Nonlocalities in Nuclear Optical-Model Potentials

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The energy dependence and surface geometry of an optical-model equation with a nonlocal potential are compared with corresponding properties of optical-model equations with local and velocity-dependent potentials which are used to generate approximate solutions to the nonlocal-potential equation.

In a recent paper,[1] Scherk has proposed an effective-mass equation that is suggested to be completely compatible with a nonlocal-potential equation used by Perey and Buck[2] and Perey[3] in their neutron optical-model studies. Scherk has also stated that the energy dependence and surface geometry of a local potential used by Perey and Buck to generate approximate solutions to the nonlocal equation is unphysical. We disagree with these statements, and in this note we show that the Perey-Buck local-potential approximation is substantially correct. In particular, the surface properties of the Perey-Buck approximation are almost identical to the surface properties of the original nonlocal equation. The surface properties of the effective-mass equation differ from those of the nonlocal equation and the local-potential approximation.

Fiedeldey[4] has already shown that the Perey-Buck local approximation is a good one in the neighborhood of the nuclear surface, and our statement to this effect is by no means new. However, more recent work[5-7] on the properties of nonlocal and local equations can be applied to delineate the differences just mentioned in a simple manner, and the main purpose of this note is to illustrate one aspect of a method for systematically studying the differences between nonlocal, local, and velocity-dependent equations. While we restrict this discussion to s-wave scattering of neutrons on ^{40}Ca for purposes of comparison with Ref. 1, the method used is reasonably general and can be applied to most short-range, nonlocal or velocity-dependent interactions.

We consider the s-wave radial equation for a nonlocal potential:

$$-\frac{\hbar^2}{2m}u''(k,r)+\int_0^\infty V(r,s)u(k,s)ds=Eu(k,r). \quad (1)$$

Throughout this Letter a prime will indicate differentiation with respect to r. The nonlocal potential $V(r,s)$ is taken to be real and symmetric and E is related to k by $E=\hbar^2k^2/2m$. We choose as two independent solutions of Eq. (1) those defined by the boundary conditions

$$\lim_{r\to\infty} e^{\mp ikr}f_\pm(k,r)=1; \quad (2)$$

these solutions satisfy

$$f_-(k,r)=f_+{}^*(k,r). \quad (3)$$

We define a radial flux[8]

$$S(k,r)=(\hbar/2mi)\times[f_-(k,r)f_+'(k,r)-f_+(k,r)f_-'(k,r)] \quad (4)$$

by analogy with the flux for a one-dimensional Schrödinger equation. It follows from Eq. (1) that

$$S'(k,r)=(2/\hbar)\int_0^\infty V(r,s)\,\mathrm{Im}[\rho(k,r,s)]ds, \quad (5)$$

where

$$\rho(k,r,s)=f_-(k,r)f_+(k,s) \quad (6)$$

is a radial mixed probability density; and from the boundary conditions (2) that

$$S(k,r)=\frac{\hbar k}{m}-\frac{2}{\hbar}\int_r^\infty\int_0^\infty V(s,t)\,\mathrm{Im}[\rho(k,s,t)]dtds. \quad (7)$$

The radial flux satisfies

$$S(k,0)=S(k,\infty)=\hbar k/m, \quad (8)$$

as is evident from Eq. (7) and the symmetry of the nonlocal potential. Equation (8) is the radial-equation analog of the statement that real, symmetric nonlocal potentials conserve flux globally. In general, $S(k,r)$ is a function of both k and r; but in the local-potential limit, $S(k,r)=\hbar k/m$ and $S'(k,r)=0$ for all k and r. Real local potentials conserve flux locally whereas real, symmetric nonlocal potentials do not. This is a basic difference between nonlocal and local potentials, and it is convenient to define a relative radial flux $J(k,r)$ by

$$S(k,r)=J(k,r)\hbar k/m. \quad (9)$$

$J(k,r)$ represents the radial flux associated with a nonlocal potential relative to the radial flux for *any* real local potential. Deviations of $J(k,r)$

from unity are a measure of the nonlocality of a potential. Note that $J(k,r)$ is unity in both of the limits as $r\to 0$ and $r\to\infty$.

According to our definition (4), the radial flux is the flux associated with a source at the origin, which looks like a point source in the limit as $r\to\infty$. Then as this spherical wave[9] propagates from the origin, $S'(k,r)<0$ corresponds to a local loss of flux and $S'(k,r)>0$ corresponds to a local gain of flux. By virtue of Eq. (8), these local losses and gains must exactly compensate each other in such a manner that there is no net loss or gain of flux. The net area under an $S'(k,r)$ curve as a function of r must be zero. In some cases, $S'(k,r)$ may be very small over a substantial distance that is within the range[10] of a nonlocal potential, in which case the wave is propagating almost like a wave for a local potential even though $S(k,r)$ may be quite different from $\hbar k/m$. We illustrate these effects in Fig. 1 for the rate of change of the relative radial flux. We will discuss this figure in more detail later.

If the parameters of a local potential with approximately the same range as a nonlocal potential are adjusted so that both potentials yield the same absolute phase shift over an energy interval, then the two potentials are equivalent in the energy interval insofar as measurements depending on the phase shifts are concerned. The wave functions for the two potentials can be normalized so that they are identical in the limit as $r\to\infty$, and the behavior of the wave functions for distances within the ranges of the potentials can be compared. This procedure was used by Perey,[3] and he observed that the wave function for the nonlocal optical-model potential was systematically smaller than the wave function for a corresponding local optical-model potential inside the nucleus. This damping of nonlocal wave functions relative to local wave functions is called the Perey effect, and it has been interpreted by Austern[11] as a depletion of flux in the elastic channel due to virtual excitations of the incident nucleon to other channels.

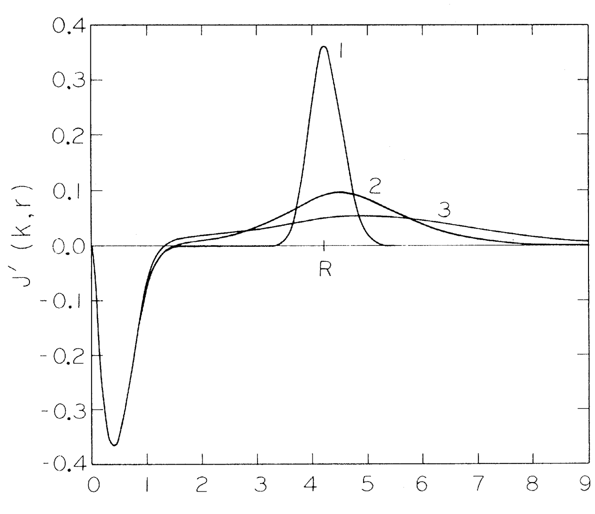

FIG. 1. Rate of change of the relative radial flux for the nonlocal potential given in Eq. (13). The curve labeled 1 is for a diffuseness of 0.10 F; the curve labeled 2, for a diffuseness of 0.65 F; and the curve labeled 3, for a diffuseness of 1.20 F.

The Perey effect can be treated more explicitly by relating the wave functions obtained from Perey's procedure[3] according to $u_N(k,r)=A(k,r)u_L(k,r)$. $A(k,r)$ is called the damping function, and the Perey effect corresponds to deviations of $A(k,r)$ from unity. In previous work,[5-7] we imposed the requirement that $A(k,r)$ be independent of the boundary conditions used to normalize the wave functions. For the solutions considered here, this requirement takes the form

$$f_{N\pm}(k,r)=A(k,r)f_{L\pm}(k,r) \tag{10}$$

An immediate consequence of this requirement is

$$S(k,r)=A^2(k,r)\hbar k/m, \tag{11}$$

since the radial flux for any real local potential is a constant. Thus, we can equate the relative radial flux $J(k,r)$ to the square of the damping function; similarly, we can identify the relative radial flux with the ratio of the nonlocal and local radial probability densities:

$$J(k,r)=A^2(k,r)=\rho_N(k,r)/\rho_L(k,r), \tag{12}$$

where $\rho(k,r)=\rho(k,r,r)$ as defined in Eq. (6). These relations and the requirement used to obtain them are discussed in more detail in Refs. 6 and 7.

We will now use the preceding analysis to compare the properties of the nonlocal equation (1) with the potential used by Perey and Buck,[2,3] their local approximation to this equation, and the the effective-mass equation with the parameters proposed by Scherk.[1] For s waves, the nonlocal potential is

$$V(r,r')=V(\tfrac{1}{2}(r+r'))h_\beta(r,r'), \tag{13}$$

where

$$V(r)=V_0\{1+\exp[(r-R)/a]\}^{-1} \tag{14}$$

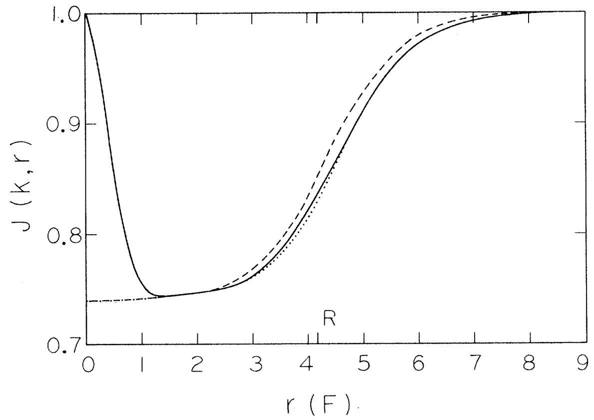

FIG. 2. Relative radial flux for the nonlocal potential in Eq. (13) (solid line), the Perey-Buck local-potential approximation given by Eq. (17) (dotted line), and the effective-mass equation given by Eq. (19) (dashed line).

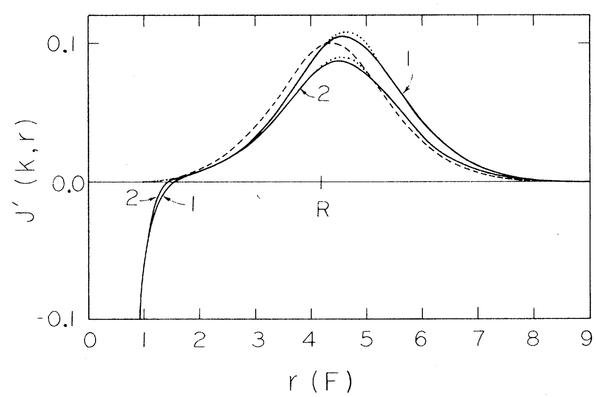

FIG. 3. Rate of change of the relative radial flux for the nonlocal potential (solid lines), the Perey-Buck local-potential approximation (dotted lines), and the effective-mass equation (dashed line). The curve labeled 1 is for $E = 5$ MeV, and the curve labeled 2 is for $E = 45$ MeV; the effective-mass equation curve is independent of energy.

and

$$h_\beta(r, r') = \frac{\exp[-(r-r')^2/\beta^2] - \exp[-(r+r')^2/\beta^2]}{\pi^{1/2}\beta}. \qquad (15)$$

The parameters used in this study are taken from Perey and Buck[2] unless otherwise indicated. The radius $R = R_0 A^{1/3}$ was taken to be 4.172 F and $\hbar^2/2m$ was taken to be 20.734 MeV F^2. The local-potential approximation to the nonlocal potential $V_{PB}(k, r)$ is given by

$$V_{PB}(k, r) \exp\{(m\beta^2/2\hbar^2)[E - V_{PB}(k, r)]\} = V(r), \quad (16)$$

and the relative radial flux obtained from Perey's empirical damping function[3] is

$$J_{PB}(k, r) = [1 - (m\beta^2/2\hbar^2) V_{PB}(k, r)]^{-1}. \qquad (17)$$

The approach we have followed for the radial equation with a nonlocal potential can also be used for the radial effective-mass equation. The relative radial flux for the effective-mass equation is

$$J_{em}(k, r) = [1 + c(E_0)\rho(r)]^{-1} \qquad (18)$$

in the notation of Ref. 1, and

$$J_{em}(k, r) = [1 - (m\beta^2/2\hbar^2) V_{PB}(k_0, 0) V(r)/V_0]^{-1} \qquad (19)$$

in the present notation. The energy $E_0 = \hbar^2 k_0^2/2m$ is taken to be 25 MeV. Note that the relative radial flux for the effective-mass equation turns out to be identical to the effective mass used in Ref. 1.[12]

J, J_{PB}, and J_{em} are compared in Fig. 2 for $E = 25$ MeV. J is the solid curve, J_{PB} is the dotted curve, and J_{em} is the dashed curve. Figure 3 of Ref. 1 is a comparison of the quantities $V_{PB}(k, r)/V_{PB}(k, 0)$ and $V(r)/V_0$ that was used to point out deficiencies in the Perey-Buck approximation. It follows from Eqs. (17) and (19) that Fig. 2 of this note represents a comparison of the same quantities. It is clear that J_{PB} is a much better approximation to J than J_{em}. Figure 3 of this note shows a comparison of J_{PB}' (dotted line) and J' (solid line) for $E = 5$ and 45 MeV. The dashed line in Fig. 3 is J_{em}' which is independent of energy. Again, it is clear that J_{PB} is a better approximation to J than J_{em}. We have made similar comparisons of the Perey-Buck local-potential approximation and the numerically exact results for $A = 27$-216 and $E = 5$-45 MeV. In no case are the differences between the exact results and the approximation any greater than is indicated in Figs. 2 and 3. Fiedeldey[4] has shown that the Perey-Buck approximation does not deteriorate for higher angular momenta. We conclude that the Perey-Buck approximation adequately reflects the energy dependence and surface properties of the nonlocal potential, and that the effective-mass equation with the parameters proposed by Scherk[1] does not.

In this writer's opinion, Scherk[1] has underestimated the effect of nonlocality on the surface properties and energy dependence of the nonlocal nuclear optical-model interaction. The effect of the nonlocality on the surface properties can be seen in Fig. 1 which shows $J'(k, r)$ for the nonlocal potential (13) with different values of the diffuseness. As the diffuseness is increased, the maximum of $J'(k,r)$ shifts to larger values of r. Curves 1 and 2 of Fig. 3 show that the surface properties are also dependent on the energy. However, these differences between the effective-mass equation and the nonlocal-potential equation do not constitute a basis for rejecting an effective-mass equation as an optical model. The relevance of an effective-mass equation as a nuclear optical model can be based on analogies with classical optics.

It is apparent from Figs. 2 and 3 that the properties of the two approximations differ from the properties of the nonlocal equation for small r. Since both approximations are thought to be exact in the nuclear interior,[1,4] these differences are of interest, and we conclude this note with a few remarks about them.

The approximations are based on an extrapolation of a model[13] in which the radius R in Eq. (14) is taken to be infinite (infinite nuclear matter). In that model, translational invariance requires $J(k,r)$ to be independent of r, and flux is conserved locally. However, flux is not conserved locally if R is finite. For finite R, the differences between the exact and approximate results for small r are due to the fact that the exact $J(k,r)$ satisfies a global radial-flux conservation requirement (8) whereas the approximations to $J(k,r)$ do not. According to Eq. (8), the relative radial flux must be 1 at the origin for *any* real, symmetric nonlocal interaction with a finite range.[10] We have shown previously[5,6] that Eq. (8) must be satisfied in each partial wave of a partial-wave expansion with a nonlocal interaction.[14] The global conservation of radial flux is a general property of interactions (local or nonlocal) which have spherical symmetry. Hence, the behavior of $J(k,r)$ near the origin and its correlation with similar behavior near the surface ($r \sim R$) is a geometrical effect that is characteristic of nonlocal interactions which have spherical symmetry.

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[1] L. R. Scherk, Phys. Rev. Lett. 26, 574 (1971).

[2] F. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

[3] F. G. Perey, in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach, New York, 1963), p. 125.

[4] H. Fiedeldey, Nucl. Phys. A96, 463 (1967).

[5] M. Coz, A. D. MacKellar, and L. G. Arnold, Ann. Phys. (New York) 58, 504 (1970).

[6] M. Coz, L. G. Arnold, and A. D. MacKellar, Ann. Phys. (New York) 59, 219 (1970).

[7] L. G. Arnold and A. D. MacKellar, Phys. Rev. C 3, 1095 (1971).

[8] The radial flux is defined in the context of a single partial wave. It is not equivalent to the radial component of a three-dimensional flux vector.

[9] Since Eq. (1) is taken as the starting point in this discussion, the usual $1/r$ dependence of a spherical wave does not appear.

[10] The range of a nonlocal potential is defined to be that value of $r=r_0$ such that $V(r,r')$ is negligible for all r' and $r>r_0$.

[11] N. Austern, Phys. Rev. 137, B752 (1965).

[12] There are several ways to interpret $J(k,r)$, and our use of the term radial flux as opposed to effective mass is purely a matter of convention. The important point is that the quantity $J(k,r)$ is a meaningful basis for comparing nonlocal, local, and velocity-dependent potentials.

[13] W. E. Frahn, Nuovo Cimento 4, 313 (1956).

[14] The behavior of $J(k,r)$ near $r=0$ is not encountered for $J(k,r)$ near $x=0$ in one-dimensional problems since the corresponding global flux conservation condition involves the limits as $x \to \pm\infty$.