the identity

$$(\partial P/\partial x)_y = \rho(\partial \mu/\partial x)_y + s(\partial T/\partial x)_y \quad (3)$$

with respect to y at fixed x, and average the resulting equation at neighboring points on opposite sides of the coexistence curve, we find that

$$(\partial^2 P/\partial x\, \partial y)_d = \tfrac{1}{2}[(\partial^2 P/\partial x\, \partial y)_{x_c^+, y} + (\partial^2 P/\partial x\, \partial y)_{x_c^-, y}] \doteq [\dot{\rho}_d(\partial \mu/\partial x)_y + \dot{s}_d(\partial T/\partial x)_y](\partial T/\partial y)_x, \quad (4)$$

where by $a \doteq b$ we mean that $|a-b|$ remains bounded as $T \to T_c$.

We interpret the Griffiths-Wheeler hypothesis to mean that apart from accidental cancelations or peculiarities introduced by special symmetries, any singular structure in the pressure is independent of the particular form chosen for the function $y(\mu, T)$.[7] But the choice $y \equiv T$ (permissible if the tangent to the coexistence curve is not parallel to the μ axis at T_c) gives

$$(\partial^2 P/\partial x\, \partial y)_d \doteq \dot{\rho}_d(\partial \mu/\partial x)_T, \quad y(\mu, T) \equiv T, \quad (5)$$

and the choice $y \equiv \mu$ (permissible if the tangent to the coexistence curve is not parallel to the T axis at T_c) gives

$$(\partial^2 P/\partial x\, \partial y)_d \doteq \dot{s}_d(\partial T/\partial x)_\mu \dot{\mu}^{-1}, \quad y(\mu, T) \equiv \mu. \quad (6)$$

Thus the Griffiths-Wheeler hypothesis requires in general that $\dot{\rho}_d$ and $\dot{s}_d$ have the same singularity at T_c, provided that the tangent to the coexistence curve at T_c is not parallel to either axis of the μ-T plane. In view of the exact identity (2), this singularity must be at least as large as that of the constant-volume specific heat.

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‡National Science Foundation Pre-Doctoral Fellow.

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[6]A similar argument taken together with (1) implies that any possible divergence in $\dot{\mu}(T)$ cannot be stronger that $(T_c-T)^{-(\alpha'+\beta)}$.

[7]Like the use of the hypothesis in Ref. 1, this is a somewhat stronger application than those made by Griffiths and Wheeler, who apply it primarily to the leading singularity in the second derivatives of P.

Production of Thermonuclear Power by Non-Maxwellian Ions in a Closed Magnetic Field Configuration*

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A toroidal plasma heated by an energetic neutral beam (thus consisting of an energetic ion component and a lower-energy bulk plasma) can produce net thermonuclear power under conditions far less restrictive than Lawson's criterion.

A thermonuclear plasma with a Maxwellian ion distribution must meet Lawson's minimum condition[1] on $n_i\tau_E$ (the product of ion-density and plasma-energy confinement time) in order to generate sufficient thermal power to recreate the electrical power invested in producing the plasma. We will consider the generalization of Lawson's criterion for the case where the bulk-plasma temperatures T_e and T_i are maintained by injection of a neutral beam that gives rise to an energetic ion component of density $n_h \ll n_i$ and initial energy $W_0 \gg T_e, T_i$.

If, on the average, an injected ion produces F times its initial energy in fusion reactions, before slowing down in the bulk plasma, then the condition for net electrical-power generation can be written approximately in the form

$$\eta_n F > 1 - \eta_p. \quad (1)$$

The efficiencies of conversion to electrical energy from fusion-reaction energy and plasma energy are given, respectively, by η_n and η_p. For simplicity, we neglect the fusion reactions of the bulk-plasma ions with each other and of the energetic ions with each other, as well as the favorable effects arising from the deposition of charged fusion-reaction products within the plasma. On the other hand, we also neglect the rate of energetic ion loss, τ_{hl}^{-1}, from the plasma volume, assuming a choice of parameters such that the rate of thermalization, τ_{hc}^{-1}, within the plasma is several times higher. We also assume that the neutral-beam injection efficiency is very high.[2]

We find that Eq. (1) is readily satisfied for an energetic deuteron plasma within a bulk plasma of tritium ions: The obtainable F factors are as large as 2-3, assuming the standard fusion energy release of 22.4 MeV, and could be increased by an appropriate blanket design. From the thermodynamic point of view, η_p could be close to unity,[2] but even for the value $\frac{2}{3}$ and the standard choice $\eta_n = \frac{1}{3}$, Eq. (1) can be met for $F > 1$.

To permit appreciable F values, the electron temperature must be in the keV range. The requirement to maintain this temperature establishes threshold conditions for both the energetic-ion component and the bulk plasma, which resemble Lawson's criterion but are considerably relaxed. Surprisingly, it is found that even with a bulk plasma of *cold* tritium ions the conditions for net power production are favorable. For sufficiently high n_i/n_h, the bulk plasma must satisfy $T_i \approx T_e$. To realize the largest possible F values, both T_e and T_i then enter the thermonuclear range. In this limit the bulk-plasma reactions become dominant, but the proper choice of neutral-beam parameters still facilitates the attainment of the thermonuclear-ignition condition.

While the reactor regime described here may appear rather unusual, it is closely related to previous literature. The condition that the injected deuterons should multiply their injection energy while thermalizing is similar to the basic condition for open-ended thermonuclear reactors.[2] The possibility of obtaining an appreciable energy multiplication factor with an energetic-ion beam and a hot-electron target is mentioned explicitly by Powell,[3] but the process is judged to be of no practical value. We have also encountered two early unpublished proposals[4,5] for target-plasma fusion reactors, with somewhat unrealistic numbers and configurations.

For the practical implementation of the two-component approach, a neutral-beam-injected Tokamak configuration[6] seems to be ideally suitable. The use of adiabatic compression and decompression[7] should permit the attainment of efficiencies η_p at least as large as $\frac{2}{3}$.

An ion injected into the bulk plasma at initial energy W_0 slows down, by Coulomb scattering, approximately according to the equation

$$\frac{dW}{dt} = -\frac{4\pi e^4 \ln\Lambda}{v}\left[\frac{n_i}{m_i} + \frac{4}{3\sqrt{\pi}}\frac{n_e}{m_e}\left(\frac{m_e W}{m_h T_e}\right)^{3/2}\right]. \quad (2)$$

Here we assume singly charged ions, with energetic- and target-ion masses m_h and m_i, respectively. The velocity v of the energetic ions is assumed large compared with that of a bulk ion and compared with that of an electron at temperature T_e. The electron density n_e is given by $n_e = n_i + n_h$, but we will assume $n_h \ll n_i$, so that $n_e = n_i$. The quantity $\ln\Lambda$ is about 20 for the parameter range of present interest.

An illustrative case calculated from the exact relation[8] for dW/dt is given in Fig. 1. A 180-keV deuteron is decelerated by a tritium target plasma with $n_i = n_e$, $T_e = 5$ keV, and $T_i = 0$. The instantaneous probability of fusion is also shown; the time integral of this curve gives a fusion probability factoı $f = 0.0115$. With a total energy

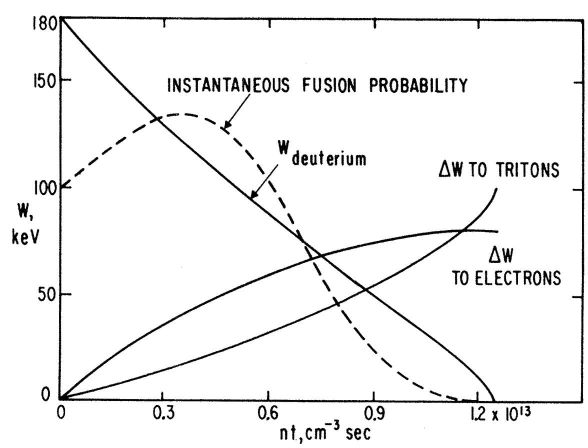

FIG. 1. 180-keV deuteron slows down in bulk tritium plasma of $T_e = 5$ keV, $T_i = 0$. Instantaneous probability of fusion and division of deuteron energy between electrons and tritons are also shown. (The $\ln\Lambda$ factors are calculated at $n_p = 3\times10^{13}$ cm^{-3}.)

yield of $E = 22.4$ MeV per fusion reaction, the ratio of mean-energy release to injected energy is given by $F = fE/W_0 = 1.43$. Figure 1 also shows the relative magnitude of the energy losses to the target electrons and ions. As can be seen from Eq. (2), the electron-drag term can be eliminated by going to sufficiently large T_e.

Results for a variety of injection energies and electron temperatures are summarized in Fig. 2. We see that energy multiplication factors greater than 1 are readily obtained, even for cold target ions and for target-electron temperatures well below the characteristic range (10-20 keV) of one-component toroidal-reactor regimes.

The time scale for thermalization of the energetic ions may be written in the form

$$n_i \tau_{hc} = (\sigma_c v)_0^{-1}, \tag{3}$$

with the latter quantity to be calculated from Eq. (2) at the energy of injection: $(\sigma_c v)_0 = (dW/dt)_0 \times (n_i W_0)^{-1}$. If we write Eq. (3) in terms of the energetic-ion loss time τ_{hl} rather than in terms of thermalization time τ_{hc} (initially assumed to be shorter), we arrive at the more severe condition $n_i \tau_{hl} \gg (\sigma_c v)_0^{-1}$.

A second "$n\tau$-type" criterion can be derived from the balance of neutral-beam input power and bulk-plasma energy loss rate τ_E^{-1}:

$$n_h W_0/\tau_{hc} \geqslant n_i (T_i + T_e)/\tau_E. \tag{4}$$

Using Eq. (3), we find that

$$n_h \tau_E \geqslant (\sigma_c v)_0^{-1} (T_i + T_e)/W_0. \tag{5}$$

There is a range of possible operating regimes,

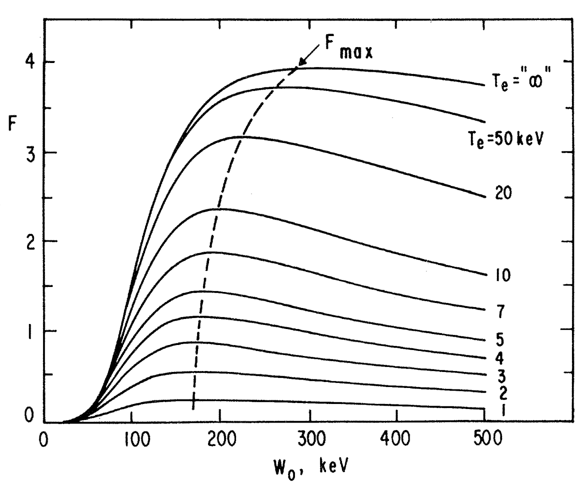

FIG. 2. Energy multiplication factor F as a function of deuteron injection energy W_0 for various electron temperatures of cold-triton-target plasma, assuming total energy release of 22.4 MeV.

characterized by the free parameter $\Gamma = n_h W_0 / n_i (T_i + T_e)$, which measures the relative energy content of the hot ions and the bulk plasma. Thus, we may write the conditions

$$n_i \tau_E \geqslant \Gamma^{-1} (\sigma_c v)_0^{-1}, \tag{6a}$$

$$n_h \tau_{hl} \gg n_h \tau_{hc} = \Gamma (\sigma_c v)_0^{-1} (T_i + T_e)/W_0, \tag{6b}$$

which characterize the separate "$n\tau$" of each component.

If we take $\Gamma < 1$, then the condition on $n_h \tau_{hl}$ is relaxed at the expense of $n_i \tau_E$. Another consequence is that we must have $\tau_{hc} < \tau_E$, and since the electron-ion equilibration time of the bulk plasma is comparable with τ_{hc}, it follows that $T_i \approx T_e$.

If we take $\Gamma > 1$, then the condition on $n_i \tau_E$ is relaxed at the expense of $n_h \tau_{hl}$. In this case it becomes possible to have $T_i \ll T_e$, since $\tau_{hc} \geqslant \tau_E$ is permissible. [Strictly speaking, we must then replace Eq. (4) by separate equations for the bulk ions and electrons, but for the low-T_e range the neutral-beam heat input rate into the electrons remains close to τ_{hc}^{-1}.]

The most interesting operating range appears to be $\Gamma \gtrsim 1$. The requirements on the bulk plasma are then conveniently relaxed, in respect to both $n_i \tau_E$ and T_i/T_e, while the requirements on the energetic-ion parameter $n_h \tau_{hl}$ remain very lenient. We note that $\Gamma = 1$ maximizes the fusion power density for fixed plasma energy density, giving optimal ratios slightly above those for a Maxwellian. The limit $\Gamma \ll 1$ is of interest chiefly as a transition stage on the way to an ordinary Maxwellian thermonuclear reactor.

In Fig. 3, we compare Eqs. (3) and (5) with the standard Lawson criterion. The two-component "$n\tau$" values are very favorable. (The improvement with decreasing T_e, however, is exaggerated by this graph, since F also drops, as shown, thus requiring higher η_p.)

In the preceding analysis we have assumed that the injected ions lose their energy predominantly to the background plasma, and that the loss rate is classical. We now examine these assumptions briefly.

If the ions are injected as a single directed beam, the excitation of various microinstabilities is highly probable for the parameter range of interest here. Consequently, the injection of a nearly isotropic distribution function is indicated. In contrast to the case of open-ended systems, such a distribution can be confined in a closed configuration. In an axisymmetric torus

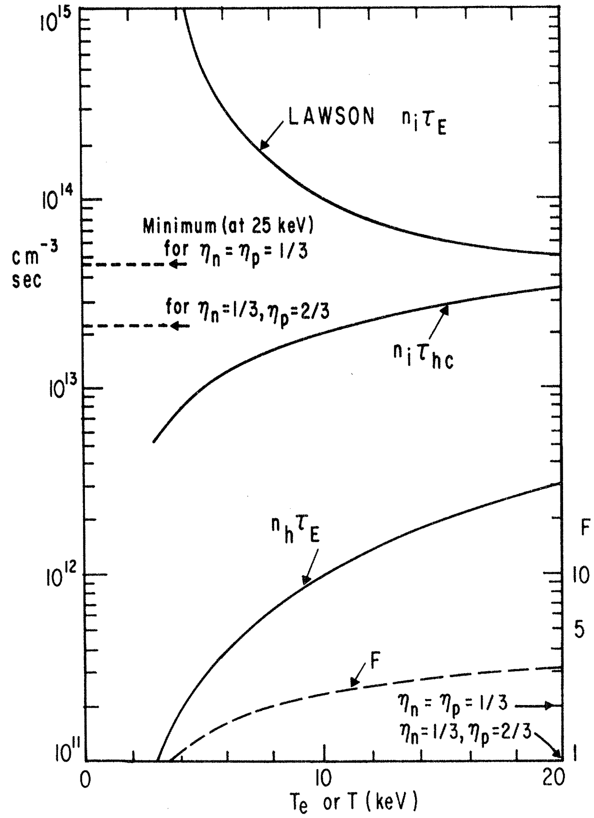

FIG. 3. Lawson criterion for $n_i\tau_E$, assuming $\eta_n = \eta_p = \frac{1}{3}$, with Maxwellian plasma (equal parts of deuterium and tritium) and $T_e = T_i = T$. Minima of $n_i\tau_E$ are shown for both $\eta_p = \frac{1}{3}$ and $\frac{2}{3}$. $n_i\tau_{hc}$ and $n_h\tau_E$ are given as functions of T_e, assuming deuterons with W_0 corresponding to maxima of F in Fig. 2, and cold bulk tritons. Corresponding F values are shown: $F \geq 1$ is required for $\eta_n = \frac{1}{3}$ and $\eta_p = \frac{2}{3}$, and $F \geq 2$ for $\eta_n = \eta_p = \frac{1}{3}$.

the isotropy is limited only by the requirement that the ion "banana" orbits[9] should be small compared with the plasma radius. In a stellarator the avoidance of loss cones in velocity space[9] imposes difficult design conditions, and may require operation with an imperfectly isotropic energetic-ion component. In that case the use of a mirror machine with toroidal target plasma may actually be preferable. To avoid loss-cone phenomena, the most practical confinement geometry is the Tokamak,[6] and this choice will be assumed in the following discussion. Even in the Tokamak, the finite size of "banana" orbits causes some velocity-space anisotropy, and so a confining field based on several megamperes of plasma current may be desirable.

The presence of a departure from the Maxwellian-ion energy distribution is implicit in the present scheme, but the required distribution need not be unstable against microinstabilities. Assuming again that we have $\tau_{hi} \gg \tau_{hc}$, the steady-state distribution function f_h in terms of energetic-ion velocity v follows from $f_h v dW/dt = \text{const}$ and Eq. (2), and is seen to be monotonically decreasing with v.

In respect to hydromagnetic equilibrium and stability, the two-component system has some advantages. The required total plasma pressure can be substantially lower than for the one-component case: This is mainly because the plasma energy can be concentrated in the energetic deuterons, rather than being distributed among deuterons, tritons, and electrons. While the usual one-component reactor design for a Tokamak is slightly marginal in respect to hydromagnetic modes,[10] the two-component design appears amply safe.

In respect to limitations by various density-gradient and temperature-gradient–driven microinstabilities, including trapped-particle modes,[9] the principal advantage of the two-component approach is the ability to operate with relatively low target-plasma temperatures and short energy-confinement times. Similar considerations apply with respect to radiation and charge-exchange losses. The target-plasma parameters required are not far removed from those already realized in the present small-sized Tokamak experiments,[11,12] and should be readily attainable in the next generation of experimental devices (designed to operate with megampere plasma currents).

The conventional toroidal-reactor design has the advantage of providing very large energy multiplication factors (of order 10^2). In this sense, the conventional design may be considered to have a greater margin of safety for its ultimate success. The conventional design also does not require energy input after thermonuclear ignition—while the cold-target system merely acts to multiply the injected power by moderate factors. In this sense, the conventional design has an economic advantage for the production of very large levels of output power.

The principal usefulness of the cold-target system emerges in the range of moderate output power (below 1000 MW of electrical power), which is virtually inaccessible to the standard toroidal reactor. The highly efficient production of neutral beams, with 100- to 1000-A currents at 100- to 300-keV energies, seems to be within the reach of present technology, thus providing the possibility of reactors in the 10- to 300-MW range.

We note, finally, that there is a natural evolution from the cold-target reactor to the conventional thermonuclear reactor. The first approach creates the possibility of conveniently sized fusion-power producers; as the target plasmas become larger and hotter, the operation becomes increasingly advantageous for low-cost power.

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Transport of Intense Relativistic Electron Beams in a z Pinch*

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We have demonstrated efficient guidance and transport of high-ν/γ electron beams along a z pinch. Pinch currents substantially smaller than the beam current are sufficient to contain and propagate the beam. The data suggest single-particle motion of beam electrons in the pinch magnetic field. Numerical calculations of possible trajectories correlate with observed beam expansion and corroborate the single-particle analysis.

Intense relativistic electron beams ($\geqslant 10^5$ A) in neutral gases suffer losses due to the initial lack of charge neutralization and to the back emf induced by the rapidly rising current ($\sim 10^{13}$ A/sec).[1] A solution to these problems is the preionization of the background gas, which results in rapid charge neutralization and nearly complete current neutralization (due to backstreaming plasma electrons).[2] In this way electric fields are quenched. However, the beam's self-magnetic field is also quenched, and electrons escape radially[3] with a transverse motion that arises from the beam's self-magnetic interaction in the generating diode.

A way to avoid this latter problem while retaining the advantages of preionization is to inject the beam into a plasma with a trapped azimuthal magnetic field. This field, an externally applied substitute for the self-fields, would presumably result in radial containment of the beam. Roberts and Bennett[4] have reported the transport in a z pinch of a beam with a 5° divergence half-angle (little transverse energy, $\nu/\gamma \sim 0.2$[5]). We have applied this technique to beams with an ~60° divergence half-angle (high transverse energy, $\nu/\gamma \approx 7$) and have propagated them with negligible loss using magnetic fields substantially less than the self-field of the beam in the diode.

Figure 1 shows the configuration of a z pinch and a beam-generating diode. The z pinch, which utilized argon at 300 μm, had a peak current of 225 kA at $t = 4.6$ μsec, and maximum compression occurred at $t = 3.6$ μsec. The Pyrex discharge tube was 60 cm long and 10.2 cm