ratio completely decouples ω and f from the nucleon for $\Delta\lambda = 1$. The first ratio predicts a 3:1 ratio between the $\Delta\lambda = 0$ couplings of f or ω and ρ or A_2 to the nucleon. For a discussion see, e.g., R. Odorico *et al.*, Phys. Lett. 32B, 375 (1970); C. Michael and R. Odorico, to be published.

[16]This discussion is consistent with the usual pole-model description of these amplitudes. See, e.g., M. Krammer and U. Maor, Nucl. Phys. B13, 651 (1969).

[17]Since the ρNN coupling involves some nonflip contribution, our qualitative statements in cases (c), (d), and (f) are actually weaker than our other predictions. Other problems in these processes are the possible contributions of exchanges other than ρ and the observed deviation from the $\alpha = \frac{1}{2} + t$ effective trajectory. Only a complete s-channel helicity analysis of these processes can give a complete picture. Until the data allow such an analysis, we must regard our conclusions on these reactions as tentative.

Can One See Isobars in the Deuteron?*

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The reaction $\pi d \rightarrow N\Delta$ is studied as a probe of the $\Delta\Delta$ content of the deuteron. Assuming a Δ-exchange mechanism, the cross section is expected to be 10–100 μb at 1 GeV for an estimated probability of about 1% for the $\Delta\Delta$ configuration.

Until recently, most analyses of nuclear phenomena ignored the internal structure of the nucleon and the possibility of its excitation. A few recent papers[1-3] have shown that inclusion of configurations containing an excited nucleon (isobar) can improve agreement between theory and experiment; in particular, the interpretation[1] of the backward peak in p-d elastic scattering at ~1 GeV in terms of the currently fashionable Reggeized nucleon-exchange model implies the presence of $N^*(1688)$, the $\frac{5}{2}^+$ Regge recurrence of the nucleon, in the deuteron with a probability of about 1%. It would be useful, nonetheless, to have *direct* evidence for presence of nulceon isobars in nuclei. In this note, we discuss reactions which we expect to be unimportant if the occurrence of $\Delta(1236)$ in the deuteron is disregarded.

The forward production of protons in $\pi^- d$ collisions requires exchange of baryon number as well as two units of charge. Since the lowest known state possessing these quantum numbers is $\Delta^{++}(1236)$, the simplest mechanism for forward proton production,

$$\pi^- + d \rightarrow p + X^-, \tag{1}$$

would be via Δ^{++} exchange, and we expect its contribution to be very important at pion energies above a few hundred MeV. If quasi two-body processes predominate, as in most high-energy reactions, we should expect reaction (1) to proceed mainly through

$$\pi^- + d \rightarrow p + \Delta^-. \tag{2}$$

This could be verified by momentum analysis of the proton in reaction (1). Under the hypothesis that Δ^{++} exchange [Fig. 1(a)] dominates reaction (2), the forward differential cross section directly measures a certain average of squared $d\Delta\Delta$ form factors. If spins could be disregarded, this would just be the probability of finding the target deuteron in the configuration $\Delta^-(r)\Delta^{++}(q)$, where $\Delta^-(r)$ is a "physical" Δ of four-momentum r while the off-mass-shell Δ^{++} has energy $m_d - r_0$ and three-momentum $-\vec{r}$ in the deuteron rest frame (laboratory system).

The "mirror" reaction

$$\pi^+ + d \rightarrow n + \Delta^{++} \tag{3}$$

is required on the basis of charge symmetry to have the same cross section as (2), if electromagnetic mass differences are ignored, as will be done hereafter. The related reactions $\pi^- + d \rightarrow n + \Delta^0$ and $\pi^+ + d \rightarrow p + \Delta^+$ only require single charge exchange, but if isospin invariance is assumed the simplest mechanisms for these in-

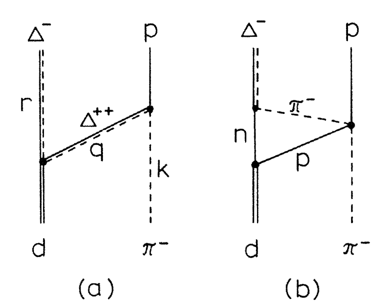

FIG. 1. (a) The Δ-exchange contribution to $\pi^- d \rightarrow p\Delta^-$; (b) triangle mechanism.

volve Δ^+ and Δ^0 exchange, respectively; if all other contributions are neglected, their cross sections are $\frac{1}{3}$ of the cross sections calculated in the corresponding approximation for (2) and (3).

It will therefore suffice to calculate the cross section for one of the cases, e.g., (2). If we define as usual $t=(d-r)^2$, $u=(d-p)^2$, and $s=(d+k)^2 = m_d^2+\mu^2+2m_dE_\pi$, then the pole corresponding to Δ exchange is at $t=M^2$, where we take Δ to have the well-defined mass[4] $M=1236$ MeV. For forward proton emission, $t=t_{\max}$, the distance to the u-channel nucleon pole at $u=m^2$ increases almost linearly with s as the energy is increased above threshold while the distance to the Δ pole remains almost constant, increasing slowly from $m_d(2M-m_d)$ to M^2 as $s\to\infty$. The contribution of nucleon exchange is further strongly suppressed by the rapid decrease of the deuteron form factor with increasing momentum transfer. One may also hope that the neglect of initial- and final-state interactions, represented in our calculation by the disregard of all interaction mechanisms other than the one depicted in Fig. 1(a), may be justified at sufficiently high energy.[5]

The matrix element corresponding to Fig. 1 may be written as

$$M=\sum_\Lambda\langle p\nu|G|q\Lambda\rangle\langle q\Lambda;r\lambda|F|dm_z\rangle(t-M^2)^{-1}, \quad (4)$$

where F and G are the interaction operators at the deuteron and pion vertices, while m_z, λ, Λ, and ν are the helicities of the deuteron, Δ^-, exchanged Δ^{++}, and outgoing proton, respectively. If we work in the deuteron rest frame and take m_z to be the projection of the deuteron spin along $\vec{q}$, the summation over Λ reduces to the single term $\Lambda=m_z+\lambda$: There is no interference between contributions from different values of Λ. Taking account of reflection invariance, there are five independent helicity amplitudes for the deuteron vertex, which we may choose as $A_1=a_0(\frac{3}{2};\frac{3}{2})$, $A_2=a_0(\frac{1}{2};\frac{1}{2})$, $A_3=a_1(\frac{1}{2};-\frac{1}{2})$, $A_4=a_1(\frac{3}{2};\frac{1}{2})$, and $A_5 = a_1(-\frac{1}{2};-\frac{3}{2})$, where

$$a_m(\Lambda;\lambda)=\langle q\Lambda;r\lambda|F|m\rangle. \quad (5)$$

These vertex functions contain all accessible information about the $\Delta\Delta$ content of the deuteron and are linearly related to the amplitudes of $\Delta\Delta$ components in the relativistic wave function of the deuteron. When $r^2=q^2$, A_4 and A_5 must be equal because of the equivalence of the Δ's (under the assumption of isospin invariance). The four independent vertex functions then correspond to the ^3S, ^3D, ^7D, and ^7G wave functions which are obtained in a nonrelativistic treatment.[6] The fifth function arises when the two Δ's are unequally off the mass shell, i.e., it depends on the "relative" energy variable (which is linearly related to t), and corresponds to an $S=2$ component in the $\Delta\Delta$ wavefunction. In a nonrelativistic treatment, where the concept of relative time does not enter, there can be no explicit dependence on the "relative energy" and there are only four independent vertex functions.

The pion vertex is taken to have the usual form of the $\Delta N\pi$ coupling:

$$G=g\bar{u}(p)k_\alpha U^\alpha(q), \quad (6)$$

where $\bar{u}(p)$ is the Dirac spinor representing the outgoing proton and $U^\alpha(q)$ is a Rarita-Schwinger spinor representing the Δ. The coupling constant g is fixed by the $\Delta\to N\pi$ width. There is no unambigous definition of what constitutes an off-mass-shell Δ; for definiteness, we choose to represent the exchanged Δ (with helicity Λ) by

$$U_\Lambda^\alpha(q)=\sum_{\lambda_1,\lambda_2}C(1,\lambda_1;\tfrac{1}{2},\lambda_2|\tfrac{3}{2},\Lambda)\epsilon_{\lambda_1}^\alpha(q)u_{\lambda_2}(q), \quad (7)$$

where

$$\epsilon_\pm=(0;\mp i,1,0)/\sqrt{2}, \quad \epsilon_0=(|\vec{q}|;0,9,q_0)/M,$$

and $u_{\lambda_2}(q)$ is a positive-energy Dirac spinor. With these prescriptions, we find

$$\sum_{m,\lambda,\nu}|M|^2=\frac{g^2}{(t-M^2)^2}\sum_{m,\lambda}|\langle q,(m+\lambda);r\lambda|F|m\rangle|^2[\textstyle\sum_\nu|\langle p\nu|G|q,(m+\lambda)\rangle|^2]$$

$$=\frac{2g^2}{(t-M^2)^2}[(|A_1|^2+|A_4|^2)\Sigma_{3/2}+(|A_2|^2+|A_3|^2+|A_5|^2)\Sigma_{1/2}], \quad (8)$$

where

$$\Sigma_\Lambda=\textstyle\sum_\nu|\langle p\nu|G|q\Lambda\rangle|^2=Z\sigma_\Lambda, \quad (9)$$

with

$$Z=\frac{(q_0+M)}{2M}\frac{(p_0+m)}{2m}\left[1+\frac{\vec{q}^2\vec{p}^2}{(q_0+M)^2(p_0+m)^2}+\frac{2|\vec{q}||\vec{p}|}{(q_0+M)(p_0+m)}\cos\theta_{pr}\right], \quad (9a)$$

and

$$\sigma_\Lambda = \tfrac{1}{2}\vec{p}^2 \sin^2\theta_{pr}, \quad \text{for } |\Lambda| = \tfrac{3}{2}, \tag{9b}$$

$$\sigma_\Lambda = (2/3M^2)(p_0|\vec{q}| + |\vec{p}|q_0 \cos\theta_{pr})^2 + \tfrac{1}{6}\vec{p}^2 \sin^2\theta_{pr}, \quad \text{for } |\Lambda| = \tfrac{1}{2}. \tag{9c}$$

The differential cross section on an unpolarized deuteron target is then given by

$$\frac{d\sigma}{dt} = \frac{mM^3}{3\pi[s-(m_d+\mu)^2][s-(m_d-\mu)^2]}\sum |M|^2. \tag{10}$$

We estimated the deuteron vertex functions (5) by calculating the $\Delta\Delta$ amplitudes generated in perturbation theory by one-pion exchange[8] [through the interaction (6) acting in second order] from the NN amplitudes in the deuteron. Since the usual description in nuclear physics of the NN amplitudes is given by a Schrödinger wavefunction φ_0, the same description was adopted for the $\Delta\Delta$ amplitudes. We therefore took the momentum-space amplitudes, at $|\vec{q}| = [(m_d+\mu)^2-t]^{1/2} \times [(m_d-\mu)^2-t]^{1/2}/2m_d$, of the $\Delta\Delta$ components of the deuteron wavefunction to be given by

$$\tilde{\psi}(\vec{q}) = -\frac{M}{(\vec{q}^2+MB')}\int\frac{d^3k}{(2\pi)^3}V(\vec{q}-\vec{k})\tilde{\varphi}_0(\vec{k}), \tag{11}$$

with $B' = 2M - m_d$. The $NN \to \Delta\Delta$ transition potential is taken to be

$$V(\vec{k}) = \sqrt{2}g^2(\vec{k}^2+\mu^2)^{-1}(\vec{S}_1\cdot\vec{k})(\vec{S}_2\cdot\vec{k}), \tag{12}$$

where the spin-transition operators $\vec{S}$ are extensions[8] of the Pauli spin operators, represented by 4×2 matrices, such that $S_{+1}\chi_{1/2,1/2} = \chi_{3/2,3/2}$. In our approximation, the various components of $\tilde{\psi}(\vec{q})$ in the helicity representation are, up to normalization factors, exactly the terms $(t-M^2)^{-1}A_i$ which enter Eq. (4). For the q values of interest, the vertex functions A_1 and $A_4 = A_5$ are found to be significantly smaller than A_2 and A_3. This can be understood simply as follows. If we consider $NN \to \Delta\Delta$ transitions from two nucleons initially at rest, the exchange of a spin-0 pion cannot transmit any helicity along $\vec{q}$; in that case, therefore, the interaction (12) only generates $\Delta\Delta$ amplitudes for helicity values of $\pm\frac{1}{2}$. Since the characteristic momentum of nucleons in the deuteron [of order $(mB)^{1/2}$] is small compared to q, we are close to this situation, and $\Delta\Delta$ states in which either isobar has helicity $\pm\frac{3}{2}$ are only weakly excited. The components of $\tilde{\psi}(\vec{q})$ in the more conventional LS representation are shown in Fig. 2, where the initial deuteron wave function $\tilde{\varphi}_0(\vec{k})$ was taken to have the four-pole parametric form given by Le Bellac, Renard, and Tran Thanh Van.[9] The one-pion exchange interaction (12) is not expected to provide a realistic description of the $NN \to \Delta\Delta$ interaction for $|\vec{k}| \gg \mu$; we therefore suppress its contribution at high momentum transfers by multiplying $V(\vec{k})$ by the regulating factor $\Lambda^2/(\Lambda^2+\vec{k}^2)$ with Λ arbitrarily chosen as 500 MeV. With these assumptions, the probability[10] of the $\Delta\Delta$ configuration, viz.,

$$P_\Delta = (2\pi)^{-3}\int d^3q\,|\tilde{\psi}(\vec{q})|^2,$$

is found to be ~1.5%, a value similar to the estimate of Arenhövel, Danos, and Williams.[6]

The differential cross sections for reaction (2) corresponding to these values of $\tilde{\psi}(\vec{q})$ are shown in Fig. 3 when the spinor u in Eq. (7) is taken to be the mass-shell spinor corresponding to spatial momentum $-\vec{r}$, the prescription which seems to us to correspond most closely to a nonrelativistic treatment. The ambiguity in the treatment of the exchanged Δ is probably the chief source of uncertainty in our calculation, which otherwise contains essentially no free parameters. Had we, for example, used the actual value of q_0 in Eq.

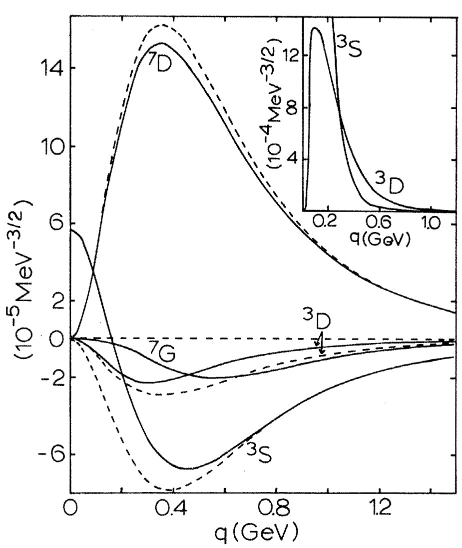

FIG. 2. Wave functions in momentum space for the $\Delta\Delta$ configuration of the deuteron. The full curves are calculated from the deuteron wave function φ_0 of Ref. 9 with $\beta = 14\alpha$, $\gamma = 11.13\alpha$, and $\delta = 8.55\alpha$, which is shown in the inset. The broken curves are obtained when the D-state component of φ_0 is omitted.

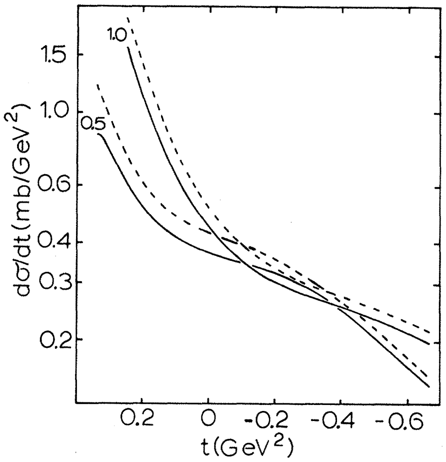

FIG. 3. The differential cross section for $\pi^- d \to p\Delta^-$ for pion lab energies 0.5 and 1.0 GeV. The solid and dashed curves have the same significance as in Fig. 2.

(9) instead of the value $(M^2 + \vec{r}^2)^{1/2}$ of our preferred prescription, we should have found differential cross sections which were reduced by about an order of magnitude and an angular distribution which had its maximum away from the forward direction. We do not know of any theoretical argument to resolve this ambiguity; the calculated predictions shown in Fig. 3 must therefore be regarded as a qualitative indication of the order of magnitude of the cross sections to be expected.

Another feature of our calculation should also be mentioned: Equation (10) predicts a cross section which increases with increasing pion energy. This is a well-known disease[11] of peripheral-model calculations involving higher-spin particles, and can be cured by treating the exchanged Δ as a Reggeon[12] rather than as a particle of fixed spin $\frac{3}{2}$. The differential cross section is then expected to decrease asymptotically[13] with energy as a power s^{-N}, where the exponent N increases approximately linearly with t. The forward differential cross section at 10 GeV is then expected to be considerably smaller than at 1 GeV.

To conclude, we estimate a cross section of 10-100 μb for the reaction $\pi^- d \to p\Delta^-$ at an energy of about 1 GeV if the probability of the $\Delta\Delta$ configuration in the deuteron is about 1%, as we expect from a simple model. It seems to us, therefore, that the study of the reaction (2), and related $\pi d \to N\Delta$ reactions, may be useful as a means to detect the presence of Δ's in the deuteron.

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[2]H. Arenhövel and M. Danos, Phys. Lett. 28B, 299 (1968).

[3]L. S. Kisslinger, Phys. Lett. 29B, 211 (1969).

[4]The neglect of the Δ width should influence our estimates by on the order of 10%. We do not aim for such precision in this note.

[5]For the triangle mechanism [Fig. 1(b)], at energies well above threshold, the π^--p vertex will be dominated by Δ^{++} exchange. This effect is included in our treatment of Fig. 1(a).

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[7]W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).

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[9]M. Le Bellac, F. Renard, and J. Tran Thanh Van, Nuovo Cimento 33, 594 (1964).

[10]Strictly speaking, this is the probability of $\Delta\Delta$ configurations with relative momentum not much greater than 500 MeV.

[11]See, e.g., H. Brody *et al.*, Phys. Rev. Lett. 16, 828 (1966).

[12]A possible objection to such a simple remedy arises from the fact that no shrinkage is observed in $\pi^- p$ backward scattering with increasing energy. E. L. Berger and G. C. Fox, Nucl. Phys. B26, 1 (1971).

[13]If we take this behavior to be the same as in $\pi^- p$ backward scattering (Berger and Fox, Ref. 12), which is also dominated by Δ exchange, then $N \approx 3-2t/\mathrm{GeV}^2$.