Intrinsic Fluctuations in the Driven Josephson Oscillator*

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Rounding of the steps induced in the current-voltage characteristic of small-capacitance superconducting weak links by microwave radiation has been measured and found to confirm predictions of Stephen's theory of phase fluctuations in the driven Josephson oscillator.

Theoretical descriptions of the effects of thermal fluctuations on superconducting weak links have recently been developed by forming a Fokker-Planck equation for the distribution function in precise analogy to the problem of a particle in a tilted periodic potential subject to the buffeting of a Langevin force. Results have been worked out for both the dc and ac Josephson effects and for the driven Josephson oscillator.[1-3] Recent measurements of the dc current-voltage characteristics of weak links confirm the applicability of this approach to the dc Josephson effect.[4] However, the linewidth of the radiation emitted by a Josephson oscillator has been reported to be a factor of 2 wider than predicted[5]; this result suggests some difficulties with the ac effects. Therefore, we have undertaken to measure the current-voltage characteristic of a driven Josephson oscillator where the dc and ac effects meet in order to explore the scope of this appealing theoretical approach to fluctuations in weakly coupled superconductors.

We report here the results of observations on a superconducting weak link designed to provide a quantitative test of the corresponding theory, due to Stephen,[3] for the shape of induced steps in the current-voltage characteristic. Application of rf or microwave radiation of frequency ν_{rf} locks the phase of the Josephson oscillator, producing steps of nearly constant voltage in the dc current-voltage characteristic of the junction at integral multiples of the voltage $h\nu_{rf}/2e$ as shown in Fig. 1(a). Intrinsic fluctuations in the quasiparticle and pair currents are supposed to degrade this phase locking, leading to an observable tilting and rounding of the steps. We have measured many complete step profiles and have found quite good agreement with Stephen's theoretical profiles in a one-parameter fit. The fitting parameter was independently checked by measuring the shape of the "zero-voltage step," i.e., the dc Josephson effect in the absence of radiation. These measurements incidentally confirm the observations of Simmonds and Parker[4] on the dc Josephson effect and their agreement with the theory of Ambegaokar and Halperin.[1]

Stephen has shown that the description of fluctuations in the driven Josephson oscillator can be put into exactly the same form as the description of fluctuations in the dc Josephson effect.

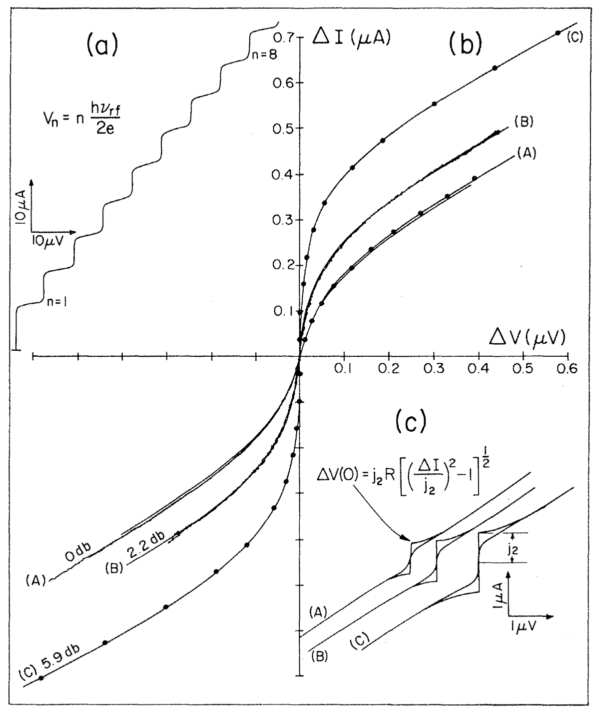

FIG. 1. (a) Typical *I-V* tracing with sufficient rf (3.14 GHz) power applied to the junction to induce many steps. (b) Development of a step as rf power is first increased (relative powers shown) for $T = 3.801$ K, $\nu_{rf} = 3.14$ GHz, and step number $n = 1$. Data are represented as actual tracings and as points (cf. text). Smooth solid lines are one-parameter (j_2) fits to data using Eq. (1). Relevant parameters: Curve *A* (entire step), $j_2 = 0.3450$ μA, $a_2 = 1.986$; (step center) $j_2 = 0.3353$ μA, $a_2 = 1.930$. Curve *B*, $j_2 = 0.4588$ μA, $a_2 = 2.642$. Curve *C*, $j_2 = 0.6621$ μA, $a_2 = 3.814$. (c) Lower-resolution tracings of same steps as in (b), along with theoretical curves for the corresponding cases with no fluctuations [$\Delta V(0)$], using the fitted j_2 valves.

In the limit of low junction capacitance the Fokker-Planck equation leads directly to an equation for the current-voltage relationship at an induced step in terms of the dimensionless parameters a_2 (which provides a measure of the barrier height of the periodic potential relative to the thermal energy k_BT) and a_1 (which provides a measure of the tilt of the potential) that is the driving force for uncoupling of the phase locking. Stephen's result may be written as[6]

$$\Delta V = (Rj_2/2a_2)\sinh(\pi a_1) \times \left[\int_0^{\pi/2} \cosh(2a_1 y) I_0(2a_2 \cos y)\,dy\right]^{-1}, \qquad (1)$$

where

$$a_2 = (\hbar j_2/2ek_BT)V_0/RJ$$

and

$$a_1 = (\hbar \Delta I/2ek_BT)V_0/RJ;$$

ΔV and ΔI respectively express the voltage and current along the step measured relative to the step center, $2j_2$ is the step height in the absence of noise, R is the dynamic resistance of the junction, J and V_0 are, respectively, the current and voltage at the step center, T is the temperature, and I_0 is a modified Bessel function.

Notice that j_2 is the natural fitting parameter since it is the only parameter in Eq. (1) not *directly* determinable from experiment. At the step center $\Delta I \to 0$ and Eq. (1) gives the minimum dynamic resistance $R_c = R/I_0^2(a_2)$. At low temperatures and high rf power, a_2 can become very large and the step extremely sharp. For example, Clarke's experiments[7] on h/e suggest values of $R_c \lesssim 10^{-14}\ \Omega$ for $a_2 \approx 300$. However, broader-step profiles that are rather easily measured over the entire step can be obtained by selecting temperature and rf power such that a_2 is relatively small.

We have measured step profiles of sequences of steps in the range $0.2 \lesssim j_2 \lesssim 2\ \mu A$ at several temperatures, frequencies of 0.50 to 3 GHz at power levels varied by a factor of 10^2 in a thoroughly shielded apparatus with filtering similar to that found effective in other sensitive fluctuation experiments.[8] Temperature control was adequate at $\pm 10\ \mu K$ with thermometer and heater well shielded from the sample, and variations in the rf drive oscillators were negligible. A mechanically and thermally stable niobium point-contact junction with shunt capacitance[9] $C < 0.5$ pF and normal resistance $R_N = 1.46\ \Omega$ at 4 K was fabricated for this experiment following Buhrman, Lukens, Strait, and Webb.[10] Radiation was coupled into the shielded enclosure containing the junction via a coaxial cable to an antenna loop positioned to radiate an electric field parallel to the junction axis. Thermal radiation transmitted from room temperature was negligible.

Figure 1 displays some representative results. A typical sequence of the steps induced by microwave radiation in the I-V characteristic of a junction are shown in the upper left corner, Fig. 1(a). The set of curves in the center, Fig. 1(b), shows the development of a step as rf power is increased. In Fig. 1(b) case (B) displays tracings of the actual data recorded going both up and down a step in a time span of ≈3 min. The bottom half of case (A) is also a tracing (up only). For clarity in comparison with theory curves, the rest of the data are displayed as points. The smooth solid lines are one-parameter fits of Eq. (1) to the data, using only j_2 as the fitting parameter. Values of T, V_0, R, and J were independently measured for each step. For case (A), two theoretical curves are shown as a measure of the uncertainty of the fit; the longer one is the least-squares fit to the entire data curve, the shorter one is the best fit to points near the step center. The latter value of j_2 is the smaller by ≈3% here and in no case by more than 5%. These results indicate quite good agreement between theory and experiment, the slight imperfections in the fits being just barely resolvable. In Fig. 1(c) we have plotted, along with lower-resolution tracings of the steps, the corresponding theoretical curves for the hypothetical case of no fluctuations using the fitted values of j_2.

We have varied a_2 (which determines the sharpness of the step) by varying temperature, rf power (hence j_2), and the ratio of the absolute resistance (V_0/J) to the dynamic resistance R. (This ratio is a function of the step number and radiation frequency.) Figure 2 summarizes results of these parameter variations for several steps. The circles and triangles are points taken from experimental step profiles and normalized with the step height; the solid lines are again the best theoretical fits to these points. This semilogarithmic plot provides a sensitive display of the fit of theory to experiment and demonstrates good agreement over the entire range of parameters. Note that curve (2) corresponds to case (C) of Fig. 1. For the points along (7) and (8), curve (8) is the best fit to the bottom two points (i.e., the step center) and (7) the best fit to all points (the entire step). Curves (9) and (10) are exam-

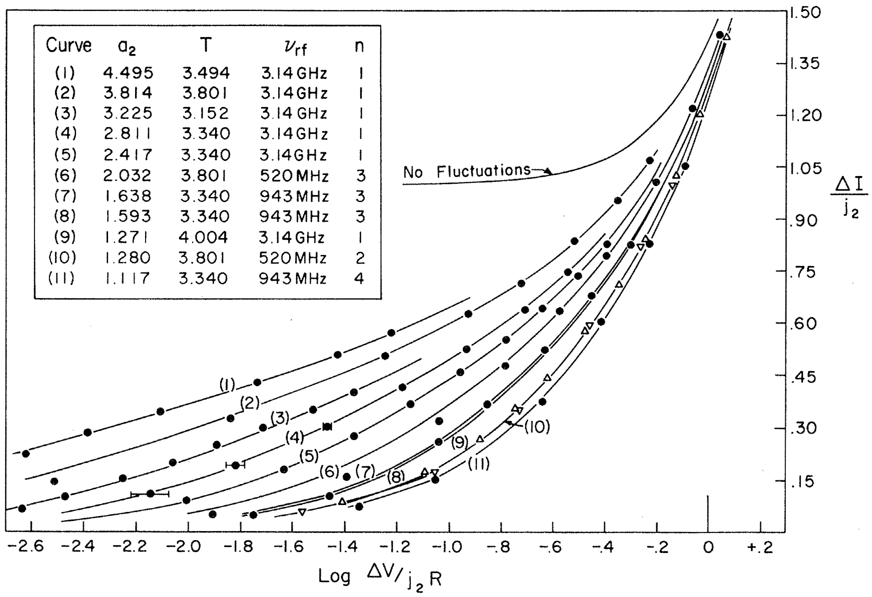

FIG. 2. Semilog plot of normalized current and voltage for several steps (upper halves only) for various parameter variations. Circles and triangles are data points. Solid curves are again best theoretical fits to the data points. Error bars correspond to the experimental voltage uncertainty of ±1.5 nV.

ples of steps of nearly the same height but frequencies near opposite ends of the range studied. Only the step in curve (6) shows a significant deviation from the theory. This sort of deviation occurred in cases where the critical current was depressed very nearly to zero and appears to be due merely to distortion of the step profiles by mutual overlap.

There is close correspondence between the theory of intrinsic noise in the dc Josephson effect and the driven ac effect that suggests that agreement with the theory in one case should be accompanied by agreement in the other. Therefore, we measured the I-V characteristic to determine the fluctuation rounding of the dc Josephson effect in our case and found close agreement with the corresponding theory (using only I_1, the critical current without noise, as the fitting parameter). These results confirm the measurements of Simmonds and Parker on low-capacitance thin-film weak links, including the small discrepancy in the shape of the I-V characteristic that they observed near its point of maximum curvature in the case of large coupling parameter $\gamma = \hbar I_1/ek_BT$.[11]

Direct connection between the dc and driven-ac data can be made through the phenomenological formula[12] relating the step amplitude j_2 to the critical current I_1 and the rf voltage in the junction:

$$j_2(n) = I_1(t)|J_n(2eV_{rf}/h\nu_{rf})|. \qquad (2)$$

The Bessel-function dependences of the step amplitudes j_2 were readily observed and provided reasonable measures of V_{rf} which were used to compare j_2 and I_1 at three temperatures. These values correspond to within 5%, well within the experimental uncertainty. The variation of j_2 with T, V_{rf}, and I_1 was predictable, and we saw no indication of recently reported anomalies in V_{rf}.[13]

Several minor points raised by Lee's recent extension of Stephen's theory require comment.[14] Lee has treated the effect of finite bandwidth of the external oscillator. In our case this effect was entirely negligible. In addition, Lee has indicated that the proper criterion to reach the low-junction-capacitance limit (where the simple theory applies) is $\Omega^2 \equiv (RC)^2 2eI_1/\hbar C \ll 1$ or $\Omega_1{}^2 \equiv (RC)^2 \times 2ej_2/\hbar C \ll 1$. Both these criteria and Stephen's are satisfied. Lee also avoided the explicit assumption $2eV_{rf}/h\nu_{rf} \ll 1$ that appeared in Stephen's derivation. Our results seem to confirm that this assumption is not necessary since good fits of the step profiles were obtained throughout the range $0.45 \lesssim 2eV_{rf}/h\nu_{rf} \lesssim 4.5$. Finally we note that the interesting region of large a_2, i.e., large j_2 or low T in the driven Josephson oscillator, corresponds to large γ in the dc effect where noticeable deviations from theory appeared. This regime has not been studied in our ac experiments although it would be accessible using a superconducting quantum voltmeter if a motive arose to pursue the small deviations.

In summary, we have measured step profiles of the driven Josephson effect in low-capacitance point-contact junctions in a regime where intrinsic thermal fluctuation rounding is substantial and external noise was effectively excluded. We have found that the data agree well with Stephen's theory and its extensions by Lee for intrinsic noise in a Josephson junction, within the range of their approximations and the accuracy of the data. Single-parameter fits yielded step heights j_2 corresponding to the dc critical currents I_1 and noise temperatures equal to the actual experimental temperatures. These results indicate that the simple theory in which thermal noise is introduced as a Langevin force and a periodic potential is introduced by phase locking to the incident radiation provides a rather good description of the driven Josephson oscillator that is consistent with the corresponding phenomenon in the dc Josephson effect. Thus the general theory appears to be basically correct, and we presume that the factor-of-2 discrepancy in the previously reported linewidths of the Josephson radiation must be due either to experimental error (perhaps external noise) or to an aspect of the theory special to the radiation-emission problem.

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Landau-Level Structure in Magnetoreflection at the 2-eV Saddle Point in InSb†

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The first observation of Landau levels at an M_1 saddle point is reported at the E_1 edge of InSb using a circular-polarization modulation magnetoreflection technique. The transverse reduced mass is measured to be $m/\mu_t^* = 19.7 \pm 1.3$. The importance of the electron-hole interaction is discussed.

A new magnetoreflection technique[1] is applied to observe quantized harmonic-oscillator (Landau) levels in InSb at an energy threshold higher than the fundamental edge. In this method an optically polished and freshly etched[2] sample of undoped InSb[3] is mounted in a strain-free manner in the central field region of a superconducting solenoid. Monochromatic light is reflected from a (111) face of the sample at near-normal incidence. The light is circular-polarization modulated before reflection by passing it through a plane polarizer and a resonant fused-quartz block