using the branching ratios in M. Russo *et al.*, Phys. Lett. <u>33B</u>, 1 (1970). For further details see G. S. Abrams *et al.*, UCRL Report No. UCRL-20067 (to be published).

<sup>10</sup>For  $K\overline{K}$  decay: We observe  $17 \pm 8$  events in the  $K^+\overline{K}^0$  decay mode of the  $A_2^+$  where the error includes uncertainty in estimating the background level. This when corrected for unseen  $\overline{K}^0$  decays gives the total number of events shown in Table I. For further details see W. R. Butler, UCRL Report No. UCRL-19845, 1970 (unpublished).

<sup>11</sup>See Butler, Ref. 10.

<sup>12</sup>For  $\eta'\pi$  decay: We observe a strong  $\eta$  signal in the missing-mass distribution for Reaction (3) and a clear  $\eta'$  signal in the  $\pi^+\pi^-\eta$  mass distribution for those events in this channel which are fitted by the one-constraint hypothesis  $\pi^+p \rightarrow p\pi^+\pi^+\pi^-\eta$ . We do not however see any compelling evidence for the  $A_2^{+} \rightarrow \eta'\pi^+$  decay, and can only set a rough limit at  $2\pm 2$  events. The corrections for unseen decays allow for the presence of 75% of the  $\pi^0\pi^0\eta$  decay of the  $\eta'$  where  $\eta$  decays to

 $\pi^+\pi^-\pi^0$ , as this fraction of such decays has a  $\pi^0\pi^0\pi^0$  mass within our  $\eta$ -missing-mass limits. For further details see Abrams *et al.*, Ref. 9.

<sup>13</sup>Abrams et al., Ref. 9.

<sup>14</sup>Russo et al., Ref. 9.

<sup>15</sup>K. W. J. Barnham and G. Goldhaber, UCRL Report No. UCRL-20293 (to be published), and in Proceedings of the Caltech Conference on Phenomenology of Particle Physics, March 1971 (to be published).

<sup>16</sup>D. G. Coyne *et al.*, UCRL Report No. UCRL-20088 (to be published).

 ${}^{17}d\sigma/dm \sim (M-M_0)^2 [(M-M_0)^2 + (\Gamma-2)^2]^{-2}$ . We regard this purely as a convenient empirical parametrization of a dip.

<sup>18</sup>D. G. Coyne, EXTRACT, Lawrence Radiation Laboratory Trilling-Goldhaber Group Physics Note No. TG-175, 1969 (unpublished).

<sup>19</sup>The confidence levels in Table II were calculated for the mass region 1220 to 1380 MeV as was done in Refs. 1 and 4. The question of confidence levels is further discussed in Ref. 15.

## Momentum Dependence of the $180^{\circ} \pi p$ Charge-Exchange Cross Section

V. Kistiakowsky, F. A. Triantis, and R. K. Yamamoto Massachusetts Institute of Technology, \* Cambridge, Massachusetts 02139

and

R. D. Klem, P. Marcato<sup>†</sup>, I. A. Pless,<sup>‡</sup> and I. Spirn Argonne National Laboratory, Argonne, Illinois 60439

and

E. F. Anelli, C. N. DeMarzo, and A. Romano Istituto di Fisica dell'Università di Bari, Bari, Italy, and Istituto Nazionale di Fisica Nucleare, Sottosezione di Bari, Bari, Italy and

> D. G. Crabb, A. C. Meyers, III, and J. R. O'Fallon St. Louis University, St. Louis, Missouri 61303 (Received 22 March 1971)

The differential cross sections for  $\pi^- p$  charge exchange at 180° are given as a function of  $\pi^-$  momentum from 1.8 to 6.0 GeV/c. Below 4 GeV/c there is a resonancelike structure. Above 4 GeV/c the data are in excellent agreement with a smooth exponential drop. A modification of the Igi model gives a good fit to the data. The momentum dependence of the moduli of the isospin $-\frac{1}{2}$  and  $-\frac{3}{2}$  amplitudes and their relative phase is presented.

We have measured the differential cross section for pion charge exchange,

 $\pi^- + p \rightarrow \pi^0 + n,$ 

with the  $\pi^0$  produced at approximately 180°, for 52 values of  $\pi^-$  momentum from 1.8 to 6.0 GeV/c. A previous paper<sup>1</sup> reported preliminary values for some of the momenta up to 5.0 GeV/c. This

paper presents our final results which below 4.0 GeV/c display a structure of maxima and minima but above 4.0 GeV/c fall off exponentially. From these and other charge-exchange data and a compilation of  $\pi^{\pm}p$  elastic-scattering data we have calculated the moduli of the isospin- $\frac{1}{2}$ and  $-\frac{3}{2}$  amplitudes and their relative phase.

This experiment was performed in the 17°

negative-pion beam at Argonne National Laboratory. The experimental details and the analysis have been briefly discussed in our previous publication<sup>1,2</sup> and remained unchanged for the completion of the experiment with the following exceptions. (1) Subsequent to the preliminary publication we have experimentally determined the efficiency of our neutron counter to be better than 99% for neutrons that interact in the counter. [98% of all neutrons incident on our detector at  $0^{\circ}$  (lab) interact in the detector.] Therefore, the data which we present in this paper are those for which both a neutron trigger and a  $\pi^0$  trigger (pulses from at least two counters of the  $\pi^0$ hodoscope) were required. Since the emptytarget subtraction is a significantly smaller percentage of the full-target data for which the neutron trigger is required than of the data where it is not required, this results in a reduction of statistical uncertainties. (2) A subsequent experimental study of accidental-coincidence rates disclosed a contribution from accidental coincidences between a  $\pi^0$  produced in the liquid hydrogen from which only one photon triggers a  $\pi^0$  hodoscope counter and a background event not from the liquid hydrogen which also triggers a hodoscope counter. Since this is not removed by an empty-target subtraction, a separate correction was made for this effect. This was approximately 5% at all momenta for the data where a neutron coincidence was required and 10% for those where a neutron coincidence was not required.

As previously mentioned, the presence of false events from sources such as multiple-pion production or accidental coincidences would affect the data where neutron coincidence is required less than those where the neutron coincidence is not required. Therefore, we have calculated the differential cross sections corrected for accidentals for both sets of data at all momenta. The average deviation between the two sets of data is 0.12  $\mu$ b/sr and the  $\chi^2$  is 32.8 for 52 points. The excellent agreement indicates that any residual effects of background or accidentals in our data are much smaller than our statistical uncertainties.

Our results are presented in Fig. 1 as a function of  $\pi^-$  momentum. The cosine of the average center-of-mass angle decreases from -0.9953at 1.8 GeV/c to -0.9972 at 6.0 GeV/c.<sup>1,2</sup> The uncertainties given are statistical. In addition, we estimate that there is a 10% systematic uncertainty in these differential cross sections



FIG. 1. (a) The center-of-mass differential cross section for  $\pi^- p \rightarrow \pi^0 n$  as a function of pion momentum (lab). The solid curve is a five-parameter fit of the Igi model, modified by introducing an imaginary part into all Regge trajectories, to our data. The dashed curve is a fit to our data from 4.0 to 6.0 GeV/c. inclusive, by  $d\sigma/d\Omega = Ae^{-bp} [A = 85.2 \pm 2.1 \ \mu b/sr, b = 0.368]$  $\pm 0.025$  (GeV/c)<sup>-1</sup>,  $\chi^2 = 11$ ]. (b) Compilation of centerof-mass differential cross sections for  $\pi^- p \rightarrow \pi^0 n$ . Plain error bars, this experiment; solid squares, Ref. 3; solid circles, Ref. 4; inverted triangles, Ref. 5; open circles, Ref. 6 ( $\cos\theta \sim 0.999$ ); crosses, Ref. 6  $(\cos\theta \sim 0.995)$ ; open squares, Ref. 7; upright triangles, Ref. 8. The solid curve is the exponential fit to our data and is shown in the insert compared with two differential cross sections for higher momenta.

due mainly to edge effects in the  $\pi^0$  hodoscope. This is an uncertainty in normalization and does not effect the relative values for different  $\pi^$ momenta.

Figure 1(a) shows  $(d\sigma/d\Omega)_{c.m.}$  plotted as a function of  $\pi^-$  momentum. In the momentum region below 4 GeV/c the data display well-defined structure. The minimum at 2.1 GeV/c could be due to the 2190-MeV  $T = \frac{1}{2}$  resonance, but 180° is equivalent to t = -3 (GeV/c)<sup>2</sup> for a  $\pi^-$  momentum 2.1 GeV/c, and so this minimum also corresponds to the dip observed in the charge-exchange t distributions at t = -3 (GeV/c)<sup>2</sup>.<sup>9</sup> There is no evidence for the 2650-MeV  $T = \frac{1}{2}$  resonance at 3.25 GeV/c. The maxima at 2.64 and 3.85 GeV/c correspond to the 2420- and 2850-MeV  $T = \frac{3}{2}$  resonances, but there is no evidence of a maximum at ~5.1 GeV/c such as that observed in  $\pi^{-}p$  elastic-scattering data. In fact, above 4 GeV/c the differential cross section decreases smoothly within the uncertainties. We fitted an exponential  $\left[ \frac{d\sigma}{d\Omega} = Ae^{-bp} \right]$  to our values from 4 to 6 GeV/c inclusive and obtained A = 85.2 $\pm 2.1 \ \mu b/sr$ ,  $b = 0.368 \pm 0.025 \ (GeV/c)^{-1}$ , with  $\chi^2 = 11$  for twenty points, or a confidence level of  $\sim 100\%$ . This exponential fit is shown by the dashed line in Fig. 1(a).

The solid line in Fig. 1(a) shows a fit to all the data of a modification of a dual-resonance model,<sup>10</sup> in which the  $\pi N$  amplitudes are a linear combination of Veneziano terms,<sup>11</sup> one term for each pair of Regge trajectories allowed in the *s*, *t*, and *u* channels. This modification consisted of introducing a common linear imaginary part into all the Regge trajectories<sup>2,12</sup> and resulted in a five-parameter expression which fits the data with  $\chi^2 = 79$  for 52 points. (The point at 1.8 GeV/*c* contributes  $\chi^2 = 21$ ; excluding this, the confidence level is 15%). This fit is seen to represent the data very well except in the regions of the minima at 2.1 and at 3.0-3.5 GeV/*c*.

In Fig. 1(b) we present a compilation of results from other experiments for the differential cross section for charge exchange at angles close to 180° together with the results of this experiment. The agreement is generally within the uncertainties except for the data of Chase *et al.*,<sup>8</sup> which are systematically lower than our data and the data of Schneider *et al.*<sup>6</sup> and Boright *et al.*<sup>7</sup> While determining our neutron-detector efficiency, we measured the detection efficiency of a neutron counter similar to that used by Chase *et al.* This measurement was made with a tagged neutron beam of known dimensions and energy.<sup>2</sup>



FIG. 2. (a) The absolute value of the *s*-channel phase between the  $T = \frac{1}{2}$  and  $T = \frac{3}{2}$  amplitudes from  $\pi^+ p$  elastic and  $\pi^- p$  charge-exchange scattering as a function of pion momentum (lab). (b) The modulus of the *s*-channel  $T = \frac{1}{2}$  amplitude. (c) The modulus of the *s*-channel  $T = \frac{3}{2}$  amplitude. (d)-(f) The corresponding *u*-channel values.

We found that the efficiency for detecting neutrons incident within 2 or 3 in. of the edge of the detector was low for low neutron momentum (2-3 GeV/c), but improved with increasing momentum. This may explain some of the discrepancy between the results of Chase *et al.* and the other experiments cited above.

The *s*- and *u*-channel amplitudes for  $\pi^+ p$  elas-

tic scattering  $(A^+)$ ,  $\pi^- p$  elastic scattering  $(A^-)$ , and charge exchange  $(A^0)$  are linear combinations of the corresponding isospin- $\frac{1}{2}$  and  $-\frac{3}{2}$  amplitudes  $A_{1/2}$  and  $A_{3/2}$ , respectively. One can thus calculate  $|A_{1/2}|$ ,  $|A_{3/2}|$ , and the absolute value of their relative phase  $|\delta|$  from the  $\pi^+ p$  elastic  $(\sigma^+)$ , the  $\pi^- p$  elastic  $(\sigma^-)$ , and charge-exchange  $(\sigma^0)$  differential cross sections at 180°. Compilations of the  $\pi^+ p^{13-25}$  and  $\pi^- p^{18-27}$  elastic-scattering differential cross sections at ~180° were made. From these and the data in Fig. 1(b), the results for  $|\delta|$ ,  $|A_{1/2}|$ , and  $|A_{3/2}|$  presented in Fig. 2 were obtained. The spread on the points corresponds to cross-section values one standard deviation above and below the central values. In regions where there were contradictory data a visual fit was made, and in regions where there were no data, points were estimated by interpolation. The Regge model predicts a constant angle for the *u*-channel value of  $|\delta|$ , but gives two possible values for this angle. If only the nucleon and  $\Delta$  trajectories are considered, this prediction is  $|\delta| \sim 50^\circ$  or  $\sim 130^\circ$ . It is seen in Fig. 2(d) that the *u*-channel value of  $|\delta|$  has several maxima and minima and then above 4 GeV/c appears to decrease slowly. The values in the region from 5 to 6 GeV/c are not inconsistent with the Regge prediction of ~50°. However, the precision of this determination is low, mainly because of the large uncertainties in the  $\pi^+ p$  elastic-scattering data. The *s*-channel value of  $|\delta|$  has a constant value of  $105 \pm 15^{\circ}$  above 3 GeV/c. There are no indications of the 2190and 2650-MeV  $T = \frac{1}{2}$  resonances in the s-channel  $|A_{1/2}|$  values.

We thank the staff of the AD and HEF divisions of Argonne National Laboratory for their invaluable assistance, without which this experiment would not have been possible. \*Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT (30-1) 2098.

<sup>†</sup>Present address: Massachusetts Institute of Technology, Cambridge, Mass. 02139.

<sup>‡</sup>Permanent address: Massachusetts Institute of Technology, Cambridge, Mass. 02139.

<sup>1</sup>V. Kistiakowsky *et al.*, Phys. Rev. Lett. <u>22</u>, 618 (1969).

<sup>2</sup>A detailed discussion of this experiment will be submitted for publication elsewhere. Tabular data available on request.

<sup>3</sup>F. Bulos et al., Phys. Rev. Lett. 13, 558 (1964).

<sup>4</sup>H. R. Crouch et al., Phys. Rev. Lett. 21, 849 (1968).

<sup>5</sup>V. D. Antopolsky *et al.*, Phys. Lett. 28B, 223 (1968).

<sup>6</sup>J. Schneider *et al.*, Phys. Rev. Lett. 23, 1068 (1969).

<sup>7</sup>J. P. Boright et al., Phys. Rev. Lett. 24, 964 (1970).

<sup>8</sup>R. C. Chase et al., Phys. Rev. D 2, 2588 (1970).

<sup>9</sup>B. Brabson *et al.*, to be published.

<sup>10</sup>K. Igi, Phys. Lett. <u>28B</u>, 330 (1968).

<sup>11</sup>G. Veneziano, Nuovo Cimento <u>57A</u>, 190 (1968).

<sup>12</sup>Thus the Regge trajectories have the form  $\alpha(s) = \alpha_0 + \alpha_1 s + i\beta s$ .

<sup>13</sup>A. S. Carroll *et al.*, Phys. Rev. Lett. 20, 607 (1968).
<sup>14</sup>T. Dobrowolski *et al.*, Phys. Lett. <u>24B</u>, 203 (1967).
<sup>15</sup>I. A. Savin *et al.*, Phys. Lett. <u>17</u>, 68 (1965).

<sup>16</sup>Aachen-Berlin-Birmingham-Bonn-Hamburg-London

(I.C.)-München Collaboration, Phys. Lett. <u>10</u>, 248 (1964). <sup>17</sup>V. D. Antopolsky *et al.*, Phys. Lett. <u>28B</u>, 220 (1968).

<sup>18</sup>W. F. Baker *et al.*, Phys. Lett. 23, 605 (1966), and 25B, 361 (1967).

<sup>19</sup>P. J. Duke et al., Phys. Rev. <u>149</u>, 1077 (1966).

<sup>20</sup>A. Ashmore *et al.*, Phys. Rev. Lett. <u>19</u>, 460 (1967). <sup>21</sup>W. R. Frisken *et al.*, Phys. Rev. Lett. <u>15</u>, 313 (1965).

<sup>22</sup>J. Orear et al., Phys. Rev. <u>152</u>, 1162 (1966).

<sup>23</sup>P. M. Ogden et al., Phys. Rev. <u>137</u>, B1115 (1965).

<sup>24</sup>J. A. Helland *et al.*, Phys. Rev. <u>134</u>, B1062, B1079 (1964).

<sup>25</sup>D. P. Owen et al., Phys. Rev. 181, 1794 (1969).

<sup>26</sup>S. W. Kormanyos, A. D. Krisch, J. R. O'Fallon, and K. Ruddick, Phys. Rev. Lett. <u>16</u>, 709 (1966), and Phys. Rev. 164, 1661 (1967).

<sup>27</sup>E. W. Anderson *et al.*, Phys. Rev. Lett. <u>20</u>, 1529 (1968).