β Decay of ^{8}Li and ^{8}B: The Second-Class Current Problem

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Measurement of the excitation spectra of ^{8}Be following the decays of ^{8}Li, ^{8}B permits the comparison of mirror Gamow-Teller transitions over a wide range of $W_0^+ + W_0^-$, largely free of uncertainties associated with possible T_z dependencies of nuclear overlap integrals. We find no evidence for second-class currents: Using the conventional parametrization we find that $|g_{IT}| < 7\times 10^{-4}$ at the 99% confidence level as against $g_{IT} \approx 2\times 10^{-3}$ that comes from the direct comparison of absolute ft values.

Comparison of mirror Gamow-Teller transitions reveals that the ft value, $(ft)^+$, for the positron emitters are systematically larger than those, $(ft)^-$, for the associated negatron emitters.[1] Furthermore, the relationship is not inconsistent with a linear dependence of $\delta = [(ft)^+/(ft)^-] - 1$ on the energy release $W_0^+ + W_0^-$. This may be expected if the β interaction contains second-class currents,[2] i.e., components that reverse their sign relative to their associated principal components in passing from the negatron-emitting to the positron-emitting body. The most important such possible term is due to the induced tensor coupling; the constant g_{IT}, if not purely electromagnetic, may be expected to be of gross order M^{-1} where M is the nucleon mass. Specifically, we expect[3]

$$\delta \approx \tfrac{4}{3}|g_V/g_A| g_{IT}(W_0^+ + W_0^-). \qquad (1)$$

The experimental data[1] imply, naively, that $g_{IT} \approx 2\times 10^{-3}$. However, as we have repeatedly stressed, we must, before claiming a demonstration of second-class currents, be sure that "trivial" nuclear-structure effects are not destroying the mirror symmetry sufficiently to give rise to the observed values of δ. Such "trivial" effects are of many kinds,[4] but by far the most important appears to be the possible T_z dependence of the nuclear overlap integrals. Theoretical studies[4] of these effects for the $A = 12$ system do not account for its value $\delta \approx 0.11$. It is critically important to resolve this question of the possible responsibility of the nuclear overlap integrals.

We have made a new type of approach, independent of lifetime measurements, through the $A = 8$ system which affords a unique opportunity to get away from the uncertainty associated with the nuclear overlap. Our data are totally inconsistent with expression (1) using $g_{IT} \approx 2\times 10^{-3}$; we may say, in terms of expression (1), $|g_{IT}| < 7\times 10^{-4}$ at the 99% confidence level. We have also repeated the conventional approach through lifetime measurements finding $\delta = 0.107 \pm 0.011$ or, via expression (1), $g_{IT} = (1.93 \pm 0.19)\times 10^{-3}$ which is consistent with the earlier data.[1] We conclude that this finite value of δ cannot be due to a "conventional" second-class current, proportional to the momentum transfer, and we are faced either with a "trivial" nuclear-structure effect of surprising magnitude or with a fundamental effect in the β interaction of a type so far unforeseen.

The nuclear overlap effect, that we seek to eliminate, arises because the "last proton" in the positron-emitting nucleus is less tightly bound than the associated "last neutron" in the negatron-emitting mirror. The wave function of this "last proton" is, in the light nuclei, despite the Coulomb barrier, more spread out radially than that of the "last neutron," and so their overlaps with the nucleons into which they transform on β decay differ, contrary to the hypothesis of mirror symmetry. This effect sets in rapidly as the separation energy approaches zero, and so the overlap involving the "last proton" is poorer than that involving the "last neutron": $(ft)^+ > (ft)^-$ as observed.

Consider the case of a common residual even-even nucleus of $T_z = 0$. Neutrons and protons are both tightly bound in the final $T_z = 0$ state, but in the initial $T_z = \pm 1$ states the "last nucleons" are feebly bound, the more feebly bound ones in the proton-rich body, hence the different overlaps with $T_z = 0$. Unfortunately, we have no present means of measuring these overlaps, hence the impasse in understanding the origin of δ. However, consider the situation if it were possible to change gradually the mass for $T_z = 0$ leaving unchanged the masses for $T_z = \pm 1$. So long as the nucleons in a $T_z = 0$ state remained strongly bound, their wave functions would change little and so their (different) overlaps with $T_z = \pm 1$ would change little. But we should be changing $W_0^+ + W_0^-$; so if the finite value of δ were due to second-class currents, δ should change proportionally to $W_0^+ + W_0^-$. If, however, the finite value of δ were due to the difference of the nuclear overlaps it would change little as we changed the $T_z = 0$ mass. Unfortunately, we do not usually have the mass of the $T_z = 0$ nucleus under our control in this way but in one case we do: ^{8}Be.

^{8}Li, in which the "last neutron" is bound by 2.03 MeV, and ^{8}B, in which the "last proton" is bound by 0.14 MeV, are of $J^\pi = 2^+$; they decay to the first excited, $J^\pi = 2^+$, state of ^{8}Be, nominally at 2.90 MeV, where neutrons and protons are bound by 16.00 and 14.35 MeV, respectively. The situation is therefore as described above: The experimental value $\delta \approx 0.11$ may be due perhaps to the reality of second-class currents or perhaps to the difference of the nuclear overlaps. But the $J^\pi = 2^+$ state in ^{8}Be is tremendously broad and, in effect, extends for many MeV in excitation. As seen in the only open heavy-particle channel, ^{4}He + ^{4}He, the phase shift rises through 90° at an excitation in ^{8}Be of $E_x \approx 3.3$ MeV, reaches only 115° at $E_x \approx 5$ MeV, and then slowly sinks, passing back through 90° at $E_x \approx 10$ MeV [5] (other $J^\pi = 2^+$ states may occur within this region of E_x but, if so, are automatically included in our analysis). We may therefore, by determining the excitation in ^{8}Be to which the β transitions lead, effectively change the mass of the $T_z = 0$ nucleus in the way fancifully described above. Let us call the spectra of ^{8}Be excitation by positron (negatron) transitions $N^\pm(E_x)$ without reference to normalization, i.e., without reference to absolute lifetimes. If we have full mirror symmetry,

$$\frac{[N^-(E_x)/f(W_0^-)]}{[N^+(E_x)/f(W_0^+)]} = \text{const} \times \frac{(ft)^+}{(ft)^-} = \text{const},$$

where $W_0^\pm$ belong to E_x. This expression should hold (almost) good even for $\delta \neq 0$ if the failure of mirror symmetry is due to the difference of nuclear overlaps. If, however, $\delta \neq 0$ is due to second-class currents, we should find[6]

$$\frac{[N^-(E_x)/f(W_0^-)]}{[N^+(E_x)/f(W_0^+)]} = \text{const} \times [1 + \xi(W_0^+ + W_0^-)], \quad (2)$$

where $\xi = \delta/(W_0^+ + W_0^-)_w$. Here δ refers to the overall effect as conventionally determined through the ^{8}Li, ^{8}B lifetimes and $(W_0^+ + W_0^-)_w$ refers to the weighted sum over the spectra of excitation.

Now ^{8}Be is everywhere unstable against breakup into two α particles, so the excitation E_x is signaled by an α particle of energy $\frac{1}{2}(E_x + Q)$, where Q is the energy release (0.095 MeV) in the breakup of the ground state of ^{8}Be into two α particles.[7] Our experiment therefore consists simply of an accurate measurement of the α-particle spectra following the β decay of ^{8}Li and ^{8}B and, from them, the construction of the functions $N^\pm(E_x)/f(W_0^\pm)$.

The quintessential element of our experiment is the knowledge that the $N^\pm(E_x)$ refer to the same mean excitations $\overline{E}_x$ for all values of $\overline{E}_x$. This we must be assured of to better than 10 keV in order to interpret our data to the desired precision. This demands either that the ^{8}Li, ^{8}B sources be very thin or that the effective *relative* distributions of activity within the sources be sufficiently well known. We have tackled this problem as follows: The reactions ^{7}Li$(d,p)^8$Li and ^{6}Li$(^3$He$,n)^8$B respectively produced ^{8}Li and ^{8}B which recoiled downstream and impinged upon a movable catcher, the same for both targets, consisting of either 100- or 200-μg/cm^2-thick Au evaporated onto 20-μg/cm^2-thick C (the "thin" and "thick" catchers, respectively). After impregnation of the catcher, the bombardment was stopped and the catcher swung in front of a Si detector of thickness just sufficient to stop the most energetic α particles from the ^{8}Be breakup. The cycle was repeated many thousands of times over periods of hours (for ^{8}Li) or days (for ^{8}B). The arm that bore the catcher was masked from the target spot; we checked, both for ^{8}Li and for ^{8}B, by removing the catcher from the arm, that the activity from the arm itself was effectively zero. The same test showed that the counter registered nothing except from the catcher. Owing to the stringent stability requirement the gain was monitored by a continuously running precision pulser at $E_\alpha \approx 9$ MeV; tiny gain changes were made as necessary. We also performed frequent calibrations using an ^{241}Am source, permanently mounted in the target-counting chamber, which could be exposed to the counter in similar geometry to the catcher foil. The long ^{8}B runs were frequently interrupted for printout. In these ways, we significantly bettered our target of 10-keV stability overall.

The catchers were considerably thicker than

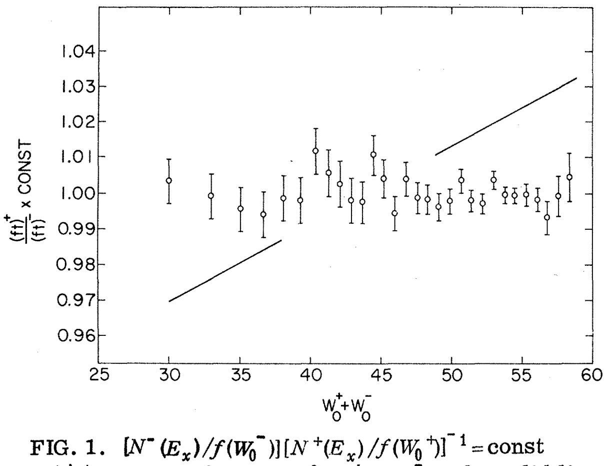

FIG. 1. $[N^-(E_x)/f(W_0^-)][N^+(E_x)/f(W_0^+)]^{-1}$ = const $\times (ft)^+/(ft)^-$ as a function of $W_0^+ + W_0^-$. The solid lines show the slope to which the points should conform for a second-class current of $g_{IT} = 2\times10^{-3}$ such as is indicated by the absolute lifetimes.

10 keV: For the 5.5-MeV α particles of ^{241}Am the thin and thick catchers absorbed 35 and 60 keV, respectively. However, only 22% of the ^{8}B recoils that strike the thin catcher stick in it (as directly determined by comparison with an infinitely thick catcher), so their distribution in depth must be rather uniform.[8] We were able, by comparison of our spectra in the region of their peaks at $E_\alpha \approx 1.5$ MeV, to determine the relative effective source-depth distributions.[9] We find, for the thin catcher, that the ^{8}Li distribution is effectively centered 6 keV closer to the catcher surface (for $E_\alpha \approx 5.5$ MeV); for the thick catcher the corresponding figure is 5 keV. These very small differences may be accurately allowed for.[10]

The final α-particle spectra, which covered about 800 channels, were divided by $f(W_0^\pm)$[11] channel by channel; the channels were then grouped so as to preserve a statistical accuracy in the final ratios of everywhere better than $\pm\frac{2}{3}$%. The results are displayed in Fig. 1 which combines the concordant data from both catchers.

We have also remeasured the lifetimes of ^{8}Li and ^{8}B via the α particles. We find $t_{1/2}(^8\text{Li}) = 838\pm6$ msec; $t_{1/2}(^8\text{B}) = 762\pm5$ msec. Combining these numbers with our spectra yields $\delta = 0.107 \pm 0.011$, or, via expression (1), $g_{IT} = (1.93\pm0.19)\times10^{-3}$.

Figure 1 also shows the slope to which our points should conform if a "conventional" second-class current with $g_{IT}\approx 2\times10^{-3}$ were responsible for the δ values of $A=8$ and the other, concordant, high-energy cases ($A = 9$, 12, 13); it is absolutely excluded. We have analyzed our data in terms of expression (1), least-squares fitting our points by slopes parametrized by g_{IT}. Figure 2 results, from which we quote $|g_{IT}| < 7\times10^{-4}$ at the 99% confidence level.[12]

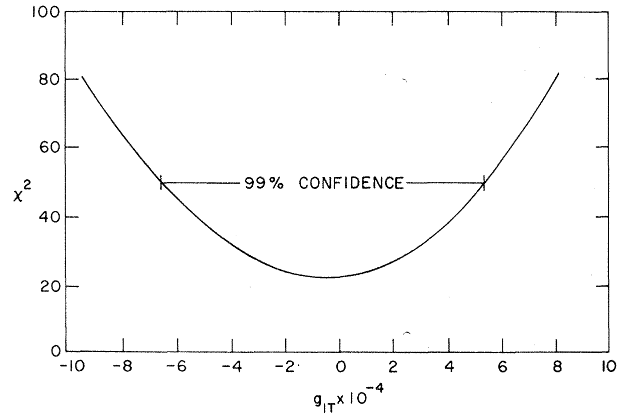

FIG. 2. χ^2 values from the fitting of the points of Fig. 1 by slopes parametrized by g_{IT} through expression (1) of the text.

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[1]D. H. Wilkinson, Phys. Lett. 31B, 447 (1970); D. H. Wilkinson and D. E. Alburger, Phys. Rev. Lett. 24, 1134 (1970); D. E. Alburger and D. H. Wilkinson, Phys. Phys. Lett. 32B, 190 (1970).

[2]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[3]H. J. Lipkin (private communication) has stressed that, even assuming perfect nuclear symmetry, $\delta \neq 0$ need not imply the existence of second-class currents and that circumstances could arise in which δ would not be proportional to $W_0^+ + W_0^-$.

[4]R. J. Blin-Stoyle and M. Rosina, Nucl. Phys. 70, 321 (1965).

[5]See, e.g., T. A. Tombrello and L. S. Senhouse, Phys. Rev. 129, 2252 (1963).

[6]Gamow-Teller transitions to $T=1$ components of ^{8}Be vitiate this expression because they interfere with opposite sign for positron and negatron emission; the effect of the mixed $T=0,1$ $J^\pi=2^+$ states at 16.6 and 16.9 MeV which contain the analog of the ^{8}Li, ^{8}B ground states must be particularly considered. They turn out to be of negligible importance because, for E_x well below them, their $T=1$ transitions cancel. We have followed the analysis of F. C. Barker [Aust. J. Phys. 22, 293 (1969)] using empirical data from W. D. Callender and C. P. Browne [Phys. Rev. C 2, 1 (1970)] and the theoretical strength of the $T=1$ β transition from F. C. Barker [Nucl. Phys. 83, 418 (1966)].

[7]This is not strictly true owing to the lepton recoil. If $E_{\alpha0}$ is the α energy in the absence of lepton recoil,

the recoil converts the line into the spectrum (assuming axial-vector coupling) $P(E_\alpha) = \frac{5}{6}(1-3\varphi+2\varphi^2)(2E_{\alpha 0} \times E_m)^{-1/2}$, where E_m is the maximum recoil energy imparted to the ^{8}Be and $\varphi = E_m^{-1}(E_\alpha^{1/2} - E_{\alpha 0}^{1/2})^2$. We have used this function together with the empirical spectra to compute the importance of this effect. The effect itself is large but the associated differential correction is small because the lepton recoils for ^{8}Li, ^{8}B are almost equal. The differential correction (which we have applied) amounts to 0.60 % *across* the range of our analysis.

[8]This hypothesis is not critical since accurate knowledge of the absolute value of $\bar{E}_x$ is not needed; an error of Δ (keV) in $\bar{E}_x$ means an error of only $6\times10^{-5}\Delta$ in $(ft)^+/(ft)^-$ *across* our whole range. *Relative* $\bar{E}_x$ values following ^{8}Li and ^{8}B decay are critically important but this depends on the *relative* depth distributions of the sources which are accurately known from the peak positions of the α spectra.

[9]One must allow for the shift in the peaks due to the changes of $f(W_0^\pm)$ across them; this moves the ^{8}Li peak downwards by 6 keV in E_α relative to ^{8}B.

[10]The observed spectra are changed by the source thickness but the essence of our experiment is the *comparison* of the ^{8}Li and ^{8}B spectra where this effect is closely the same for both and may be accurately allowed for. (The analogous small effect of electron-addition pulses is identical in the two cases.)

[11]Allowance must be made in $f(W_0^\pm)$ for the effects of finite nuclear size [D. H. Wilkinson, Nucl. Phys. A158, 476 (1970)] which give a correction to the *slope* of the points in Fig. 1 of $\frac{1}{5}R\{\frac{11}{3}\alpha Z - \frac{4}{7}CR\} = 1.25\times10^{-4}$, where R is the nuclear radius and $C = W_0^+ - W_0^-$. Allowance has also been made for radiative corrections [D. H. Wilkinson and B. E. F. Macefield, Nucl. Phys. A158, 110 (1970)]. The difference between these corrections for ^{8}Li and ^{8}B is $3\alpha C/2\pi W_0^-$: 2×10^{-4} *across* Fig. 1. The effect of errors in the standard masses is an uncertainty of 7×10^{-4} across Fig. 1.

[12]We must ask about second-forbidden corrections; those containing r^2 in the operator give a *slope* to Fig. 1. We take as an upper limit to such terms one single-particle unit of the $r^2\sigma$ operator $(1p_{3/2} \to 1p_{3/2})$ whose interference with the allowed matrix element would give a *slope* of 1.4×10^{-4} to Fig. 1 or a "pseudo g_{IT}" of only 1.3×10^{-4}.

Master Equation for the P Representation

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A master equation for the P representation in the Schrödinger picture is derived for a general class of Hamiltonians.

The coherent-state basis has been shown to be very suitable for describing the evolution of the density matrix.[1] Furthermore, it is very useful to find a diagonal representation of the density matrix in terms of coherent states, provided this is possible. This is the so-called P representation.[2] Instead of solving the equation of motion for the density matrix and then trying to find its diagonal representation, it would be of interest to write down directly an equation describing the time evolution of the P representation.

We recall briefly that the coherent states $|\alpha\rangle$ are defined as the eigenstates of the annihilation operator a with complex eigenvalues α,

$$a|\alpha\rangle = \alpha|\alpha\rangle, \tag{1}$$

and that they form a complete (but not orthogonal) set of states:

$$\pi^{-1}\int d^2\alpha |\alpha\rangle\langle\alpha| = 1. \tag{2}$$

The equation of motion for the density matrix in the Schrödinger picture is

$$i\partial\rho/\partial t = [H, \rho]. \tag{3}$$

Let us assume that the Hamiltonian H can be represented by a convergent, ordered series in the creation and annihilation operators $a^\dagger$ and a,

$$H = \sum_{mn} H_{mn}(a^\dagger)^m a^n. \tag{4}$$

If the matrix elements between the coherent states for an operator A are written as

$$A(\alpha^*, \beta) = \frac{\langle\alpha|A(a^\dagger, a)|\beta\rangle}{\langle\alpha|\beta\rangle}, \tag{5}$$

the following equation for $\rho(\alpha^*, \alpha)$ can be derived