molecular internal field F if the third-order polarizability has already been evaluated by another method (optical Kerr effect).[7]

We tried to study benzene and tetraethylene glycol under the above conditions. In both cases, important luminescence signals of intensity proportional to the square of the incident intensity (two-photon absorption of the fundamental wave) were recorded at 4000 and 4400 Å, as already noted by Maker[8] in the case of benzene. They invalidate our scattering measurements for these two compounds. An experimental setup, now being assembled, will allow the use of an inductive wavelength of 1.06 μm. In avoiding the two-photon absorption of the fundamental wave, we hope to open the door to the study of benzene and other aromatic centrosymmetric derivatives.

---

*On leave from Institute of Physics, University of Poznan, Poznan, Poland. Present address: University of Bordeaux, 33 Talence, France.

[1]R. W. Terhune, P. D. Maker, and C. M. Savage, Phys. Rev. Lett. 14, 681 (1965).

[2]S. Kielich, Acta Phys. Pol. 33, 89 (1968), and IEEE J. Quantum Electron. 4, 744 (1968).

[3]R. Bershon, Y.-H. Pao, and H. L. Frisch, J. Chem. Phys. 45, 3184 (1966).

[4]S. Kielich, Acta Phys. Pol. 26, 135 (1964).

[5]S. Kielich, Chem. Phys. Lett. 7, 347 (1970).

[6]J. R. Lalanne and J. J. Piaud, to be published.

[7]S. Kielich, J. R. Lalanne, and F. B. Martin, C. R. Acad. Sci. (Paris), Ser. B 272, 731 (1971).

[8]P D. Maker, Phys. Rev. A 1, 923 (1970).

Effect of a Toroidal Field on Plasma Transport in a Magnetically Perturbed Multipole*

T. Jernigan, J. Rudmin, and D. M. Meade
Plasma Studies, University of Wisconsin, Madison, Wisconsin 53706
(Received 5 April 1971)

Field errors have been observed to cause anomalous loss to an internal hoop and produce convection patterns which have potentials $\sim kT_e/e$. A simple model of plasma flowing along field lines accounts semiquantitatively for the observed anomalous loss and the production of convection patterns. The addition of a small (5%) toroidal field reduces the anomalous loss by a factor of 60 to the level given by classical ion-neutral diffusion across the field.

Toroidal multipoles derive many of their favorable confinement properties directly from axial symmetry. However in actual experiments, all of the magnetic fields are slightly asymmetric. It is therefore important to determine the allowable level for magnetic field asymmetries in confinement experiments. The addition of small magnetic field errors to a shearless axisymmetric multipole magnetic field can cause the field lines to spiral across the entire confinement region since the shearless multipole magnetic field has no structural stability.[1] Previous experiments[2] with controlled field errors have shown that small (1%) externally applied field errors can increase the plasma loss to the internal hoop of a multipole-like device by over an order of magnitude. The plasma losses were increased when the magnetic field errors were arranged to cause field lines to cross the entire confinement region and the increased losses were negligible when the perturbation was oriented to keep the field lines closed. Reductions in plasma loss to the internal hoop by a factor of 60 have been observed with the addition of a small (5%) toroidal field. Azimuthal variations in plasma density have been produced by the addition of field errors and then subsequently reduced by the addition of a toroidal field. The sensitivity of plasma confinement in multipoles to field errors has been demonstrated[3] and may be responsible for the anomalous loss of plasma to the internal hoops of other devices.[4-6]

The apparatus consists of an internal hoop energized with dc current and an external set of coils used to guide the externally produced plasma near the internal hoop in an azimuthally symmetric manner (Fig. 1). A hydrogen plasma is continuously produced at one end of the machine by electron cyclotron breakdown in a Lisitano coil. This arrangement simulates that found in a multipole device, such as an octupole, near one of the internal conductors. Dipole coils situated outside the vacuum tank on the plane of symmetry produced the error magnetic fields. The toroidal field B_θ was produced by a current I_z flowing along the symmetry axis of the device.

The plasma loss to the internal hoop as a function of the toroidal field and applied error field

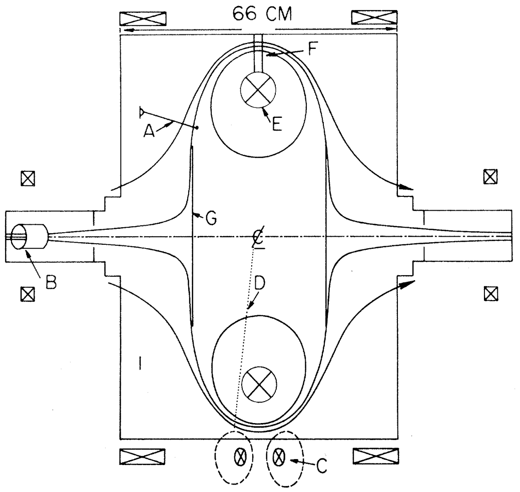

FIG. 1. Schematic view of the apparatus showing the rotating probe A, the Lisitano coil B, dipole perturbation C, path of a probe scan behind the hoop D, the internal hoop E, the bifilar current feed to the hoop F, and the magnetic field separatrix G. For these experiments, the internal hoop current was 17 000 A turns and the maximum dipole perturbation current was 6000 A turns.

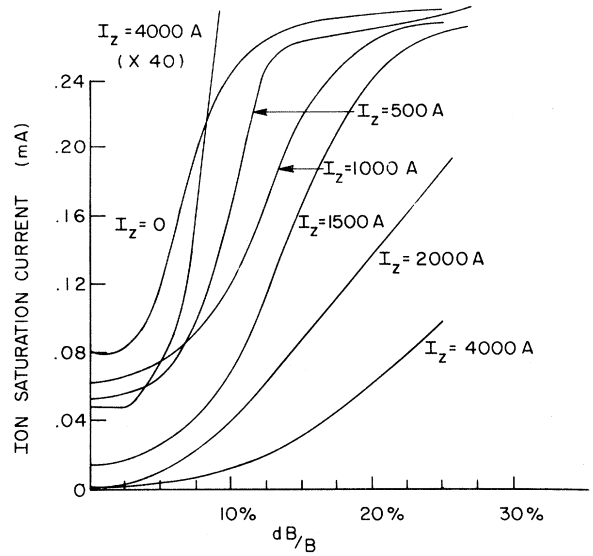

FIG. 2. Ion saturation current to the internal hoop collector as a function of the dipole error field. The term $\delta B/B$ is the ratio of the dipole error field to the multipole field evaluated at the separatrix between the hoop and the outer wall. This is the peak value of the ratio. The ratio of the error field to the multipole field averaged over a multipole field line is reduced by a factor of 20 from the peak value while $\delta B/B$ averaged over the entire machine is reduced by a factor of 200 from the peak value.

is shown in Fig. 2. The curves shown have been normalized by use of the internal hoop supports as a monitor probe. The curves for $I_z = 2000$ and 1500 A have been multiplied by 4 and 3, respectively, while the other curves in Fig. 2 required less than a 10% change due to normalization. With $I_z = 4000$ A, there is a factor of 60 reduction in loss from the case with $I_z = 0$. The toroidal field required to reduce the plasma loss to half is ≈ 0.1 of the maximum dipole error field at the separatrix behind the hoop.

The radial density profiles for no external error are similar with and without B_θ. At 16% dipole error the radial density profile was displaced towards the hoop. With the addition of B_θ ($I_z = 4000$ A), the plasma near the hoop is reduced simultaneously with the drop in plasma loss to the hoop. In earlier multipole experiments[3,5,7,8] anomalous loss of plasma across the field lines was generally accompanied by azimuthal density and potential variations (convective cells). The addition of B_θ ($I_z = 5000$ A) reduced an error-induced azimuthal density gradient by one order of magnitude.

The trajectories of magnetic field lines under the influence of a dipole field error are shown in Fig. 3. For the case $\delta B/B = 10\,\%$ and $B_\theta = 0$, the field lines are seen to move directly from the separatrix to the internal hoop. When a small 5% B_θ ($I_z = 4000$ A) is added, the trajectories of field lines change drastically and are no longer able to cross the confinement region directly under the perturbation, but instead are moved in a small step Δ in passing by the perturbation. Calculations following the field line around the device azimuthally several times indicate that the net radial excursion is limited to about $\Delta/100$. The theory of magnetic field stability indicates that the excursions are limited in a sheared field and can no longer cross the entire confinement region. The radial flux of particles of the jth species due to classical diffusion perpendicular and parallel to the perturbed field lines is given[2] by

$$\varphi_j^* = -D_{c\perp j}\left[1+\left\langle\frac{\lambda_j \delta B}{\rho_j B_p}\right\rangle^2\right]\nabla_\perp n + n\mu_{c\perp j}\left[1+\left\langle\frac{\lambda_j \delta B}{\rho_j B_p}\right\rangle^2\right]E_\perp, \qquad (1)$$

where $D_{c\perp}$ and $\mu_{c\perp}$ are the classical diffusion and mobility perpendicular to the field lines, respectively, λ is the mean free path, ρ is the gyroradius, and $\langle \delta B/B_p \rangle$ is the radial error field over the unperturbed poloidal field B_p averaged once around the unperturbed field. In these plasma

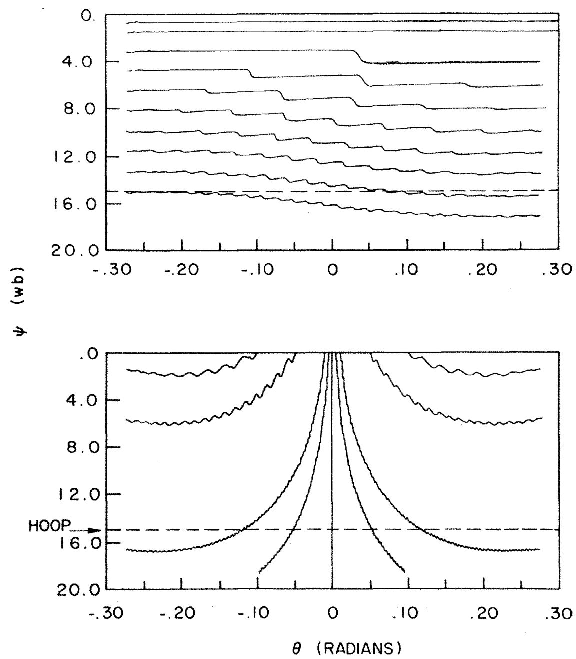

FIG. 3. The projection of the magnetic field lines onto the ψ-θ plane under the influence of a dipole error field. (ψ is the polodial magnetic flux.) The poloidal field is defined by $\vec{B}_p = \nabla\psi \times \hat{\theta}/r$. The dipole located at $\theta = 0$ produced a peak error $\delta B/B$ of 10% at the separatrix ($\psi = 0$). In the lower graph $I_z = 0$, while $I_z = 5000$ A in the upper graph.

experiments, the mean free paths of both electrons and ions were comparable to the mirror spacing, hence mirror effects were unimportant.

While the particles are being lost locally, as observed by a segmented loss detector, the plasma must maintain a quasiequilibrium in the presence of the gradient B drifts. The continuity equation requires azimuthal density gradients to exist in regions where $\varphi_e^* \neq \varphi_i^*$. This result has been verified experimentally[2] by observing the density gradient as external electric fields changed $\varphi_e^* - \varphi_i^*$. This type of analysis is the same as that used to describe the effects of local losses at supports in an octupole.[9]

An effective radial diffusion coefficient D_R^* can be written as $D_R^* = \frac{1}{3}\Delta_R^2\nu$, where Δ_R is the average distance a particle moves radially between collisions of frequency ν. In the limit of high collision frequency, $\Delta_R = \rho$, that is, $D_R^* = \frac{1}{3}\rho^2\nu$ which is just classical diffusion. At somewhat lower collision frequencies, the particle will move a radial distance $\Delta_R = \langle\lambda\delta B/B_p\rangle$ by moving parallel to the field lines, giving an effective diffusion coefficient which is the same as the diffusion coefficient in Eq. (1). For still lower collision frequencies the particle can traverse the entire perturbed region. In this case $\Delta_R \approx W\langle\delta B/B_\theta\rangle$, where W is the azimuthal extent of the perturbation and the effective diffusion coefficient is $D_R^* = \frac{1}{3}W^2\nu\langle\delta B/B_\theta\rangle^2$. The maximum diffusion coefficient would occur when the particle has one collision in traversing the perturbed region, that is, $\nu = vB_\theta/WB_p$, which gives $D_{R(\max)}^* = Wv\langle\delta B/B_p\rangle^2\langle B_p/B_\theta\rangle$. This analysis is conceptually the same as the analysis of superbanana orbits in asymmetric systems where similar physical arguments give results in semiquantitative agreement with detailed calculations.[10]

The expressions which have been derived above can be used to estimate the particle flux to the hoop. When the shearless multipole field is perturbed ($\langle\delta B/B\rangle \approx 1\%$), Eq. (1) shows that the direct ion flux due to diffusion is much less than the direct electron flux due to diffusion. Therefore a retarding electric field $E_a \simeq (kT_e/e)\nabla n/n$ would be needed to make the flow ambipolar to the internal hoop which is very close to the observed electric field. Under these conditions the calculated direct ion flux is determined by the mobility term in Eq. (1), which is a factor of 10 greater than the diffusion term. The experimental values $n = 1.5\times 10^8$ cm^{-3}, $L = 4$ cm, $\lambda_i = 50$ cm, $\nu_i = 1\times 10^4$ sec^{-1}, $T_e = 1$ eV, $T_i \approx 0.1$ eV, $\langle\delta B/B_p\rangle = 0.01$, and $dA \approx 4\times 10^2$ cm give a direct flux for a 10% peak perturbation of 0.2 mA which is in good agreement with the observed value (Fig. 2). The electric fields were roughly kT_e over the width of the perturbation giving an $\vec{E}\times\vec{B}$ convection flux of $\approx$0.2 mA. Therefore the direct ion flux and the induced convection flux are roughly equal and are comparable to the measured total flux to the hoop (Fig. 2). This induced anomalous loss is $\approx$20% of the loss calculated assuming Bohm diffusion.

When B_θ is added to the system with small field errors the field lines can now be equipotentials since the field lines are confined to a small radial region making parallel density gradients small. Therefore the mobility term is absent and a factor 10 improvement is obtained immediately. According to this model the electron loss is much greater than the ion loss, therefore the plasma loss to the hoops should be determined by the ion-neutral diffusion which gives $\varphi = \int \frac{1}{3}\rho_i^2\nu_i\,|\nabla n|\,dA \simeq 1$ μA. This value corresponds quite well with the observed value at zero applied error. As the error field is increased, our model predicts that the anomalous direct loss starts to become important at $\langle\delta B/B\rangle = \rho/\lambda_i \approx 2$

$\times 10^{-3}$ which is roughly the threshold for the curves in Fig. 2. Note that the average error field is ~0.10 of the peak error field. As the error field increases, the measured plasma loss increases as $(B)^{2.2}$ for the $I_z = 4000$ A case which is predicted by Eq. (1). At large error fields the observed loss in the sheared fields approaches the loss for the unsheared case. To explain this behavior we assume that electric fields again develop parallel to the field lines at large field errors and cause increased ion loss. The plasma loss was observed to increase as the background pressure was decreased at zero applied error and zero B_θ. This is in agreement with this model for loss due to field errors and is different from the case[11] for support induced loss in a sheared field where decreasing pressure decreased the hoop loss.

The results of these experiments and their interpretation indicate that the observed anomalous loss of plasma to the internal hoops of existing multipoles can be caused by small (<1%) field errors. Since present construction techniques cannot produce a magnetic field of arbitrarily small field errors, a toroidal field can be used to reduce the effects of the errors to a tolerable level.

The authors wish to thank J. Adney for the design and construction of the toroidal field winding.

*Work supported by the U. S. Atomic Energy Commission.

[1]G. Y. Skornykov, Zh. Tekh. Fiz. 32, 261 (1962) [Sov. Phys. Tech. Phys. 7, 187 (1962)]; D. W. Kerst, J. Nucl. Energy Part C 4, 253 (1962); A. I. Morozov and L. S. Solov'ev in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. II.

[2]T. Jernigan, R. Prater, and D. M. Meade, to be published.

[3]M. Yoshikawa, T. Ohkawa, D. M. Meade, and T. Jernigan, Bull. Amer. Phys. Soc. 15, 1454 (1970).

[4]H. K. Forsen, H. de la Fuente, D. W. Kerst, G. W. Kuswa, D. E. Lencioni, D. M. Meade, F. E. Mills, A. W. Molvik, J. W. Rudmin, J. A. Schmidt, and J. C. Sprott, in Proceedings of the International Symposium on Closed Confinement Systems, Dubna, U. S. S. R., 29 September–3 October 1969 (unpublished); D. M. Meade, Bull. Amer. Phys. Soc. 14, 1033 (1969).

[5]H. K. Forsen, D. Kerst, D. Lencioni, D. Meade, F. Mills, A. Molvik, J. Schmidt, J. Sprott, and K. Symon, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1969) Vol. I, p. 313.

[6]H. K. Forsen, D. W. Kerst, R. A. Bruen, A. J. Cavallo, J. R. Drake, and J. C. Sprott, in *Proceedings of the Fourth European Conference on Controlled Fusion and Plasma Physics, Rome, Italy, August 1970* (Comitato Nazionale per l'Energia Nucleare, Ufficio Edizione Scientifiche, Rome, Italy).

[7]A. W. Molvik and T. S. Wise, Bull. Amer. Phys. Soc. 15, 1466 (1970).

[8]J. A. Schmidt, Phys. Rev. Lett. 24, 721 (1970).

[9]J. A. Schmidt and G. L. Schmidt, Phys. Fluids 13, 1351 (1970).

[10]M. N. Rosenbluth, Bull. Amer. Phys. Soc. 15, 1422 (1970).

[11]R. Freeman, M. Okabayashi, H. Pacher, G. Pacher, and S. Yoshikawa, Phys. Rev. Lett. 23, 756 (1969).

Thermodynamics of the Heisenberg-Ising Ring for $\Delta \geqslant 1$*

M. Gaudin†
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 9 April 1971)

The thermodynamics of the Heisenberg-Ising ring is reduced to the solution of a system of recurrent nonlinear integral equations.

The energy levels E of the Hamiltonian[1]

$$H = \sum_{n=1}^{N} S_{n,x}S_{n+1,x} + S_{n,y}S_{n+1,y} + \Delta(S_{n,z}S_{n+1,z} - \tfrac{1}{4}), \qquad (1)$$

with $S_N \equiv S_1$, are given by the coupled equations (using Bethe's notation)[2]

$$E = \sum_{\alpha=1}^{N} (\cos k_\alpha - \Delta), \qquad (2)$$

$$N k_\alpha = 2\pi\lambda_\alpha + \sum_{\beta=1}^{M}{}' \psi_{\alpha\beta}, \quad \alpha = [1, M], \qquad (3)$$

where the pseudomomenta k_α and antisymmetric phases $\psi_{\alpha\beta}$ are parametrized[3] in terms of auxiliary quantities φ_α:

$$\cot\tfrac{1}{2}k_\alpha = \coth\tfrac{1}{2}\Phi \tan\tfrac{1}{2}\varphi_\alpha, \quad \Delta = \cosh\Phi \geqslant 1, \qquad (4)$$

$$\cot\tfrac{1}{2}\psi_{\alpha\beta} = \coth\Phi \tan\tfrac{1}{2}(\varphi_\alpha - \varphi_\beta), \qquad (5)$$

$$0 \leqslant k_\alpha \leqslant 2\pi, \quad -\pi \leqslant \psi_{\alpha\beta} \leqslant \pi, \quad -\pi \leqslant \varphi_\alpha \leqslant \pi. \qquad (6)$$