$(\ln s)^{-|M|}$ relative to the leading $M=0$ cut contribution for two particles of spin $>\frac{1}{2}$.

[8]This test is reminiscent (although different from) that proposed for factorization by H. D. I. Abarbanel and D. Gross (to be published), to wit, the spin independence of single-particle inclusive distributions.

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K_{l3} Form Factors and the Scaling Behavior of Spin-0 Fields

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Motivated by scaling considerations we formulate a theory of K_{l3} decay based on the Kemmer equation. We find that our theory (1) has a symmetry-breaking parameter of order 10-30% which agrees with experiment, (2) makes a *definite testable prediction* of a kinematic zero at $t=(m_K+m_\pi)^2$ in the "effective" scalar form factor, and (3) yields a modified Callan-Treiman relation which improves agreement with experiment.

It is we'' known that pseudoscalar particles can be described by at least two different covariant field equations, namely, the Klein-Gordon (K-G) and Kemmer equations.[1] For many classes of processes (such as the quantum electrodynamics of spin-0 mesons) calculations based on either equation yield identical results.[2] Therefore, it has often been tacitly assumed that the two equations will yield identical results in all cases.

We note, however, that the K-G and Kemmer fields behave differently under scale transformations [see Eqs. (2) and (3) below]. Since symmetry-breaking effects are sensitive to the scale dimensionalities of the respective fields,[3] it follows that in principle the corresponding field theories could lead to qualitatively different results in processes, such as K_{l3} decays, involving two or more pseudoscalar mesons of different mass.

To date this possibility has not been studied in a formal field theory owing to the difficulties involved in treating the strong interactions. Nonetheless, the scaling argument suggests that even in a *phenomenological* treatment of such a process, the same assumptions could yield qualitatively different results from analyses based on the K-G and Kemmer equations, respectively. If different results were obtained, we could then ascertain which equation gives the better phenomenological description of the particular process in question.

We present in this Letter the results of just such an analysis of K_{l3} decays. Our main conclusions are the following:

(1) When compared with experiment, our theory yields a symmetry-breaking parameter ρ (analogous to the K-G parameter ξ) whose experimental magnitude is $\rho=0.28\pm0.20$. Since SU(3)-symmetry breaking is expected to be of this

order (a typical theoretical calculation predicts $\rho \cong 0.20 \pm 0.10$), this number is much more understandable than the K-G value[4] of $\xi = -0.85 \pm 0.20$. The "explanation" of the large experimental value of ξ is seen when the Kemmer matrix element is expressed in terms of the functional form of the K-G matrix element. One then obtains an "effective" K-G parameter $\hat{\xi} = -\rho - 0.57$, the 0.57 being a kinematic factor.

(2) Perhaps most importantly, we make the *definite prediction* that the "effective" scalar form factor $\hat{f}_0$ has a kinematic zero at $t = (m_K + m_\pi)^2$.

(3) The analysis can be extended to K_{l2} and π_{l2} decays and leads to a modified Callan-Treiman relation which improves agreement with experiment.

The free Kemmer equation for a particle of mass m is given by[1,2]

$$(\beta \cdot \partial + m)\psi(x) = 0, \quad \bar{\psi}(x)(-\beta \cdot \partial + m) = 0, \tag{1}$$

where the β's satisfy an algebraic relation given in Refs. (1) and (2). The equal-time commutator (ETC) of the Kemmer fields $\psi(x)$ and $\bar{\psi}(x)$ is given by

$$[\psi(\vec{x}, 0), \bar{\psi}(\vec{x}', 0)] = -\beta_4 \delta^3(\vec{x}-\vec{x}') + m^{-1}(\beta_4\beta_k + \beta_k\beta_4)(\partial/\partial x_k)\delta^3(\vec{x}-\vec{x}'). \tag{2}$$

By contrast the ETC of the K-G field $\varphi(x)$ and the canonically conjugate field $\pi(x) = \partial_0 \varphi^\dagger(x)$ is

$$[\pi(\vec{x}, 0), \varphi(\vec{x}', 0)] = -i\delta^3(\vec{x}-\vec{x}'). \tag{3}$$

From Eqs. (2) and (3) it follows that the Kemmer field $\psi(x)$ transforms under the scaling operator D as the sum of two terms having dimensions $(\text{mass})^{3/2}$ and $(\text{mass})^2$, respectively, while the K-G field $\varphi(x)$ transforms under D with a single-dimension $(\text{mass})^1$. As stated previously, this difference in the scaling behavior of the K-G and Kemmer fields is the motivation for our calculation.

The matrix element T for the decays $K(p) \to \pi(p') + l(k) + \nu(q)$ is given by

$$T = (G/\sqrt{2}) \sin\theta \langle \pi(p') | V_\lambda(0) | K(p) \rangle l_\lambda, \tag{4}$$

where $G = 10^{-5}/m_p^2$ is the weak decay constant, $\theta \cong 0.2$ is the Cabibbo angle, and l_λ is the lepton current. In the Kemmer formulation the hadronic matrix element is[5]

$$\langle \pi(p') | V_\lambda(0) | K(p) \rangle = i(m\mu/P_0 P_0' V^2)^{1/2} \bar{u}_\pi(p')\{\beta_\lambda g_V(t) + [iq_\lambda/(m+\mu)] g_S(t)\} u_K(p), \tag{5}$$
$$t \equiv -(p-p')^2 = -q^2,$$

where m and μ are the masses of the kaon and pion, respectively, and the five-component free-particle spinors $u_K(p)$ and $\bar{u}_\pi(p')$ are normalized such that $\bar{u}(p)u(p) = 1$. In the limit of exact SU(3) symmetry $g_S(t) = 0$ and hence the parameter $\rho(t) \equiv g_S(t)/g_V(t)$ measures the extent of symmetry breaking in the K_{l3} system. For purposes of calculating $\rho(t)$ it is convenient to introduce the scalar form factor $g_0(t) \equiv g_V(t) - [t/(m^2-\mu^2)] g_S(t)$.

It is a straightforward matter to show that the form factors $g_V(t)$ and $g_0(t)$ are determined (in the sense of dispersion integrals) by $J^P = 1^-$ and $J^P = 0^+$ intermediate states, respectively. As is conventional, we assume that in the physical region $[m_l^2 \le t \le (m-\mu)^2]$ the form factors $g_i(t)$ $(i = V, S, 0)$ can be approximated by a linear t dependence, $g_i(t) = g_i(0)(1 + \gamma_i t/\mu^2)$. The parameters $g_V(0)$, $g_S(0)$, $\rho = \rho(0)$, γ_V, γ_S, and γ_0 are, respectively, the Kemmer analogs of the conventional K-G parameters $f_+(0)$, $f_-(0)$, $\xi = \xi(0)$, λ_+, λ_-, and λ_0 which are defined in Ref. (4).

The parameter ρ can be obtained from experiment by directly squaring T in Eqs. (4) and (5) using the relation

$$u(p)\bar{u}(p) = (1/2m^2) i\beta \cdot p (i\beta \cdot p - m) \tag{6}$$

and various trace theorems for the Kemmer β matrices.[1,2] It is more instructive, however, to note that the matrix element of Eq. (5) can be cast into a K-G form if we define *effective* form factors $\hat{f}_\pm(t)$ and $\hat{f}_0(t)$ given by

$$\hat{f}_+(t) = [(m+\mu)/2(m\mu)^{1/2}] g_V(t), \tag{7a}$$

$$\hat{f}_-(t) = [(m+\mu)/2(m\mu)^{1/2}] g_V(t)\{\delta - \rho(t)[1 - t/(m+\mu)^2]\}, \tag{7b}$$

$$\hat{f}_0(t) = [(m+\mu)/2(m\mu)^{1/2}][1 - t/(m+\mu)^2] g_0(t), \tag{7c}$$

$$\hat{\xi}(t) \equiv \hat{f}_-(t)/\hat{f}_+(t); \quad \delta \equiv -(m-\mu)/(m+\mu). \tag{7d}$$

From Eqs. (7) it follows that the *effective* K-G symmetry-breaking parameter $\hat{\xi}$ is given in terms of ρ by

$$\hat{\xi} \equiv \hat{\xi}(0) = \delta - \rho = -0.57 - \rho. \tag{8}$$

Hence, a small positive symmetry-breaking parameter ρ in the Kemmer formulation of K_{l3} decay would look like a large negative symmetry-breaking parameter $\hat{\xi}$ in the conventional K-G formulation:

$$\rho_{\rm expt} \cong +0.28 \pm 0.20, \quad \hat{\xi}_{\rm expt} \cong -0.85 \pm 0.20. \tag{9}$$

ρ can also be calculated theoretically in a manner similar to Fuchs,[6] who assumes a once-subtracted dispersion relation for both the K-G $f_+(t)$ and $f_0(t)$. Taking into account various possibilities for the S-wave $K\pi$ phase shifts he finds $\xi \cong -0.20 \pm 0.10$, which value is representative of the results obtained from other calculations as well. Proceeding analogously and assuming a once-subtracted dispersion relation for $g_V(t)$ and $g_0(t)$, we obtain

$$\rho_{\rm theor} \cong +0.20 \pm 0.10, \quad \hat{\xi}_{\rm theor} \cong -0.77 \pm 0.10. \tag{10}$$

Using Eqs. (7), many expressions of interest, such as the K_{l3} Dalitz-plot distribution, can be calculated exactly for the Kemmer case from already existing K-G expressions simply by replacing the K-G form factors $f_{\pm,0}(t)$ by $\hat{f}_{\pm,0}(t)$. In computing the K_{l3} rates, however, care must be taken in treating the t dependence of the effective form factors $\hat{f}_\pm(t)$ which are *not* to be approximated by a linear t dependence. Only the $g_i(t)$ should be linearly expanded while the remaining t dependence (which represents the "memory" of the Kemmer wave functions) must be treated exactly. Consequently the expressions for the K_{l3} decay rates have had to be integrated anew, and the results will be given elsewhere.

From Eq. (7c) we see that the effective scalar form factor $\hat{f}_0(t)$ has a kinematic zero at $t = (m+\mu)^2 \equiv t_0$ [barring the highly unlikely possibility of $g_0(t)$ having an accidental pole at t_0]. The presence of such a zero is an *unambiguous prediction of the Kemmer formulation.* It can be tested for either by a careful extrapolation of $\hat{f}_0(t)$ outside the physical region (when sufficiently accurate data become available) or by looking for its effect in $K\pi$ scattering.[7] In Fig. 1 we plot the function $\hat{f}_0(t)$ for several values of ρ along with the conventional scalar form factor $f_0(t)$ which is shown for several values of ξ.

We turn next to a discussion of the hypothesis of partialy conserved axial-vector current (PCAC) and the Callan-Treiman (C-T) relation.[8] In formulating the Kemmer generalization of PCAC we are motivated by two observations. First, the Kemmer (ψ_a, $a = 1, 2, 3, 4, 5$) and K-G (φ) solutions for both the free-particle and π mesonic atom are such that $\psi_5(x) \propto \varphi(x)$. Further, in the Kemmer formulation of $NN\pi$ coupling,[9] the representation of the π field reduces to that of the $a = 5$ component of ψ_a, and is thus the same as the usual $\bar{N}\gamma_5 N\varphi$ coupling. This leads us to assume PCAC in the form

$$\partial_\lambda A_{\lambda a}{}^i(x) = G_\pi \psi_{\pi a}{}^i(x), \tag{11}$$

where $i = 1, 2, 3$ is the isospin label and a is the Kemmer index. Then Eq. (11) maintains the correspondence between the fifth component of the Kemmer field and the K-G field since the $a = 5$ part of Eq. (11) is the usual PCAC relation [with $A_{\lambda 5}(x) = -A_\lambda(x)/\sqrt{2}$]. $a = 1, 2, 3, 4$ corresponds to four-derivatives of the usual relation. The matrix element for one meson to vacuum in the Kemmer formulation is

$$\langle 0|A_{\lambda a}{}^{1-i2}(0)|\pi^+(p)\rangle = ip_\lambda g_\pi(\mu/p_0 V)^{1/2} u_{\pi a}(p), \tag{12}$$

From Eq. (6) we then find $g_\pi = f_\pi/(2\mu)^{1/2}$ and hence $G_\pi = \frac{1}{2}\mu^{3/2} f_\pi$, where f_π is the usual pion-decay constant. To derive the Kemmer form of the C-T relation for K^+ decay we begin by contracting the pion field:

$$\langle \pi^0(p')|V_\lambda{}^{4-i5}(0)|K^+(p)\rangle = i(\mu/p_0'V)^{1/2}\int d^4x\, e^{-ip'\cdot x}\bar{u}(p')(i\beta\cdot p' + \mu)\langle 0|T[\psi_\pi{}^3(x)V_\lambda{}^{4-i5}(0)]|K^+(p)\rangle. \tag{13}$$

Using Eq. (11) and the relations

$$\bar{u}_a(p' = 0) = -\delta_{a5}/\sqrt{2}, \quad A_{\lambda 5}{}^3(x) = -A_\lambda{}^3(x)/\sqrt{2} \tag{14}$$

we find, in the soft-pion limit,

$$\hat{f}_+(m^2) + \hat{f}_-(m^2) = f_K/\sqrt{2}f_\pi \quad \text{(revised C-T relation).} \tag{15}$$

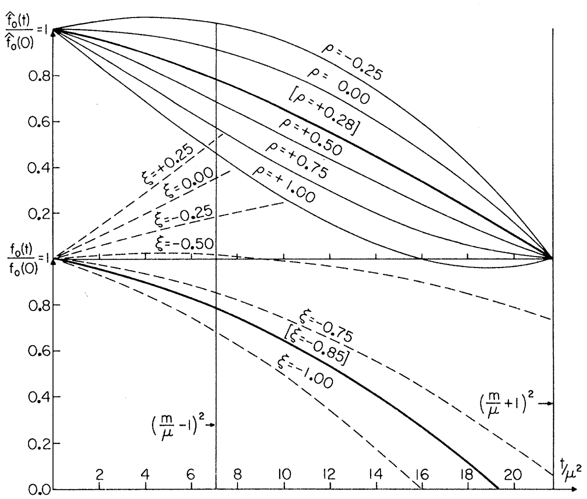

FIG. 1. The Kemmer "effective K-G" scalar form factor $\hat{f}_0(t)$ and the K-G scalar form factor $f_0(t)$, plotted for various values of the symmetry-breaking parameters ρ and ξ with $\gamma_V=\lambda_+=0.05$ and $\gamma_S=\lambda_-=0.025$. In each case the heavy lines correspond to the best fit to experiment. The best fits agree in the physical region $m_l^2\leq t\leq(m-\mu)^2$, but only with widely different magnitudes of the symmetry-breaking parameters. Note the clear prediction of a kinematic zero at $t=(m+\mu)^2$ in the Kemmer case, *irrespective of the values of* ρ, γ_V, and γ_S.

For the sake of comparison, the conventional C-T relation is[8]

$$f_+(m^2)+f_-(m^2)=f_K/\sqrt{2}f_\pi. \tag{16}$$

If we assume that the form factors $\hat{f}_\pm(t)$ and $f_\pm(t)$ are sufficiently slowly varying functions (so that we can replace the various form factors by their values at $t=0$), then the numerical values of the left-hand side of Eqs. (15) and (16) are, respectively, $(0.39\pm0.20)/\sqrt{2}$ and $(0.15\pm0.20)/\sqrt{2}$ while the right-hand side of each equation is (1.28 ± 0.06). Thus, although the agreement between the revised C-T relation (15) and experiment is not particularly good, it is nonetheless better than the standard relation (16) by a factor of ~2.6. Presumably the remaining discrepancy between Eq. (15) and experiment can be attributed to the variation of the form factors between $t=m^2$ and $t=0$. A more complete discussion of the Kemmer formulation of K_{l3} decays will be given elsewhere.

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