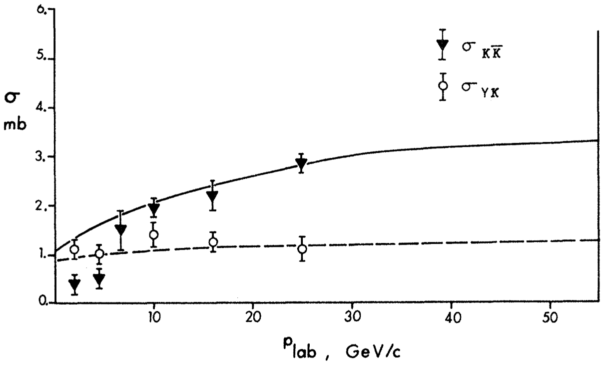

FIG. 2. The partial cross section $\sigma_{K\bar{K}}$ (σ_{YK}) for π^-+p interactions is graphed against laboratory momenta. The unbroken (dashed) line is the one-parameter fit described in the text. The data for σ_{YK} have been obtained by adding the two cross sections for $\sigma_{Y^{\pm}K}$ and σ_{Y^0K} given in Ref. 5.

cluding strange mesons in the kernel will be to boost the leading intercept by approximately 0.1 (for $\delta = 0$). This boost will be larger (smaller) for $\delta > 0$ (<0).

We thank Dr. K. C. Wali and Dr. D. Majumdar for helpful discussions.

*Work supported by the U. S. Atomic Energy Commission.

[1]G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968), and Phys. Rev. Lett. 20, 1078 (1968).

[2]Chan H.-M., J. Loskiewicz, and W. W. Allison, Nuovo Cimento 57, 93 (1968); L. Caneschi and A. Pignotti, Phys. Rev. Lett. 22, 1219 (1969); D. Silverman and C.-I Tan, Phys. Rev. D 2, 233 (1970); C. DeTar, Phys. Rev. D 3, 128 (1971); L. Caneschi and A. Schwimmer, Phys. Lett. 33B, 577 (1970).

[3]F. Turkot, in CERN Report No. 68-7, 1968 (unpublished), Vol. 1, p. 316.

[4]R. Amann, to be published.

[5]J. W. Waters, W. D. Walker, A. R. Erwin, and J. W. Elbert, Nucl. Phys. B17, 445 (1970).

[6]J. W. Elbert, A. R. Erwin, W. D. Walker, and J. W. Waters, Nucl. Phys. B19, 85 (1970). Here our complete neglect of the charges of the produced particles prevents detailed comparison.

[7]Y. Fujimoto and S. Hayakawa, in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1967); H. Pilkuhn, Nucl. Phys. B4, 439.

[8]D. M. Tow, Phys. Rev. D 2, 154 (1970); G. F. Chew and D. R. Snider, Phys. Rev. D 1, 3453 (1970); G. F. Chew, T. Rogers, and D. R. Snider, Phys. Rev. D 2, 765 (1970).

Sixth-Order Magnetic Moment of the Electron

M. J. Levine*
Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 51213

and

Jon Wright†
Physics Department, University of Illinois, Urbana, Illinois 61801
(Received 12 April 1971)

A calculation of the sixth-order anomalous magnetic moment of the electron is presented. We obtain $a_e = \frac{1}{2}\alpha/\pi - 0.32847(\alpha/\pi)^2 + 1.49(\alpha/\pi)^3$ which is in good agreement with the present experimental answer.

Recently an experiment[1] to measure the anomalous magnetic moment of the electron was performed with sufficient accuracy to justify a theoretical calculation. Several groups have published partial calculations,[2,6] and we present here the result for the sum of all remaining graphs. Our theoretical answer for the anomalous magnetic moment through sixth order is

$$a_e^{\text{theor}} = \tfrac{1}{2}\alpha/\pi - 0.328\,479(\alpha/\pi)^2 + 1.49(\alpha/\pi)^3$$

which is to be compared with the present experimental answer[1] of

$$a_e^{\text{exp}} = \tfrac{1}{2}\alpha/\pi - 0.328\,479(\alpha/\pi)^2 + (1.68 \pm 0.33)(\alpha/\pi)^3.$$

For a review of the present status of quantum electrodynamics see the review by Brodsky and Drell.[7] We use $\alpha^{-1} = 137.036\,08(26)$.[8]

There are 72 graphs for the sixth-order magnetic moment of the electron, of which 40 are distinct. They are shown in Figs. 1 and 2. Of the forty graphs, the twelve involving fermion loops (Fig. 1) have been previously calculated. Seven of these have second-order vacuum-polar-

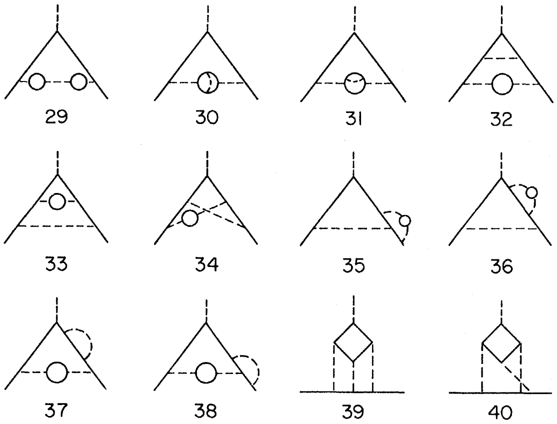

FIG. 1. The twelve graphs involving fermion loops.

ization insertions,[2,3] three have fourth-order vacuum-polarization insertions,[2,4] and two have light-light–scattering insertions.[5] There are 28 distinct graphs with no fermion loops, only 3 of which have been previously calculated.[6]

In our work we took advantage of the fact that some contributions canceled among the various graphs and we therefore report here only the answer for the sum of graphs 1-28 shown in Fig. 2. Graphs 1-3 were also calculated in Ref. 6. The appropriate factors of 2 are of course included for the nonsymmetric graphs. In a future paper we will present results for individual graphs and give more details of our methods.

The method we use to evaluate the graphs[9] is straightforward. Almost all of the work is done by the computer. Feynman parameters are introduced in the usual way and the computer then does all of the necessary Dirac algebra and collecting of like monomials in the Feynman parameters using a program developed by one of us.[10] The integrals over the Feynman parameters are on an eight-dimensional simplex which we map onto a seven-dimensional hypercube. In some diagrams there is a trivial simplification to reduce the integration region to a five- or six-dimensional hypercube. After renormalization and removal of infrared divergences the integrals are all finite although the integrand has integrable singularities on the faces of the cube. We experimentally find a polynomial mapping to remove these singularities and make the integrand relatively smooth.

At this stage we have a choice of integration schemes, and we chose to do straightforward numerical integration using Gaussian weights and points in each variable. This is to be contrasted with the use of C. Sheppey's routine by Brodsky[2,3,5] and co-workers, which is an adaptive

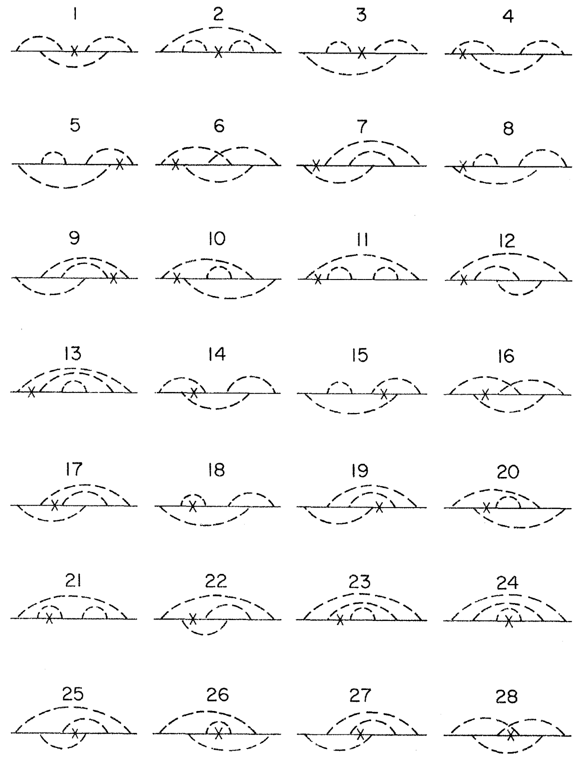

FIG. 2. The 28 graphs calculated in this paper.

Monte Carlo method that puts more integration points where they are needed. We have not tried their method extensively. About half of our points are in regions where the integrand was very small. However, Gaussian integration normally works much better for reasonable functions if accurate answers are needed. We made several runs varying the points in each dimension to test for convergence. As a typical example, we show the results for the sum of graphs 16 and 28 in Table I. We estimate our error for the individual graphs to be on the order of 0.02 and the error for the sum to be <0.2. As an additional test of convergence, we have summed the results of our numerical integrations for individual graphs for various meshes. These sums are 1.54, 1.64, 1.67, 1.77, and 1.75, with the number of mesh points increasing by more than a factor of 10 from first to last. Those contributions which we have computed exactly are not included in these sums. The large percentage fluctuations and estimated total error result from the 90% cancelation between graphs. The result for the sixth-order magnetic moment of the electron is

$$a_e^{(6)} = [(1.23 \pm 0.2) + (0.0554) - (0.154 \pm 0.009) + (0.36 \pm 0.04)](\alpha/\pi)^3,$$

where the first term is our answer for graphs 1-

Table I. Convergence of the integral for increasing number of integration points for graph 28 plus 2×graph 16. The number in column one is half of that obtained from column two because we only integrate over half of the hypercube. The nonuniform number of points for some of the runs represents an "educated guess" as to the best way to distribute the points.

Approximate number of integration points	Number of points in each dimension on hypercube	Coefficient of $-(\alpha/\pi)^3$
64	2, 2, 2, 2, 2, 2, 2	4.6
10^3	3, 3, 3, 3, 3, 3, 3	1.3
8×10^3	4, 4, 4, 4, 4, 4, 4	1.63
4×10^4	5, 5, 5, 5, 5, 5, 5	1.58
1.4×10^5	6, 6, 6, 6, 6, 6, 6	1.75
2×10^5	5, 5, 5, 7, 10, 7, 7	1.85
4×10^5	7, 7, 7, 7, 7, 7, 7	1.78
7×10^5	6, 6, 6, 8, 12, 8, 8	1.807
1×10^6	8, 8, 8, 8, 8, 8, 8	1.788
1.7×10^6	7, 7, 7, 9, 14, 9, 9	1.786

28, the second number is for graphs 29-31,[2,4] the third is for graphs 32-38,[2,3] and the fourth is for graphs 39 and 40.[5]

Our calculation also contributes a small correction to the magnetic moment of the muon. The sixth-order muon contribution can be written

$$a_\mu^{(6)} = a_e^{(6)} + (a_\mu^{(6)} - a_e^{(6)}).$$

The contributions to $a_\mu^{(6)} - a_e^{(6)}$ have been calculated by several groups.[2,11] They give $a_\mu^{(6)} - a_e^{(6)} = (20.3 \pm 1.1)(\alpha/\pi)^3$.

One of us (M.J.L.) would like to acknowledge helpful discussions with J. S. R. Chisholm, A. Petermann, C. Sheppey, N. Kroll, S. Brodsky, and R. Cutkosky.

---

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AEC AT(30-1) 882.

†Work supported in part by the National Science Foundation under Grant No. NSF GP 13671.

[1]The experimental number is $a_2 = (1\,159\,657.7 \pm 3.5) \times 10^{-9} = \frac{1}{2}\alpha/\pi - 0.328\,48(\alpha/\pi)^2 + (1.68 \pm 0.33)(\alpha/\pi)^3$; J. Wesley and A. Rich, private communication. See also their earlier paper: J. C. Wesley and A. Rich, Phys. Rev. Lett. 24, 1320 (1970).

[2]S. J. Brodsky and T. Kinoshita, to be published.

[3]J. Calmet and M. Perrottet, to be published.

[4]J. A. Mignaco and E. Remiddi, Nuovo Cimento 60A, 519 (1969).

[5]J. Aldins, T. Kinoshita, S. J. Brodsky, and A. Dufner, Phys. Rev. Lett. 23, 441 (1969), and Phys. Rev. D 1, 2378 (1970).

[6]A. De Rujula, B. Lautrup, and A. Peterman, Phys. Lett. 33B, 605 (1970).

[7]S. J. Brodsky and S. D. Drell, Annu. Rev. Nucl. Sci. 20, 147 (1970).

[8]B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969).

[9]See for example J. S. R. Chisholm, Proc. Cambridge Phil. Soc. 48, 300 (1952).

[10]M. J. Levine, J. Comput. Phys. 1, 454 (1967).

[11]B. Lautrup, Phys. Lett. 32B, 627 (1970).