sion comes from a recent study of baryon-exchange production of various Λ and Σ states.[13] The ratio of cross sections for a backward $\Lambda(1405)$ to $\Lambda(1520)$ production was observed to be less than

$$\frac{\Lambda(1405)\text{ production}}{\Lambda(1520)\text{ production}} < \frac{3\ \mu\text{b}}{75\ \mu\text{b}} = \frac{1}{25}. \qquad (6)$$

The ratio of coupling strengths of the $\Lambda(1405)$ and $\Lambda(1520)$ to $\bar{K}N$ deduced from the Argand diagram shown in Fig. 2 is consistent with ratio (6).

In conclusion, the data presented here suggest the possibility of *directly* studying the below-threshold $\bar{K}N \rightarrow Y\pi$ amplitude using virtual nucleons in the deuteron. A partial-wave analysis indicates that the S_{01} channel has a large repulsive background that can be described by a negative scattering length and the coupling of the $\Lambda(1405)$ to the $\bar{K}N \rightarrow \Sigma\pi$ amplitude is very small relative to $\Lambda(1520)$. This latter conclusion is in agreement with the data on baryon-exchange production of the $\Lambda(1405)$ and $\Lambda(1520)$.

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Two-Particle Distributions and the Nature of the Pomeranchuk Singularity*

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We propose the measurement of reactions of the type $p_1+p_2 \rightarrow q_1+q_2+$anything at high energy in the region in which the produced particles q_1 and q_2 can be regarded as "fragments" of the incident particles p_1 and p_2, respectively. Dependence of the cross section on the angle between transverse parts of the vectors $\vec{q}_1$ and $\vec{q}_2$ would indicate that the Pomeranchuk singularity contains Regge-cut contributions.

A central problem in the theory of strong interactions is the exact nature of the Pomeranchuk singularity which governs the asymptotic behavior of high-energy collisions. Various forms of Regge cuts for the Pomeranchuk singularity have been suggested,[1] while the hypothesis of approximate dominance by a simple pole still has many supporters.[2] We present here a possible experimental probe of this question, which involves measurement of two-particle inclusive distributions of the type $p_1+p_2 \rightarrow q_1+q_2+$ anything (Fig. 1) at high energy. The measurement at a conventional accelerator would require detection of a fast, nearly forward particle in coincidence with

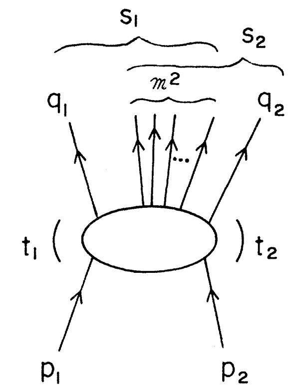

FIG. 1. Variables describing the two-particle inclusive production process.

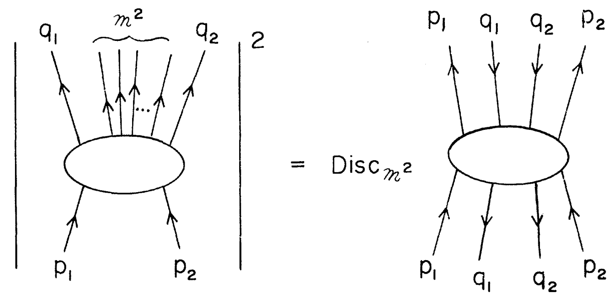

FIG. 2. A generalized optical theorem which relates the two-particle inclusive cross section to a discontinuity of an eight-point function.

a slow particle. At a colliding beam facility, both particles would be fast and nearly collinear with the beams.

The test we propose concerns the Toller[3] M-quantum-number properties of the Pomeranchuk singularity. This quantum number, as we shall discuss later, can be related to possible Regge-cut contributions to the Pomeranchukon. The development follows previous work by Mueller[4] on single-particle distributions. Extending the Mueller technique, we express the two-particle-distribution cross section as the appropriate discontinuity of an eight-point function as indicated in Fig. 2. Single- and multiple-Regge behavior for the eight-point function of Fig. 2 is assumed. It is further assumed that the Regge behavior remains after the appropriate discontinuity in the variable $\mathfrak{M}^2$ is taken. Our fundamental limit for the two-particle distribution is obtained by taking the leading high-energy limit implied by the exchange of the Toller-Lorentz plane singularity (see Fig. 3) designated by the two quantum numbers λ, M of the symmetry group O(3,1). This is the relevant symmetry of our eight-point function since we are in the forward direction.

The formula, in this case, is, in the limit of large $\mathfrak{M}^2$, with t_1, t_2, $s_1/\mathfrak{M}^2$, and $s_2/\mathfrak{M}^2$ finite,

$$\frac{d\sigma}{(d^3q_1/\omega_1)(d^3q_2/\omega_2)} = s^{-1} \sum_{M=0}^{\infty} (\mathfrak{M}^2)^{\alpha_v(0)} \cos M\varphi F_M(t_1, t_2, s_1/\mathfrak{M}^2, s_2/\mathfrak{M}^2), \tag{1}$$

where $\alpha_v(0)$ is related to the leading singularity in the Lorentz quantum number λ and coincides with the leading vacuum angular-momentum singularity. The process depends upon six independent variables, five of which are defined as follows (see also Fig. 1):

$$t_1 = (p_1 - q_1)^2, \quad t_2 = (p_2 - q_2)^2,$$
$$\mathfrak{M}^2 = (p_1 + p_2 - q_1 - q_2)^2, \quad s_1 = (p_1 + p_2 - q_2)^2,$$
$$s_2 = (p_1 + p_2 - q_1)^2. \tag{2}$$

The sixth independent variable φ has its origin in the O(3,1)-group analysis and has a simple physical description. It is, in fact, the angle between the transverse components of $\vec{q}_1$ and $\vec{q}_2$ in any Lorentz frame collinear with the beam. The variable s is not independent and can be written, in the limit of Eq. (1), as

$$s = (s_1/\mathfrak{M}^2)(s_2/\mathfrak{M}^2)\mathfrak{M}^2. \tag{3}$$

In terms of the fractional longitudinal momenta (in the c.m. system),

$$x_1 = 2q_{1L}/W, \quad x_2 = -2q_{2L}/W,$$
$$s_1/\mathfrak{M}^2 = (1-x_1)^{-1}, \quad s_2/\mathfrak{M}^2 = (1-x_2)^{-1},$$
$$s/\mathfrak{M}^2 = (1-x_1)^{-1}(1-x_2)^{-1}. \tag{4}$$

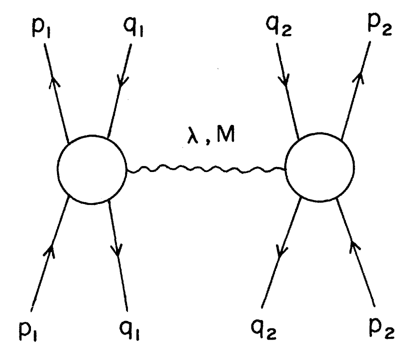

FIG. 3. Exchange of the leading Toller singularity in the eight-point function.

The following comments are made in explaining the derivation and interpretation of Eq. (1): (1) The formula follows immediately from the asymptotic expansion of the O(3,1) representation functions given by Toller.[3] The relation between O(3,1) group parameters and measured variables is particularly simple if the group analysis is referred to the rest frames of the beam and of the target particles with z axis in these frames fixed along the beam direction. (2) From Eq. (1) it is evident that the leading singularity $\alpha_v(0)$ must have a nonvanishing $M=0$ component since otherwise the cross section has negative values for some φ.[5] In the case of a simple pole, the sum over M reduces to a single term which must therefore have $M=0$.[6] (3) If the leading singularity is a cut, then in Eq. (1) $\alpha_v(0)$ is the location of the branch point, and there will be $\ln\mathfrak{M}^2$ factors present. In addition, the entire sum over the quantum number M (including zero) will contribute, since, in general, cuts do not possess a well-defined M.[7]

From remarks (2) and (3) above we have the interesting consequence that if the Pomeranchuk singularity is a simple pole, then the high-energy limit (1) will not depend upon the sixth variable φ. On the other hand, a dependence on φ implies a cut component to the Pomeranchuk singularity or some singularity configuration more complicated than a simple pole.[8]

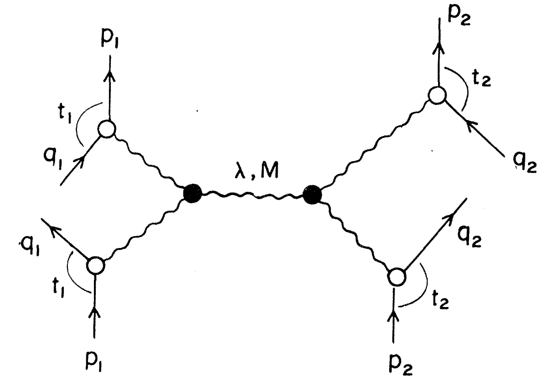

FIG. 4. Regge and Toller singularities in the eight-point function which determine the high-energy limit in Eq. (5).

A further limit can also be taken in (1) by doing a multiple Regge decomposition of the type shown in Fig. 4 corresponding to a "double-triple" Regge limit. This limit may be calculated using an O(2,1) expansion and helicity poles[9] or by an extension of the techniques of Misheloff.[10] The resulting formula is

$$\frac{d\sigma}{(d^3q_1/\omega_1)(d^3q_2/\omega_2)}=s^{-1}\sum_{M=0}^{\infty}(\mathfrak{M}^2)^{\alpha_v(0)}\cos M\varphi(s_1/\mathfrak{M}^2)^{2\alpha_1(t_1)}(s_2/\mathfrak{M}^2)^{2\alpha_2(t_2)}F_M(t_1,t_2), \tag{5}$$

with t_1 and t_2 fixed, and $s_1/\mathfrak{M}^2$, $s_2/\mathfrak{M}^2$, $\mathfrak{M}^2\to\infty$, i.e., $1-x_1\to 0$, $1-x_2\to 0$, but $(1-x_1)(1-x_2)s$ large. Here again, if the Pomeranchuk singularity is a simple pole, only the term $M=0$ will survive. The presence of a cut would require $M=0$ and $M\neq 0$ as well as logarithmic corrections to the simple power laws.

Finally we observe that if $\alpha_v(0)$ is a simple pole both Eqs. (1) and (5) scale when divided by $\sigma_T(s)$ which goes asymptotically as $s^{\alpha_v(0)-1}$. Furthermore, if Regge-pole factorization occurs in the relevant discontinuities, then, in Eq. (1),

$$F_M=\delta_{M0}f_1(t_1,s_1/\mathfrak{M}^2)f_2(t_2,s_2/\mathfrak{M}^2) \tag{6}$$

and we would expect f_1 and f_2 to be proportional to single-particle inclusive distributions in the fragmentation region.

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$(\ln s)^{-|M|}$ relative to the leading $M=0$ cut contribution for two particles of spin $>\frac{1}{2}$.

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K_{l3} Form Factors and the Scaling Behavior of Spin-0 Fields

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Motivated by scaling considerations we formulate a theory of K_{l3} decay based on the Kemmer equation. We find that our theory (1) has a symmetry-breaking parameter of order 10-30% which agrees with experiment, (2) makes a *definite testable prediction* of a kinematic zero at $t=(m_K+m_\pi)^2$ in the "effective" scalar form factor, and (3) yields a modified Callan-Treiman relation which improves agreement with experiment.

It is well known that pseudoscalar particles can be described by at least two different covariant field equations, namely, the Klein-Gordon (K-G) and Kemmer equations.[1] For many classes of processes (such as the quantum electrodynamics of spin-0 mesons) calculations based on either equation yield identical results.[2] Therefore, it has often been tacitly assumed that the two equations will yield identical results in all cases.

We note, however, that the K-G and Kemmer fields behave differently under scale transformations [see Eqs. (2) and (3) below]. Since symmetry-breaking effects are sensitive to the scale dimensionalities of the respective fields,[3] it follows that in principle the corresponding field theories could lead to qualitatively different results in processes, such as K_{l3} decays, involving two or more pseudoscalar mesons of different mass.

To date this possibility has not been studied in a formal field theory owing to the difficulties involved in treating the strong interactions. Nonetheless, the scaling argument suggests that even in a *phenomenological* treatment of such a process, the same assumptions could yield qualitatively different results from analyses based on the K-G and Kemmer equations, respectively. If different results were obtained, we could then ascertain which equation gives the better phenomenological description of the particular process in question.

We present in this Letter the results of just such an analysis of K_{l3} decays. Our main conclusions are the following:

(1) When compared with experiment, our theory yields a symmetry-breaking parameter ρ (analogous to the K-G parameter ξ) whose experimental magnitude is $\rho=0.28\pm0.20$. Since SU(3)-symmetry breaking is expected to be of this