able to produce them in very high-energy p-p collisions. Since we believe in macroscopic charge conservation —and do not doubt the principle to hold for magnetic as well as electric charges—we expect to produce monopoles only in combinations which are magnetically neutral, which we shall take to be particle-antiparticle pairs. The Dirac quantization condition[1] $eg/\hbar c = \frac{1}{2}n$ implies that monopoles interact superstrongly among themselves (the strength of the interaction is characterized by $g^2/\hbar c = \frac{137}{4}n^2$). This attractive interaction could cause the monopole and antimonopole to recombine (and annihilate) after being produced but before leaving the region of the p-p collision, thus suppressing the free-particle production rate.

It is the purpose of this paper to discuss a way of handling these superstrong final-state interactions (fsi) which can be readily incorporated into any standard model for particle production.[2] Our aim throughout will be to try, on the basis of a simple model, to extract some general features of monopole and quark production to serve as a guide in the search for these new particles. We shall state clearly what assumptions go into our final result, but shall not attempt a rigorous justification at this time. As will become obvious when results are presented, order-of-magnitude estimates of production cross sections are quite adequate for our purposes.

We begin by dividing the production process into three distinct periods, as shown in Fig. (1). These are as follows: (a) The projectile and target collide in their center-of-momentum frame (referred to hereinafter as the fireball or FB frame) and a heavy particle-antiparticle pair $A\bar{A}$ with superstrong interactions is produced with some amplitude $A(\vec{p}_1, \vec{p}_2)$, where $\vec{p}_1$ and $\vec{p}_2$ are the momenta of the constituents of the pair; (b) the particles which are created now try to escape from each other, but because of the superstrong attraction between them, only a few will escape and enter detectors as free particles; (c) in any case, the particles either escape (upper Fig. 1), or recombine and annihilate (in which case they will not be detected, lower Fig. 1).

Let us now turn our attention to the problem of producing magnetic monopoles. The interaction between A and $\bar{A}$ is extremely strong and they are produced very close together, so we expect that escape will occur only when A and $\bar{A}$ have very high relative energies: Thus we expect that relativistic effects will be important. It is extremely difficult to do a relativistic quantum-mechanical capture calculation in the strong-coupling limit, so we shall make a relativistic *classical* calculation to decide whether A and $\bar{A}$ escape from each other. We make the simplifying assumption that the monopoles are point, on-mass-shell particles that respect the classical Maxwell equations, and that they are created in pairs; but we replace the requirement of local microscopic charge conservation with macroscopic conservation over the interaction volume. We also assume that in the FB frame they are created simultaneously—other time distributions are unilluminating and inconsequential to the re-

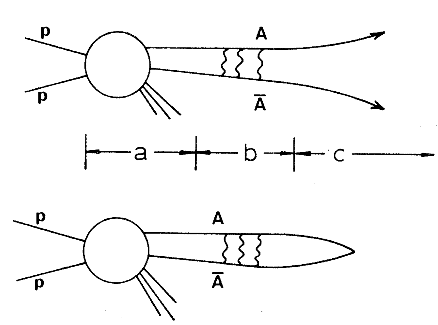

FIG. 1. Schematic representation of the process $p + p \to (A + \bar{A}) + (\text{stuff})$.

sults.

Under these assumptions, any pair $A\bar{A}$ can be completely characterized by the set of vectors $(\vec{p}_1, \vec{p}_2, \vec{r})$ in the FB frame; here $\vec{p}_1$ is the momentum of A, $\vec{p}_2$ is the momentum of $\bar{A}$, and $\vec{r}$ is the distance between A and $\bar{A}$ when they are created. [The actual distribution in $\vec{p}_1$, $\vec{p}_2$, and $\vec{r}$ is determined by the model of part (a) of the overall production process.] It is useful to define an escape function $\theta(\vec{p}_1, \vec{p}_2, \vec{r})$ such that

$$\theta(\vec{p}_1, \vec{p}_2, \vec{r}) = \begin{cases} 1 \text{ when } A \text{ and } \bar{A} \text{ escape each other} \\ 0 \text{ when } A \text{ and } \bar{A} \text{ do not escape each other.} \end{cases} \quad (1)$$

Unfortunately there is no simple closed analytic expression for $\theta(\vec{p}_1, \vec{p}_2, \vec{r})$—we must use numerical methods to solve for the orbits of A and $\bar{A}$ for any given set $(\vec{p}_1, \vec{p}_2, \vec{r})$ and see whether they escape each other or not. We do this by transforming to the center-of-momentum frame of the A-$\bar{A}$ system, and then solve for the orbits of the two particles numerically, taking exact account of retardation effects and the relativistic transformation of fields. The bremsstrahlung radiation emitted by the monopoles while they move in these orbits is calculated[3] and included in the numerical procedure. However, it is found that for almost all the monopoles which escape, very little energy is lost by this mechanism (typically, escaping monopoles will radiate less than 5% of their rest mass), so that, although radiation reaction is included in all results presented in this paper, it does not play an important role. A detailed discussion of this problem will be given in a forthcoming paper.[4]

It remains to choose a model to describe part (a) of the production process and incorporate the escape function θ of part (b) into it. In principle, any standard production model could be used for part (a), as long as it gives information regarding the distributions of $\vec{p}_1$, $\vec{p}_2$, and $\vec{r}$ for $\theta(\vec{p}_1, \vec{p}_2, \vec{r})$. However, because it is especially simple and successful, we will use the well-known Hagedorn statistical model.[5-7] Since we should like to establish lower mass or upper cross-section limits, this particular model is particularly appropriate because it gives the most conservative lower bound on the mass.

It should be noted that the division of the problem into two disjoint parts—one involving production amplitudes [part (a)] and one handling the restriction of the phase-space integrals due to recombination [part (b)]—involves the neglect of certain processes which might be present in nature. To see this, consider the model outlined above, where the production takes place in a fireball. In this model, we imagine the monopoles in the fireball (1) interacting with other magnetically neutral particles (and with each other) *hadronically* to set up the equilibrium, and (2) interacting *magnetically* with each other. The first type of interaction is incorporated in the statistical model which we shall use, and the second is handled approximately as outlined above. What is neglected is the coupling between the two—that is, the change in the way one monopole interacts with the surrounding hadrons (i.e., the way in which it contributes to the hadronic equilibrium) because of its magnetic interaction with the other monopole. The assumption that this coupling is small cannot be rigorously defended, but is can be made plausible by noting that our results are not changed significantly when we allow a large time delay between the time of production of the two constituents of the pair, rather than assuming that they are created simultaneously. A discussion of a similar problem arising in the production of antideuterons is given by Hagedorn.[8]

Particles and resonances—among them the monopole-antimonopole pair $A\bar{A}$ under consideration—coalesce from the fireball in average numbers ("multiplicities") predicted by the statistical model. In particular, it is shown in Ref. 7 that, for particles of mass M so large that the average individual multiplicity is a very small fraction, the average number of *pairs* per fireball having the particle A (mass M) momentum (referred to the FB frame) in a range d^3p_1 around $\vec{p}_1$ and the antiparticle $\bar{A}$ (mass M) momentum in a range d^3p_2 around $\vec{p}_2$ is

$$\nu(\vec{p}_1, \vec{p}_2; m)d^3p_1d^3p_2 = \frac{(\Delta V)^2}{(2\pi)^6} z^2 \exp\left[\frac{-(\vec{p}_1^{\,2}+M^2)^{1/2}-(\vec{p}_2^{\,2}+M^2)^{1/2}}{T_0}\right] d^3p_1d^3p_2. \quad (2)$$

Here the volume ΔV over which thermodynamic equilibrium is established[7] is found empirically to be $\Delta V \simeq 0.4(4\pi/3)m_\pi^{-3}$; the temperature T_0 is found empirically to be $T_0 = 160$ MeV; and $z = 2j + 1$ takes account of the spin states allowed for a particle of spin j. We shall see below that experiments place a lower limit on the order of a few GeV for the monopole or quark mass M. This is sufficiently large that the criterion above (2) is satisfied, and also that the total pair multiplicity $\int \nu(\vec{p}_1, \vec{p}_2; m) d^3p_1 d^3p_2$ is a very small fraction; this justifies our considering only one monopole-antimonopole pair $A\bar{A}$ at a time, as in Fig. (1).

The $\vec{p}_1$ and $\vec{p}_2$ distributions to be used with $\theta(\vec{p}_1, \vec{p}_2, \vec{r})$ can be taken directly from (2); we still must define an appropriate distribution for $\vec{r}$. It is simplest to take the fireball to be a sphere of radius ~1 fm (this is consistent with the ΔV given above) and assume that each monopole (antimonopole) is equally likely to be created at any point within this sphere (this is the only distribution consistent with the statistical picture). We can then write down a probability $P(r)dr$ that the distance between A and $\bar{A}$ is between r and $r + dr$.

It is now a straightforward matter to incorporate the idea of suppression due to fsi (described by θ) into this statistical model. Were it not for the influence of fsi, the average number of A's (or $\bar{A}$'s) emerging from a very high energy p-p collision would be the integral of (2) over all $\vec{p}_1$ and $\vec{p}_2$. However, with fsi present, we must be sure to count only those cases in which $\vec{p}_1$ and $\vec{p}_2$ are such that A and $\bar{A}$ will indeed escape from each other; this is taken care of by multiplying the differential pair multiplicity (2) by the escape function $\theta(\vec{p}_1, \vec{p}_2, \vec{r})$ from (1) and weighting with the probability density $P(r)$ mentioned above. Thus we must evaluate, for a given monopole mass,

$$\nu = \int P(r)\nu(\vec{p}_1, \vec{p}_2; M)\theta(\vec{p}_1, \vec{p}_2, \vec{r}) d^3p_1 d^3p_2 dr. \quad (3)$$

Here ν is the average free-particle multiplicity per fireball, i.e., per p-p collision above kinematic threshold.

In Fig. (2) we have plotted the average free-particle multiplicity ν from (3) as a function of mass for monopoles having magnetic charge such that $eg/\hbar c = \frac{1}{2}n$ with $n = 0$, 1, and 4. The $n = 0$ case corresponds to having no final-state interaction between the monopole and antimonopole; thus it is the prediction of the statistical model in ordinary application. The relation between ν and the ordinary cross section can be obtained by noting that the production cross section for monopoles from a monochromatic beam of very high-energy protons impinging on a proton target can be written

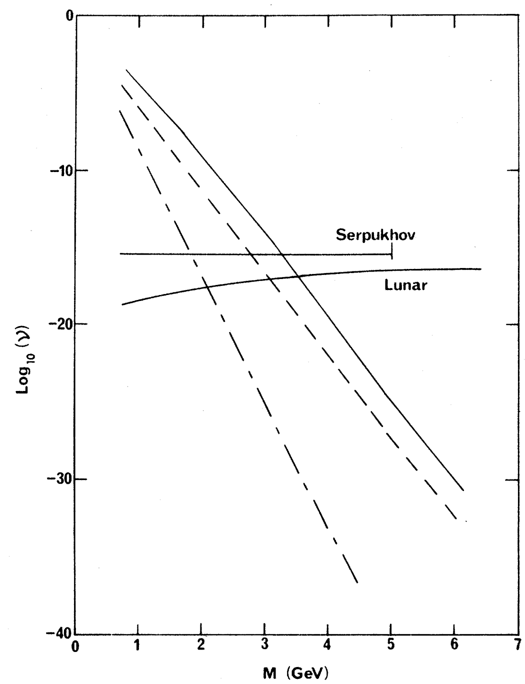

FIG. 2. Calculated free monopole and antimonopole multiplicities per collision, ν, plotted as functions of monopole mass for different fsi couplings: $g = 0$, solid line; $eg/\hbar c = \frac{1}{2}$, dashed line; $eg/\hbar c = 2$, dot-dashed line. Experimental limits are from Refs. 9 and 10.

$$\sigma_{Mp} = \sigma_{pp}\nu, \quad (4)$$

where σ_{pp} is the familiar 38-mb total p-p cross section.

The experimentally observed multiplicity at 95% confidence in two recent (null) monopole experiments is also plotted in Fig. (2).[9,10] The dual interpretation of this graph should be noted: (1) If an experiment of a certain sensitivity is performed and monopoles are not seen, then *either* the mass M of the monopole is large and the statistical factor $\exp(-2M/T_0)$ suppresses the production amplitude, *or* the magnetic charge g is large and the production is suppressed by the fsi. (Of course, if both g and M were large, we would have the worst of all possible worlds.) (2) Because of the suppression of high mass in the statistical model, and because the lunar search experiment already is near the limit of sensitivity for present experimental techniques, it is unlikely that monopoles of mass $\gtrsim$5-7 GeV will be seen directly. For monopoles in this mass range, the fsi introduce a reduction of at least two orders of magnitude in the expected production cross sections.

Finally, we conclude from our graph that lower limits can be placed on the monopole mass by present experimental data. These limits are $M_M \gtrsim 3.25$ GeV for $eg/\hbar c = \frac{1}{2}$, and $M_M \gtrsim 2.25$ GeV for $eg/\hbar c = 2$. A similar calculation for quarks gives $M_Q \gtrsim 1.75$ GeV.

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Anomalous Behavior of $\gamma\gamma \to n\pi^0$ in Effective Lagrangians*

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A recent result of Aviv, Hari Dass, and Sawyer, relating the anomalous value of $\gamma\gamma \to$ (any odd number of pions) to the anomalous rate of $\pi^0 \to 2\gamma$, is demonstrated by conventional graph techniques. It is also shown that the neutral mode $\gamma\gamma \to n\pi^0$ is suppressed for all odd n.

Aviv, Hari Dass, and Sawyer[1] (AHS) have recently shown that the matrix element for $\gamma\gamma \to n\pi$ (where n is odd) at zero pion four-momenta is related to the matrix element for $\pi^0 \to 2\gamma$, and that the anomalous value of the latter[2,3] implies that the former also does not vanish as formal ("naive") chiral symmetry arguments would lead one to believe. They observed that requiring functional derivatives of the Lagrangian $\mathcal{L}$ with respect to external pion fields to transform covariantly under SU(2)⊗SU(2) does not imply that $\mathcal{L}$ itself is a chiral scalar, but rather implies only that the nonscalar part have a particular functional dependence on the pion fields. According to AHS, the part of this noninvariant piece which contains only neutral pion fields must be of the form[4]

$$B(F_{\mu\nu}) \arcsin(2\lambda\pi^0). \tag{1}$$

The pieces containing charged pion fields can be obtained from (1) using isospin arguments.[1] By considering graphs arising directly from (1) alone ("nonpole" terms), together with (1) combined in the usual way with the conventional invariant piece of $\mathcal{L}$ (pole terms or tree graphs), one can calculate the amplitude for $\gamma\gamma \to n\pi^0$. In particular, AHS show that the total $3\pi^0$ amplitude is zero at zero momentum.

Our purpose here is to show how the AHS result may be derived in a more conventional, if less elegant, manner, and then to use these methods to demonstrate that $\gamma\gamma \to n\pi^0$ is forbidden for all $n > 1$. This note is therefore both a gloss on, and an extension of, AHS.

In what follows we several times exploit the remarkable freedom to redefine the pion fields without destroying chiral symmetry, or indeed without altering any on-shell matrix elements at all. In the first place, rather than working with the $\vec{\pi}$ fields used by AHS, it will be instructive to use the more familiar choice in which the covari-

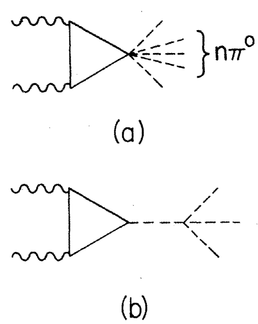

FIG. 1. Examples of (a) nonpole graphs and (b) pole graphs for $\gamma\gamma \to n\pi^0$.