limit $\sigma \ll k'c$. For typical conductors the *opposite* inequality always holds. While this difference has no effect on their conclusions, it is crucial for us. Their result implies V and A_t vanish for $\sigma \to \infty$. We would be unable to satisfy our boundary conditions if this were actually the case. [Their Eq. (5) for $\vec{E}$ is incorrect.] For the infinite-conductivity case, see also L. Bass and E. Schrödinger, Proc. Roy. Soc., Ser. A 232, 1 (1955).

[7]For σ finite, $\vec{j}$ must be finite. Hence there are no surface currents. On the other hand, for $\vec{j}\cdot\hat{n} \neq 0$ there will be surface charges. Hence $\hat{n}\cdot\nabla V$ may be discontinuous. Not all of the specified boundary conditions are independent when one takes the equations of motion and the Lorentz condition into account.

[8]The following "thin-perfect-conductor" boundary conditions also avoid a multitude of surfaces. Free-space equations of motion are assumed on both sides of the boundary. Only V and A are required to be continuous [$\hat{n}\cdot\nabla(\hat{n}\cdot\vec{A})$ is then automatically continuous]. In addition, one requires $\hat{n}\times\vec{E}$ to vanish at the boundary. All of the "thick"-perfect-conductor problems discussed in the text have also been treated as thin-conductor problems. The results for the interior fields and eigenvalues are always similar and, to first order in κ^2, identical.

[9]Combining this comment with our discussion of waveguides we see that for rectangular cavity modes with mode indices (l,m,n), Eq. (13) is inexact, but probably an excellent approximation for $(l,m,n) \neq 0$. If, however, any of these indices vanish the frequency is essentially independent of κ^2.

[10]H. Kendall (private communication) has suggested that the fact that the earth-ionosphere resonance has been observed sets some sort of limit on κ^2. It is probable that Eq. (13) has substantial corrections for the lowest mode of two concentric spheres but not so large as to eliminate the possibility of a useful determination. A manuscript on this effect is in preparation.

New Method of Search for Low-Frequency Gravitational Waves*

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It is proposed to search for low-frequency components of gravitational waves by attempting to detect intensity fluctuations of light that has traveled over transmission paths crossed by gravitational waves.

Ever since Weber's pioneering papers[1] the detection of gravitational waves has been a major objective of astrophysical experimental research. The existence of gravitational waves sufficiently intense to cause the signals observed by Weber would have profound astrophysical and cosmological implications. Weber's instrumentation is suitable for the detection of waves at frequencies of 10^3-10^4 Hz. The intensity near the high end of this range drops off drastically,[2] whereas below 10^3 Hz resonant structures will be too bulky, or else their sensitivity to incoming gravitational waves will suffer badly. The square of the ratio between the effective speed of sound within the structure and the speed of light in free space $(v/c)^2$ is a measure for the mismatch between the free gravitational wave and the resulting elastic vibrations of the detector.

Alternative methods of detection of low-frequency waves involve either the motions of free extraterrestrial bodies or the effects of gravitational fields on the transmission of electromagnetic waves through free space. For instance, Anderson[3] has reported residual motions of Mariner deep-space probes, which may represent the direct effect of passing gravitational waves on these space craft. Optical observations of distant astronomical objects may reveal fluctuations in the index of refraction of the transmission path caused by gravitational waves passing anywhere between object and observer.[4] If the distance is sufficiently large, a number of uncorrelated gravitational wave trains may be crossing the transmission path at the same time.

The theory of transmission paths with randomly fluctuating index of refraction has been treated by a number of authors in various contexts.[5,6] Earlier theoretical results, which had been based on geometrical optics, have been extended to take into account the wavelength of the transmitted radiation.[7] The purpose of this note is to suggest that observational techniques based on transmission path fluctuations deserve serious consideration. The argument is twofold: The superposition of many uncorrelated fluctuations over a sufficiently long path may build up an observable cumulative effect even when the disturbance caused by one gravitational wave train is below the threshold of detectability; and these methods are eminently suited to frequencies several decades below the range accessible to the resonant elastic detectors used by Weber.

Random fluctuations of the index of refraction are responsible for fluctuations of the time of transmission (optical path length), of the apparent location of the source (wave-front normal), and of the observed intensity of radiation. If the medium is statistically homogeneous and if the length of the transmission path is large compared to the coherence length of the deviations of the index of refraction from its mean, then the rms fluctuations in arrival time and in apparent location of the source increase with the square root of the length of transmission path. Aside from numerical factors, the variance in optical path length $\langle\sigma^2\rangle$ is given by

$$\langle\sigma^2\rangle \sim lL\langle h^2\rangle, \tag{1}$$

where L is the length of the transmission path, l the coherence length, and $\langle h^2\rangle$ the variance of the index of refraction. For the gradient of the eikonal, projected into the plane tangential to the unperturbed wave fronts, the variance is

$$\langle\nabla\sigma^2\rangle - \langle(\vec{n}\cdot\nabla\sigma)^2\rangle \sim (L/l)\langle h^2\rangle. \tag{2}$$

Intensity fluctuations obey a different type of law. The presence of a patch of altered index of refraction does not change the intensity of passing light but the derivative of the intensity in the direction of the path.[5] As a result the variance of the logarithm of the intensity (i.e., stellar magnitude) varies with path length as

$$\langle\alpha^2\rangle \sim (L/l)^3\langle h^2\rangle. \tag{3}$$

Depending on the critical ratio (L/l), and on the order of magnitude of the dimensionless variance $\langle\hbar^2\rangle$, $\langle\alpha^2\rangle$ in particular might reach the threshold of detectability.

To observe changes in the transmission path length, one would require an accurately periodic signal to emanate from the source, either a very narrow spectral line or the flashes from a pulsar. With either signal, the desired effect might be contaminated by irregularities of the source, scintillations of atmospheric origin, and interstellar plasma effects. Atmospheric scintillations can be eliminated by the use of detectors beyond the Earth's atmosphere. Plasma effects should be unimportant at visible light frequencies, and at radio frequencies can be identified by correlating fluctuations at two different frequencies. As for irregularities lying within the source of light, if signals received at two widely separated stations are correlated, the degree of correlation should drop if the two transmission paths are for the greater part more distant from each other than one correlation length of the gravitational waves. As the latter quantity is unknown, this technique involves not only considerable instrumental complications but the separation requirements are uncertain.

Intensity fluctuations may also be contaminated by atmospheric and plasma effects. These two sources of background may be dealt with as in the case of fluctuating optical path lengths. Fluctuations of the light source itself may be largely avoided by the selection of sources believed to be stable in this respect; correlation studies of the intensity fluctuations of several independent sources close to each other will also help. Assuming, however, that the stability of the sources may be taken for granted, correlation between distinct astronomical objects, whether their mutual proximity is real or apparent, will help to determine coherence length and power spectrum of gravitational waves. Such investigations will be in order once optical fluctuations have been observed and definitely traced to gravitational waves.

In view of the uncertain nature of the sources of gravitational radiation, the assignment of orders of magnitude of the effects to be observed is far from straightforward. Instead of basing such estimates on assumed mechanisms, one might extrapolate the orders of magnitude reported by Weber[1] and by Anderson.[3] Weber has estimated several hundred independent pulses of gravitational radiation per year. If one were to assume that such pulses crisscross our galaxy but are unimportant in intergalactic space, one would choose as one's optical targets objects within the galaxy, 10^3-10^5 light-years away. Even if each pulse were coherent within itself, the ratio (L/l) would lie between 10^6 and 10^8; the value would be higher if the individual pulses were incoherent.

Weber[1] has estimated $\langle h^2\rangle^{1/2}$ to be at least 10^{-16}, whereas Anderson[3] has claimed an order of magnitude of 10^{-11} in the frequency range investigated by him. If all these estimates are entered into the expression (3) for intensity fluctuations, one arrives at estimates for $\langle\alpha^2\rangle^{1/2}$ from as low as 10^{-7} to as high as 10^{+1}. The further pursuit of the search proposal sketched in this paper, and the design of a possible experiment, would seem to depend on one's degree of confidence in the realism of the higher estimate. The fact that the ratio (L/l) enters into Eq. (3) with a much higher power than into Eqs. (1) and (2) would in any case appear to make an examination of intensity fluctuations attractive, in spite of low intrinsic accuracy.

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Dual Absorptive Model for Dips in Inelastic Hadron Processes*

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We assume that the imaginary part of any inelastic hadronic amplitude is dominated by the peripheral ($l\sim qr$) resonances, and that the same imaginary part can also be described by a combination of t-channel poles and cuts. The strength of the required cut term is determined by whether or not the pole term itself is already peripheral. The real part has no reason to be peripheral and can be determined easily from the peripheral imaginary part only when the cuts happen to be relatively weak. These assumptions lead to a successful qualitative description of all $|t|\sim 0.6$-BeV^2 dip effects in vector- and tensor-exchange inelastic and elastic reactions.

In the absence of a theory of hadronic interactions, many phenomenological models have been proposed[1-4] for the observed behavior of inelastic hadronic reactions. The rise and fall of these models was often related to their ability or inability to explain the apparent erratic behavior of dips in inelastic differential cross sections. The presence of $|t|\sim 0.6$-BeV^2 dips in $\pi^-p\to\pi^0n$ and $\gamma p\to\pi^0 p$ or their absence in $\pi^+n\to\omega p$ and $\pi^-p\to\eta n$ are just a few examples of this puzzling behavior. Every one of these effects has been properly explained in some of the models, but every one of the models has failed to explain some of the effects.[5]

In this paper we present a simple dual absorptive scheme which accounts for the systematic pattern of these dips. Our model, which already has been applied to elastic scattering,[6] is still qualitative, but we feel that its overall success is sufficient to encourage the pursuit of a detailed quantitative analysis. We hope to report on such an analysis in the near future.

The starting point of our model is the recognition that the t-channel description of an inelastic hadronic amplitude $f(s,t)$ must involve Regge poles as well as cuts and that the combination of these poles and cuts is dual to the s-channel resonances. *These resonances dominate* Im$f(s,t)$ *in a local way*—namely, at a given value of s, Im$f(s,t)$ is dominated by resonances of mass $m\sim s^{1/2}$. On the other hand, Re$f(s,t)$ *is not locally controlled by the nearby resonances.*[7] It is actually fed by distant resonances, including those with $s<0$ (u-channel resonances).

Any t-channel description would tend to predict that structures in the angular distribution will occur (if at all) at approximately fixed values of t at all energies. This is supported by the data. How can the s-channel description of $f(s,t)$ reproduce such an effect? This can happen only if strong correlations exist between the different s-channel resonances. The simplest (but not the only[8]) way to guarantee a fixed, energy independent, t value for a given structure (dip, bump, etc.) is to demand that *every single prominent resonance will possess this structure.*[9] The sum of all resonance contributions will then automatically exhibit the same structure in t at any given energy. The condition that has to be obeyed by all prominent resonances in order to insure this behavior is $l\propto s^{1/2}$, where l is the spin of a resonance and $m=s^{1/2}$ is its mass.[7] Since Im$f(s,t)$ is locally dominated by the resonances, we conclude that at any value of s, the