cycle time of 36 $\mu$sec. $C_6F_6$, $SrF_2$, and $CaF_2$ were used to tune the spectrometer and check the chemical-shift scaling factor. The linewidths of polycrystalline $CaF_2$ taken at the same time as the $MgF_2$ data, 210-230 Hz, can be used to estimate the instrumental resolution.

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[1]U. Haeberlen and J. S. Waugh, Phys. Rev. 175, 453 (1968).

[2]J. S. Waugh, C. H. Wang, L. M. Huber, and R. L. Vold, J. Chem. Phys. 48, 662 (1968).

[3]J. S. Waugh, L. M. Huber, and U. Haeberlen, Phys. Rev. Lett. 20, 180 (1968).

[4]For example, see D. S. VanderHart, H. S. Gutowsky, and T. C. Farrar, J. Chem. Phys. 50, 1058 (1969), or W. Derbyshire, J. P. Strart, and D. Warner, Mol. Phys. 17, 449 (1969).

[5]For example, P. Dichl and C. L. Khetrapal, *NMR: Basis Principles and Progress* (Springer, New York, 1969), Vol. 1.

[6]P. C. Lauterbur, Phys. Rev. Lett. 1, 343 (1958).

[7]P. C. Lauterbur and J. J. Burke, J. Chem. Phys. 42, 439 (1965).

[8]M. Bensousean, J. Phys. Chem. Solids 28, 1533 (1967).

[9]R. W. G. Wyckoff, *Crystal Structures* (Interscience, New York, 1960), Vol. I.

[10]C. H. Dungan and J. R. Van Wazer, *Compilation of Reported $F^{19}$ NMR Chemical Shifts* (Interscience, New York, 1970), Appendix I.

[11]M. R. Baker, C. H. Anderson, and N. F. Ramsey, Phys. Rev. 133, A1533 (1964).

[12]S. I. Chan and A. S. Dubin, J. Chem. Phys. 46, 1745 (1967).

[13]D. Ikenberry and T. P. Das, Phys. Rev. 138, A822 (1965).

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# Generality of the Singular Diameter of the Liquid-Vapor Coexistence Curve*

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It is argued that except in special cases the diameter of the liquid-vapor coexistence curve should have a temperature derivative at least as singular as the constant-volume specific heat. No assumptions are made about the equation of state near the critical point other than the hypothesis of Griffiths and Wheeler that the only direction in the $\mu$-$T$ plane "singled out by the nature of the phase transition itself is a direction parallel to the coexistence curve."

Green, Cooper, and Levelt Sengers[1] have proposed a generalization of the parametric equation of state of Schofield[2] which, when combined with a suggestion of Griffiths and Wheeler,[3] implies in general that the derivative of the coexistence-curve diameter, $d\rho_d(T)/dT=(d/dT)\frac{1}{2}[\rho_L(T)+\rho_V(T)]$, is at least as singular as the constant-volume specific heat.[4] We wish to point out that this conclusion is independent of the equation of state proposed in Ref. 1, following from the Griffiths-Wheeler hypothesis alone.

We start with an exact equation for the constant-volume specific heat in the two-phase region:

$$C_V(\rho,T)=T[\dot{s}_d+\dot{\mu}\dot{\rho}_d+\ddot{\mu}(\rho_d-\rho)], \tag{1}$$

where $s_d$ is the entropy-density diameter, $s_d=\frac{1}{2}(s_L+s_V)$, $\mu(T)$ gives the coexistence curve in the $\mu$-$T$ plane, and a dot denotes temperature differentiation. It can be shown by a straightforward extension of Fisher's proof[5] of the Rushbrooke inequality that the specific-heat singularity in the two-phase region has the same strength whether the critical point is approached along the critical isochore or along the diameter.[6] Thus,

$$C_V(\rho_c,T)\approx C_V(\rho_d(T),T)=T(\dot{s}_d+\dot{\mu}\dot{\rho}_d), \tag{2}$$

where by $a\approx b$ we mean that $a/b$ approaches a finite nonzero limit as $T\to T_c$.

Now Griffiths and Wheeler[3] have suggested that in considering the pressure as a function of $\mu$ and $T$ near the critical point, the only significant direction in the $\mu$-$T$ plane for systems lacking special symmetries is that of the coexistence curve. Suppose then that we adopt variables $x(\mu,T)$ and $y(\mu,T)$ that are regular functions of $\mu$ and $T$, such that lines of constant $y$ intersect the coexistence curve at nonzero angles, and the coexistence curve itself is given by $x=x_c$ (so that lines of constant $x$ are "asymptotically parallel to the coexistence curve"). If we differentiate the identity

$$(\partial P/\partial x)_y = \rho(\partial\mu/\partial x)_y + s(\partial T/\partial x)_y \tag{3}$$

with respect to $y$ at fixed $x$, and average the resulting equation at neighboring points on opposite sides of the coexistence curve, we find that

$$(\partial^2 P/\partial x\,\partial y)_d = \tfrac{1}{2}[(\partial^2 P/\partial x\,\partial y)_{x_c^+,\,y} + (\partial^2 P/\partial x\,\partial y)_{x_c^-,\,y}] \doteq [\dot\rho_d(\partial\mu/\partial x)_y + \dot s_d(\partial T/\partial x)_y](\partial T/\partial y)_x, \tag{4}$$

where by $a \doteq b$ we mean that $|a-b|$ remains bounded as $T \to T_c$.

We interpret the Griffiths-Wheeler hypothesis to mean that apart from accidental cancelations or peculiarities introduced by special symmetries, any singular structure in the pressure is independent of the particular form chosen for the function $y(\mu, T)$.[7] But the choice $y \equiv T$ (permissible if the tangent to the coexistence curve is not parallel to the $\mu$ axis at $T_c$) gives

$$(\partial^2 P/\partial x\,\partial y)_d \doteq \dot\rho_d(\partial\mu/\partial x)_T, \quad y(\mu, T) \equiv T, \tag{5}$$

and the choice $y \equiv \mu$ (permissible if the tangent to the coexistence curve is not parallel to the $T$ axis at $T_c$) gives

$$(\partial^2 P/\partial x\,\partial y)_d \doteq \dot s_d(\partial T/\partial x)_\mu\,\dot\mu^{-1}, \quad y(\mu, T) \equiv \mu. \tag{6}$$

Thus the Griffiths-Wheeler hypothesis requires in general that $\dot\rho_d$ and $\dot s_d$ have the same singularity at $T_c$, provided that the tangent to the coexistence curve at $T_c$ is not parallel to either axis of the $\mu$-$T$ plane. In view of the exact identity (2), this singularity must be at least as large as that of the constant-volume specific heat.

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[1] M. S. Green, M. J. Cooper, and J. M. H. Levelt Sengers, Phys. Rev. Lett. 26, 492, 941(E) (1971).

[2] P. Schofield, Phys. Rev. Lett. 22, 606 (1969).

[3] R. B. Griffiths and J. C. Wheeler, Phys. Rev. A 2, 1047 (1970).

[4] For examples of systems with diameters whose derivatives have the specific-heat singularity see P. C. Hemmer and G. Stell, Phys. Rev. Lett. 24, 1284 (1970); B. Widom and J. S. Rowlinson, J. Chem. Phys. 52, 1670 (1970); N. D. Mermin, Phys. Rev. Lett. 26, 169, 957 (1971). Present experimental evidence neither confirms nor excludes such a singularity; see J. M. H. Levelt Sengers, J. Straub, and M. Vicentini-Missoni, to be published.

[5] M. E. Fisher, J. Math. Phys. 5, 944 (1964). We assume that $(\rho_c-\rho_V)/(\rho_L-\rho_V)$ and $(\rho_L-\rho_c)/(\rho_L-\rho_V)$ approach positive nonzero limits at $T_c$ $(\beta_L=\beta_V=\beta)$.

[6] A similar argument taken together with (1) implies that any possible divergence in $\ddot\mu(T)$ cannot be stronger that $(T_c-T)^{-(\alpha'+\beta)}$.

[7] Like the use of the hypothesis in Ref. 1, this is a somewhat stronger application than those made by Griffiths and Wheeler, who apply it primarily to the leading singularity in the second derivatives of $P$.

# Production of Thermonuclear Power by Non-Maxwellian Ions in a Closed Magnetic Field Configuration*

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A toroidal plasma heated by an energetic neutral beam (thus consisting of an energetic ion component and a lower-energy bulk plasma) can produce net thermonuclear power under conditions far less restrictive than Lawson's criterion.

A thermonuclear plasma with a Maxwellian ion distribution must meet Lawson's minimum condition[1] on $n_i\tau_E$ (the product of ion-density and plasma-energy confinement time) in order to generate sufficient thermal power to recreate the electrical power invested in producing the plasma. We will consider the generalization of Lawson's criterion for the case where the bulk-plasma temperatures $T_e$ and $T_i$ are maintained by injection of a neutral beam that gives rise to an energetic ion component of density $n_h \ll n_i$ and initial energy $W_0 \gg T_e, T_i$.