Off-Mass-Shell $K\pi$ Scattering Phase Shifts*

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The final states $K^-\pi^+n$ and $\bar{K}^0\pi^-p$ were measured in a K^-p experiment at 5.5 GeV/c. The data on the angular distribution of the outgoing K meson in the $K\pi$ rest system at low momentum transfer were analyzed by a simple model including $I=\frac{3}{2}$ s-wave and $I=\frac{1}{2}$ s-, p-, and d-wave amplitudes. The results show that the phase shift for the $I=\frac{3}{2}$ s wave is negative. The two solutions obtained for the $I=\frac{1}{2}$ s-wave off-mass-shell phase shift are consistent with an s-wave resonance at either ~0.85 or ~1.2 GeV.

We have studied the $K\pi$ system produced in the reactions,

$$K^-p \to K^-\pi^+n, \quad (1)$$

$$\to \bar{K}^0\pi^-p, \quad (2)$$

with particular emphasis on the $I=\frac{3}{2}$ s-wave phase shift δ_0^3. The magnitude of δ_0^3 has been obtained from cross-section measurements[1] for $K\pi$ scattering with pure $I=\frac{3}{2}$. This Letter describes an analysis of off-mass-shell $K\pi$ scattering in Reactions (1) and (2) which determines the sign of δ_0^3 and is also sensitive to the behavior of the $I=\frac{1}{2}$ s-wave phase shift δ_0^1.

The data come from about 40 000 two-prong events and about 17 000 two-prong-plus-V events[2] obtained using the 30-in. hydrogen bubble chamber exposed to a 5.5-GeV/c K^- beam at the zero-gradient synchrotron. The total number of events selected[3] as examples of Reactions (1) and (2) are 2875 and 2086, respectively.

Figure 1 shows the spherical-harmonic moments $\langle Y_l^0\rangle$ of the $K\pi$-decay angular distribution as a function of $K\pi$ mass from 0.65 to 1.3 GeV for events with $t'=|t-t_{\min}|<0.3$ GeV^2, where t is the four-momentum transfer from the initial to final nucleon and $t_{\min}$ is the kinematical minimum of t.

In comparing the moments of Reaction (1) to those of Reaction (2), we note marked differences for both $\langle Y_1^0\rangle$ and $\langle Y_2^0\rangle$. Since $\langle Y_2^0\rangle$ is expected to be $(5\pi)^{-1/2}$ for pure π exchange and $-0.5(5\pi)^{-1/2}$ for pure vector exchange, the observed negative $\langle Y_2^0\rangle$ moment in the $K^*(890)$ region for Reaction (2) indicates strong vector exchange. The $\langle Y_2^0\rangle$ moment in the $K^*(890)$ region for Reaction (1) is positive but less than $(5\pi)^{-1/2}$, suggesting that absorption effects and/or vector exchange may be important in $K^{*0}(890)$ production. The different $\langle Y_1^0\rangle$ moments suggest that the s- and p-wave interference differs between the two reactions. An $I=\frac{3}{2}$ s-wave component may be important in accounting for this difference.

We parametrize the moments of the two reactions using a simple one-particle-exchange model neglecting absorptive effects. We consider s, p, and d waves for $I=\frac{1}{2}$ and s wave for $I=\frac{3}{2}$, and assume that the s wave $K\pi$ systems involve only π exchange. The dominant p and d waves are put in as the $K^*(890)$ and $K^*(1420)$ resonances, re-

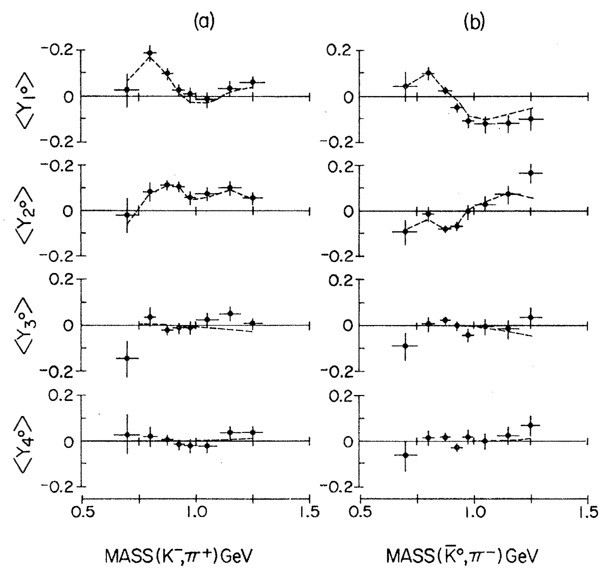

FIG. 1. Spherical harmonic moments $\langle Y_l^0\rangle$ for the outgoing K^- angular distribution in the $K\pi$ rest frame as a function of $K\pi$ mass for momentum transfer $t'=|t-t_{\min}|<0.3$ GeV^2: (a) from Reaction (1), (b) from Reaction (2). The fitted moments for the first solution (see text) are shown by the dashed lines.

spectively. We note that use of at least two reactions with different $K\pi$ charge states is essential for an analysis which includes the $I=\frac{3}{2}$ s wave.

We write the overall matrix elements $M^{\lambda\lambda'}$ in terms of (1) the s-wave π-exchange element $M_\pi^{0,\lambda\lambda'}$; (2) the π-exchange element for $K^*(890)$, $M_\pi^{1,\lambda\lambda'}$; (3) the vector-exchange element for $K^*(890)$, $M_V^{1,\lambda\lambda'}$; and (4) the π-exchange element for $K^*(1420)$, $M_\pi^{2,\lambda\lambda'}$. Then

$$M^{\lambda\lambda'}=M_\pi^{0,\lambda\lambda'}+M_\pi^{1,\lambda\lambda'}+M_V^{1,\lambda\lambda'}+M_\pi^{2,\lambda\lambda'}, \qquad (3)$$

where λ and λ' are the initial- and final-nucleon helicities, and V in the $M_V^{l,\lambda\lambda'}$ stands for ρ exchange for Reaction (1) and for ω exchange for Reaction (2).

$M_{\pi(V)}^{l,\lambda\lambda'}$ can be factorized into a nucleon vertex part $B_{\pi(V)}^{\lambda\lambda'}$ and a meson vertex part $A_{\pi(V)}^{l}$. Then A_π^l and A_V^l respectively represent the scatterings $K\pi\to K\pi$ and $KV\to K\pi$ and can be expressed in terms of off-mass-shell phase shifts δ_l^{2I} which are functions of the $K\pi$ mass m and momentum transfer t. A_π^1 and A_V^1 are given by the same $K^*(890)$ propagator and, similarly, A_π^2 is determined by the $K^*(1420)$ propagator. Thus, the parameters δ_l^1 $(l=1,2)$ are given as

$$\sin(\delta_l^1)\exp(i\delta_l^1)\simeq\frac{m_l\Gamma_l}{m^2-(m_l+\frac{1}{2}i\Gamma_l)^2}\propto A_{\pi(V)}^{l}, \qquad (4)$$

where m_l and Γ_l are the masses and the widths of the $K^*(890)$ and the $K^*(1420)$ for $l=1,2$, respectively. We decompose A_π^l into the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ amplitudes for Reactions (1) and (2) and normalize $M_\pi^{l,\lambda\lambda'}$ such that A_π^l gives the correct $K\pi$ scattering as $t\to-\mu^2$, and we require that

$$\sum_{\lambda\lambda'}|B_V^{\lambda\lambda'}|^2$$

corresponds[5] to the $K^*(890)$ density-matrix element $\rho_{11}-\rho_{1-1}\cos2\varphi$.

Then Eq. (3) is written in terms of the δ_l^{2I} for Reaction (1),

$$M^{\lambda\lambda'}=B_\pi^{\lambda\lambda'}\tfrac{1}{3}\{[\sin(\delta_0^3)\exp(i\delta_0^3)+2\sin(\delta_0^1)\exp(i\delta_0^1)]Y_0^0+2\sqrt{3}\sin(\delta_1^1)\exp(i\delta_1^1)Y_1^0$$
$$+2\gamma\sqrt{5}\sin(\delta_2^1)\exp(i\delta_2^1)Y_2^0\}+\tfrac{2}{3}\sqrt{3}B_\rho^{\lambda\lambda'}\sin(\delta_1^1)\exp(i\delta_1^1)(3/8\pi)^{1/2}\sin\theta; \qquad (5)$$

and for Reaction (2),

$$M^{\lambda\lambda'}=\tfrac{1}{2}\sqrt{2}B_\pi^{\lambda\lambda'}\tfrac{1}{3}\{\sqrt{2}[\sin(\delta_0^3)\exp(i\delta_0^3)-\sin(\delta_0^1)\exp(i\delta_0^1)]Y_0^0-\sqrt{6}\sin(\delta_1^1)\exp(i\delta_1^1)Y_1^0$$
$$-\gamma(10)^{1/2}\sin(\delta_2^1)\exp(i\delta_2^1)Y_2^0\}+\tfrac{1}{3}\sqrt{6}B_\omega^{\lambda\lambda'}\sin(\delta_1^1)\exp(i\delta_1^1)(3/8\pi)^{1/2}\sin\theta; \qquad (6)$$

where γ, the branching ratio of $K^*(1420)$ decay into $K\pi$, is taken to be 0.49. The d-wave amplitudes are approximated as in Eqs. (5) and (6). For a given $K\pi$ mass interval Δm and t' region $\Delta t'$, the $\langle Y_l^0\rangle$ moments are given by

$$\langle Y_l^0\rangle=\int_{\Delta m}dm\int_{\Delta m}dt'\int d\Omega\,\tfrac{1}{2}\sum_{\lambda\lambda'}|M^{\lambda\lambda'}|^2Y_l^0(\theta,\varphi)/\sigma, \qquad (7)$$

where $d\Omega=d\cos\theta d\varphi$ and σ_0, the cross section, is given by

$$\sigma_0=\int_{\Delta m}dm\int_{\Delta t'}dt'\int d\Omega\,\tfrac{1}{2}\sum_{\lambda\lambda'}|M^{\lambda\lambda'}|^2. \qquad (8)$$

Since $B_{\pi(V)}^{\lambda\lambda'}$ and δ_l^{2I} are smooth functions of m and t', $\langle Y_l^0\rangle$ in Eq. (7) can be expressed in terms of $\delta_l^{2I}=\delta_l^{2I}(\overline{m},\overline{t}')$, α_ρ, and α_ω, where $\overline{m}$ and $\overline{t}'$ are representative values of m and t', and the $\alpha_V(V=\rho,\omega)$ are given by

$$\alpha_V=\int_{\Delta m}dm\int_{\Delta t'}dt'\int d\Omega\sum_{\lambda\lambda'}|B_V^{\lambda\lambda'}|^2/D, \qquad (9)$$

with

$$D=2\int_{\Delta m}dm\int_{\Delta t'}dt'\int d\Omega\sum|B_\pi^{\lambda\lambda'}|^2.$$

In these calculations, since absorption is neglected, the π and vector exchange contributions do not interfere,[5] and we have the orthogonality relation

$$\sum_{\lambda\lambda'}B_\pi^{\lambda\lambda'}B_V^{\lambda\lambda'}=0, \qquad (10)$$

which was used to simplify Eqs. (7) and (8).

To study the sign of δ_0^3, we simultaneously fitted the $\langle Y_1^0\rangle$ through $\langle Y_4^0\rangle$ (i.e., a total of eight moments for each $K\pi$ mass bin) to the experimental data of Fig. 2 in the $K^*(890)$ region.[6] The phase shifts δ_1^1 and δ_2^1 were given by Eq. (4). The absolute value of the $I=\frac{3}{2}$ s-wave phase shift of Fig. 2(a) was obtained from the cross-section measurements of Cho *et al.*[1] Two cases were tried: (1) negative δ_0^3 and (2) positive δ_0^3 with three free parameters δ_0^1, α_ρ, and α_ω. For each sign of δ_0^3, we found two sets of solutions. For case (1), both solutions give $\chi^2\sim15$ for 15 degrees of freedom for the three $K\pi$ mass bins between

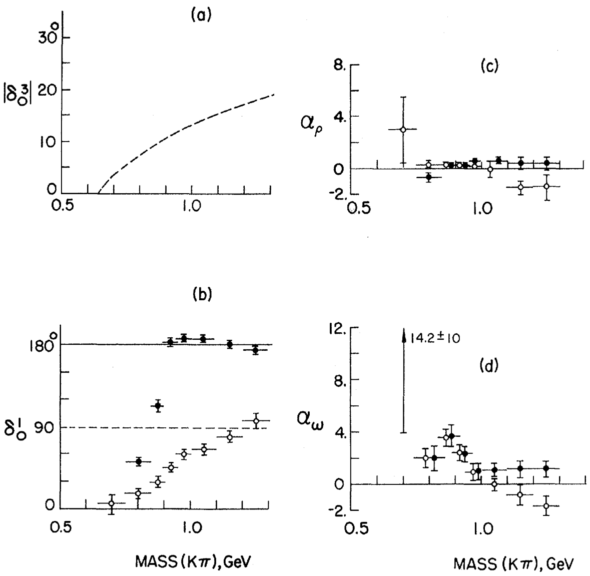

FIG. 2. (a) s-wave $I=\frac{3}{2}$ $K\pi$ phase shift for $K\pi$ mass less than ~1.3 GeV, (b) s-wave $I=\frac{1}{2}$ off-mass-shell phase shift δ_0^1, (c) α_ρ and (d) α_ω as a function of $K\pi$ mass. The open circle and the closed circle represent the first and second solutions, respectively, for $\delta_0^3<0$. The lowest datum point is common to both solutions.

0.85 and 1.0 GeV. In contrast, the fits obtained for case (2) both have $\chi^2 \sim 41$ for the same 15 degrees of freedom. We, therefore, conclude that the sign of δ_0^3 is negative[7] and note that these results are consistent with the theoretical prediction of Lovelace.[8]

For $K\pi$ masses above 1.0 GeV, this model may not hold due to the increasing inelasticity and the $I=\frac{3}{2}$ p-wave amplitude. Nevertheless, to investigate the behavior of δ_0^1, we extended the fits over the $K\pi$ mass interval from 0.65 to 1.3 GeV for the preferred case of $\delta_0^3<0$. Requiring δ_0^1 to vary smoothly from zero at the threshold, we found two sets of solutions for δ_0^1 (except for the lowest $K\pi$ mass bin where there exists only one solution with a large error).

These two solutions for the parameters δ_0^1, α_ρ, and α_ω are shown in Fig. 2 as open circles (first solution) and closed circles (second solution). The fitted $\langle Y_l^0\rangle$ moments for the first solution are shown in Fig. 1 as dashed lines. Although we have unphysically negative α_ρ and α_ω and higher χ^2 for $K\pi$ masses above 1.1 GeV, it is interesting to note that the δ_0^1 for the first solution increases smoothly from the $K^*(890)$ region and crosses 90° at about 1.2 GeV.

Trippe *et al.*[9] were the first to suggest a broad $K\pi$ $I=\frac{1}{2}$ s-wave resonance in this energy region. Further work[10] using a Chew-Low extrapolation method and including an $I=\frac{3}{2}$ wave confirmed a large $I=\frac{1}{2}$ s-wave phase shift, in agreement with the present indications.

The second solution for δ_0^1 rises sharply to cross 90° in the $K^*(890)$ mass region. In this region, no discrimination between the two solutions for δ_0^1 can be made on the basis of χ^2 although at higher masses the first solution is preferred.[11]

Since there is a phase ambiguity of 180°,[12] it is possible that the true solution is some combination of the two solutions found. For example, the second solution (closed circles) could join smoothly at about 1 GeV to the first solution (open circles) shifted by 180°.[13]

The relative intensities of ρ and ω exchange with respect to π exchange are given by $2\alpha_\rho$ and $2\alpha_\omega$. The large amount of ω exchange in the $K^{*-}(890)$ region for Reaction (2) is consistent with established results.[2] The $\bar{K}^{*0}(890)$ is produced mainly through π exchange in Reaction (1). However, the refined absorption model[14] requires some ρ-exchange amplitude to account for the observed difference in the slopes of $d\sigma/dt$ between $K^-p\rightarrow\bar{K}^{*0}n$ and $K^+n\rightarrow K^{*0}p$, which is consistent with our results.

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[1]Y. Cho *et al.*, Phys. Lett. 32B, 409 (1970); A. M. Bakker *et al.*, Nucl. Phys. B24, 211 (1970).

[2]F. Schweingruber *et al.*, Phys. Rev. 166, 1317 (1968). The 40 000 two-prong events in this Letter were measured on POLLY II.

[3]The event identification is discussed in Ref. 2 and results in backgrounds estimated to be about 7% for both reactions.

[4]The moments are defined for N events by

$$\langle Y_l^m\rangle=\sum_{i=1}^{N}[Y_l^m(\theta,\varphi)]_i/N,$$

where θ and φ are the polar and azimuthal angles, respectively, of the outgoing K meson in the $K\pi$ rest frame (Jackson frame). Here the z axis is along the incident K direction and the y axis is along the normal to the production plane, $\vec{n}\propto\vec{p}_{in}\times\vec{K}_{in}$, where $\vec{p}_{in}$ and $\vec{K}_{in}$ are momentum vectors along the incident proton and K-meson directions.

[5]J. D. Jackson and H. Pilkuhn, Nuovo Cimento 33, 906 (1964). $B_V^{\lambda\lambda'}$ depends on the helicity m of the exchanged vector meson. Since we are only interested in

$\langle Y_l^0\rangle$ moments, we omit the superscript m in $B_V^{\lambda\lambda'}$

[6]Since the effect of δ_0^3 appears as the s- and p-wave interference, it is desirable to study the sign of δ_0^3 in the $K\pi$ mass region where both s- and p-wave amplitudes are dominant.

[7]It is most unlikely that the sign of δ_0^3 could change in going from $t \sim 0.3$ GeV^2 to the pion pole.

[8]C. Lovelace, in *Proceedings of a Conference on the $\pi\pi$ and $K\pi$ Interactions at Argonne National Laboratory, 1969,* edited by F. Loeffler and E. Malamud (Argonne National Laboratory, Argonne, Ill., 1969), p. 562.

[9]T. G. Trippe *et al.*, Phys. Lett. 28B, 203 (1968).

[10]P. E. Schlein, in *Proceedings of a Conference on the $\pi\pi$ and $K\pi$ Interactions at Argonne National Laboratory, 1969,* edited by F. Loeffler and E. Malamud (Argonne National Laboratory, Argonne, Ill., 1969), p. 446; P. Antich *et al.*, *ibid.*, p. 508; W. DeBaere *et al.*, CERN Report No. CERN/D. Ph. II/PHYS. 69-17, 1969 (unpublished); P. Herquet and T. Trippe, CERN Report No. CERN/D. Ph. II/PHYS. 70-29, 1970 (unpublished).

[11]For $K\pi$ masses less than 1.1 GeV, both solutions give a χ^2 per degree of freedom of 0.9, whereas for $K\pi$ masses above 1.1 GeV, the χ^2 per degree of freedom is 1.9 for the first solution and 6.8 for the second solution. Since the model is more uncertain at the high masses, the χ^2 test may not be a reliable discriminator.

[12]This ambiguity in δ_0^1 arises because A^I is parametrized as $\sin(\delta_l^{2I}) \exp(i\delta_l^{2I})$.

[13]F. Muller, private communication.

[14]G. Kane, in *Experimental Meson Spectroscopy*, edited by C. Baltay and A. H. Rosenfeld (Columbia U. Press, New York, 1970); G. C. Fox *et al.*, "The Charge Exchange Production Mechanism for $K^*(890)$" (to be published).

Evidence for Splitting in the Q Region of $K^+\pi^+\pi^-$ Mass*

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We observe a splitting in the $K^+\pi^+\pi^-$ mass spectrum in K^+d interactions at 9 GeV/c. We find a mass of 1243 ± 8 MeV/c^2 and a width of 70_{-18}^{+26} for the lower mass state, and a mass of 1344 ± 8 and a width of less than 60 MeV/c^2 for the higher mass state. The isospin of both states is determined to be $\frac{1}{2}$. The results can be interpreted as evidence for the mixing of two $J^P = 1^+$ K^* states.

While one of the principal successes of hadron physics has been the classification of resonances into SU(3) representations, no $J^P = 1^+$ multiplets are as yet well defined. This situation is partially due to the difficulty of separating diffraction-produced[1] resonances from the background processes. Since the background in the Q region [$M(K\pi\pi) < 1400$ MeV/c^2] predominantly has $J^P = 1^+$, it is possible for $J^P = 1^+$ K^* states[2-4] to interfere not only with each other but with the background as well. Goldhaber, Firestone, and Shen, whose 9-GeV/c K^+p experiment presented strong evidence[3] for substructure in the Q region, also proposed a model for interference effects.[5] Subsequent experiments have alternately reported substructure and no substructure in the Q spectrum.

We have chosen to extend the Goldhaber experiment by running at the same beam momentum but using deuterons as targets. Our observed $K^+\pi^+\pi^-$ mass spectrum shows two distinct states upon the broad Q enhancement in both the coherent and the deuteron-breakup channels. Both states are shown to have isospin $I = \frac{1}{2}$, and spin and parity $J^P = 1^+$ is preferred for both.

The data discussed here are from a 4.3-event/μb per nucleon exposure[6] of the Brookhaven National Laboratory 80-in. deuterium bubble chamber to a beam of 9.04-GeV/c K^+ mesons. We have studied the reactions

$$K^+d \to K^+\pi^+\pi^- d \quad (714 \text{ events}) \tag{1}$$

and

$$K^+d \to K^+\pi^+\pi^- pn \quad (2060 \text{ events}) \tag{2}$$

in the four-prong topology where a clear separation of Reactions (1) and (2) is possible.[7,8]