Cross Sections for $p + p \to d + \rho^+$ from 4.0 to 12.3 GeV/c and a Search for the δ^+ †*

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Forward differential cross sections for isospin-1 bosons produced in $p+p\to d+x^+$ were measured using a deuteron missing-mass spectrometer at a small angle between 4.0- and 12.3-GeV/c incident momentum. Differential cross sections for π^+ and ρ^+ were extracted from the spectra using phase-space backgrounds. They range from 10.4 to 0.4 μb/sr for π^+ and from 1.4 to 0.3 μb/sr for ρ^+. A bump near 6 GeV/c appears in both $d\pi$ and $d\rho$ channels. No clear evidence is seen for higher-mass bosons. The possible δ^+ cross sections average less than 0.01 μb/sr.

We report here further results of our study of the isospin-1 bosons produced in reactions of the type $p+p\to d+x^+$, obtained by detecting the deuteron in a high-resolution missing-mass spectrometer. The experiment used the external proton beam of the Argonne zero-gradient synchrotron (ZGS) and covered the range of momentum for the incident proton from $p_0 = 3.4$ to 12.3 GeV/c. We have previously reported[1] on the π^+ production, giving a description of the method and apparatus. Here we report the results of our analysis of the higher-mass bosons.

Some examples of the missing-mass spectra we obtained are shown in Fig. 1. They were obtained by increasing the momentum setting of the spectrometer in 1 to 2% steps, whereas the momentum acceptance of the spectrometer was 7%. The results are given directly in terms of the differential cross section $d^2\sigma/d\omega dM^2$ in the center of mass of the colliding protons. They refer to hydrogen, with the target-empty effects subtracted. All data were taken with the spectrometer set to accept deuterons at laboratory angles in the range 4.7° to 5.3°.

In addition to the π^+ peak, a ρ^+ peak is immediately evident in all of our spectra. However, we cannot speak with confidence about peaks at higher masses. In particular, although a peak appears near $M^2 = 0.927$ GeV^2 in the 12.3-GeV/c data, at the correct mass value for the $\delta^+(962)$, it is at the limit of the sensitivity of our measurement and its presence is not confirmed by our measurements at lower values of the incident momentum. No other peaks are apparent in the spectra and we have analyzed the data for the ρ^+ cross section without including higher resonances.

The π^+ peak is broadened by the experimental resolution. We represented this with a Gaussian whose width could be calculated by combining the effects of multiple scattering, finite target size, and measurement error. The ρ^+ peak was

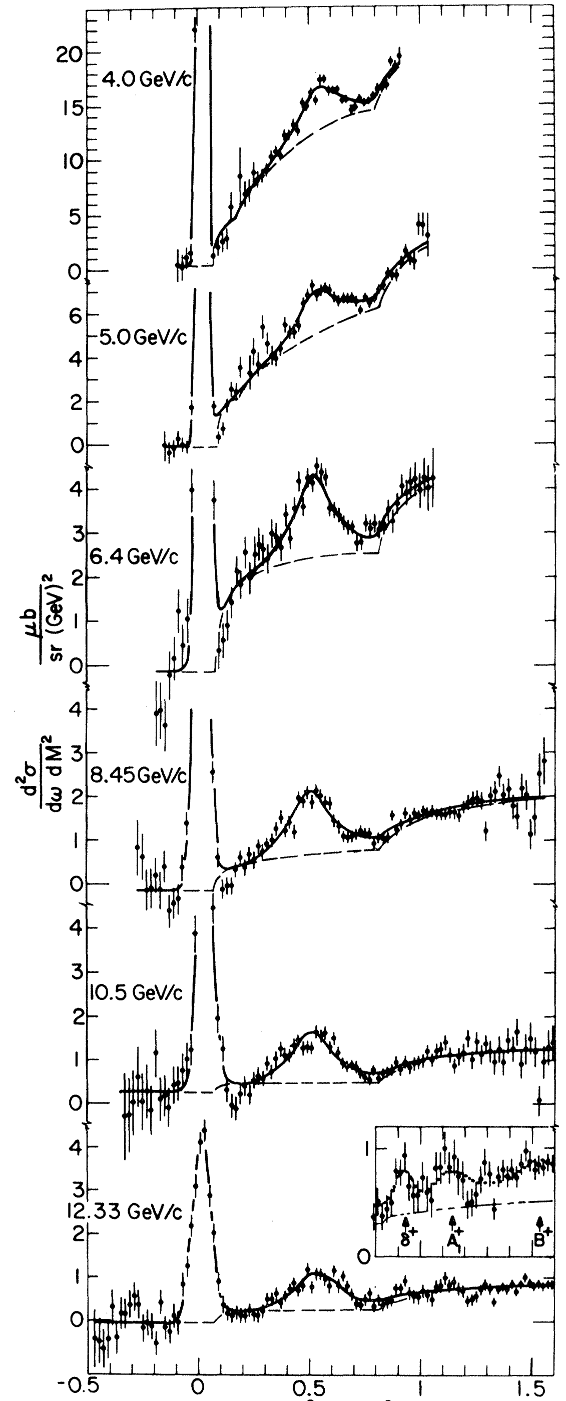

FIG. 1. Examples of missing-mass spectra from measurements of the deuteron in the reactions $p+p\rightarrow d+x^+$ at 5° in the lab. The dashed lines show the contribution of the nonresonant $d\pi\pi$, $d\pi\rho$, and $d\pi\pi\pi$ final states according to phase space and before convolving the resolution. The full line is the fit, including a pion peak of Gaussian shape and a ρ peak of relativistic Breit-Wigner shape with width fixed at 150 MeV. The insert is part of a fit with additional higher-mass resonances.

fitted with the relativistic form of the Breit-Wigner formula as given by Jackson,[2] with a $d\pi\pi$ phase-space coefficient,

$$\frac{d^2\sigma}{d\omega dM^2}=\frac{d\sigma}{d\omega}\frac{R(M^2)}{\boldsymbol{R}(M_\rho^{\,2})}\frac{M_\rho\Gamma}{(M_\rho^{\,2}-M^2)^2+(M_\rho\Gamma)^2},$$

$$M\Gamma=M_\rho\Gamma_\rho(q/q_\rho)^3,$$

$$R(M^2)=(p^*/\sqrt{s})(q/M),$$

where q is the pion momentum in the ρ^+ system, p^* is the c.m. deuteron momentum, and s is the total c.m. energy squared.

We used phase-space arguments to represent the nonresonant background. The three-body phase spaces, such as $R(M^2)$ above, rise sharply above threshold and remain essentially constant until they fall at the highest mass values which are permitted. The four- and more-body phase spaces are similar except that they rise with increasing powers of M^2. We found it sufficient to include only two three-body final states, $d\pi\pi$ and $d\pi\rho$ (the ρ mass being taken as a constant 764 MeV), and one four-body final state, $d\pi\pi\pi$. No clear distinction could be made between $d\pi\rho$ and $d\pi\omega$, although $d\pi\rho$ with its lower threshold would seem to match our data better. Had the fits been extended to the highest M^2 values measured, 3.76 GeV^2 at 12.3 GeV/c and 3.22 GeV^2 at 10.5 GeV/c, we would have had to include $d\rho\rho$ and $d\pi\pi\pi\pi$ or similar phase spaces. The fitting procedure folded in the experimental resolution. Our fits neglect possible interference effects of the ρ^+ with the nonresonant $\pi\pi$ background and were limited to $M^2<1.6$ GeV^2.

The values of the ρ^+ cross sections were found to be essentially proportional to the value used for the width of the resonance. An overall minimum χ^2 of 915 for 688 degrees of freedom was obtained for $\Gamma_\rho=180\pm10$ MeV. This rather high value of χ^2 per degree of freedom reflects the neglect of other uncertainties including those inherent in the representation of the peaks and backgrounds. To be more consistent with present values of the width,[3] we quote the cross sections obtained for $\Gamma_\rho=150$ MeV. This still gives good fits to all of our data. If the width is to be taken different from 150 MeV, the following is a good approximation:

$$d\sigma/d\omega=(\Gamma_\rho/150)(d\sigma/d\omega)_{150}.$$

The cross sections obtained for 11 values of the incident-proton momentum are listed in Table I. The value of $M_\rho^{\,2}$, used as a free parameter in the fits, was determined in each case.

Table I. Center-of-mass cross sections for $p+p\to d+\pi^+$ and $p+p\to d+\rho^+$. Cross sections refer to the c.m. backward-emitted deuteron observed at 5° in the lab. Errors shown do not include a 6% normalization error. In the case of ρ^+ there are additional uncertainties due to fitting parametrization and background subtraction.

p_0 (GeV/c)	M_ρ^2 (GeV^2)	$(d\sigma/d\omega)_\rho$ (Γ_ρ=150 MeV) (nb/sr)	$(d\sigma/d\omega)_\pi$ (nb/sr)
4.0	0.552	1444 ± 118	10447 ± 441
4.4	0.559	1024 ± 69	6678 ± 279
4.7	0.572	831 ± 77	4744 ± 200
5.0	0.541	690 ± 65	3469 ± 145
5.6	0.513	862 ± 66	2655 ± 114
6.4	0.524	649 ± 53	2411 ± 106
7.1	0.523	554 ± 50	1620 ± 73
8.45	0.512	480 ± 35	1149 ± 52
9.5	0.510	447 ± 77	864 ± 51
10.5	0.523	402 ± 36	664 ± 41
12.33	0.545	302 ± 31	440 ± 33

Our values of M_ρ^2 are lower than the present world value of 0.585 ± 0.015 GeV^2.[3] While our statistical errors are typically only 0.009 GeV^2, the deviations again reflect the other uncertainties including those involved in the selection of the ρ^+-shape parameters and the neglect of interference. A normalization error of 6% has not been included. For the ρ^+ there are additional uncertainties due to the fitting parametrization and background subtraction of perhaps 20%.

In spite of the above uncertainties, it appears meaningful to compare the ρ^+ with the π^+ cross sections. This has been done in Fig. 2, where all our π^+ cross sections are shown including those runs, not listed in Table I, in which the measurement was limited to the pion region.

The general trend of the ρ^+ cross sections is to approach the π^+ cross sections with increasing values of p_0. Allaby et al.[4] have already shown that at 21.1 GeV/c the two final states have strikingly similar cross sections. Above 7 GeV/c the c.m. cross sections are well represented by a relation of the form

$$(d\sigma/d\omega)_{\text{c.m.}}\sim s^a\exp(-p_\perp/b),$$

where s is the total c.m. energy squared. For π^+ we used the CERN value $b=0.0192$ GeV/c and obtained a good fit to our data with $a=-2.69\pm0.11$. For ρ^+ the CERN value for b was 0.0183 GeV/c and the fit to our data gave $a=-1.19\pm0.19$. The CERN points[4] agree quite well with an extrapolation to these fits. However, such an expression may be oversimplified and would have to be modified if the two cross sections were to remain equal above 21 GeV/c.

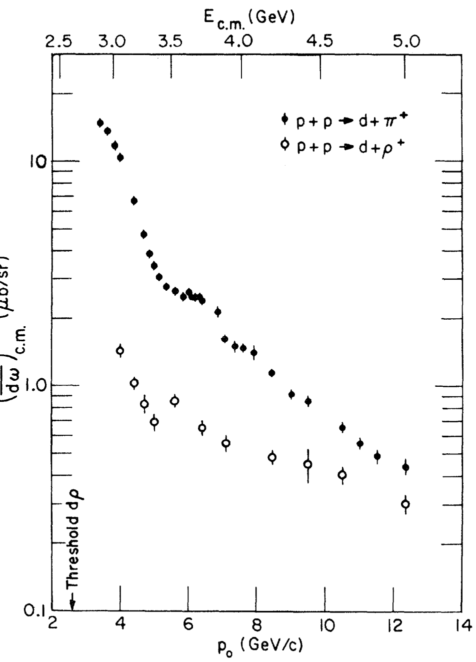

FIG. 2. Center-of-mass differential cross sections for $p+p\to d+\pi^+$ and $p+p\to d+\rho^+$ obtained by observing the backward-emitted deuteron at 5° in the lab. The ρ^+ cross section assumes a width for the ρ^+ of 150 MeV. Errors shown do not include normalization or parametrization errors.

Barger and Michael[5] and also Brown[6] have shown how the s dependence of the π^+ cross section might be accommodated within the framework of Regge theory. From this point of view the weaker s dependence of the ρ^+ cross section is unexpected and it is not clear what differences in the character of the Regge trajectories account for this behavior.

A striking feature is the appearance of a bump near 6.0 GeV/c. This is more pronounced for the ρ^+ than it is for the π^+ channel. This feature in the π^+ cross section has a ready explanation as being due to the isospin-$\frac{3}{2}$ nucleon isobar $\Delta(2420)$ in one leg of the one-pion-exchange diagram.[7,8] In this model, neglecting minor effects of the deuteron binding energy and the momentum distribution of the bound nucleons, peaks are to be expected at $s=2M_\Delta^2+M^2-2m^2-\mu^2$, where M_Δ, M, m, and μ are the masses of the nucleon isobar, the deuteron, the proton, and the boson, respectively. Here, we would expect the bump to be shifted to lower values of s by the difference, 0.566 GeV^2, between the squares of the ρ^+ and π^+ masses. If, on the other hand, the bumps were due to alternative decay modes of a common direct-channel dibaryon resonance, we would expect the s value to be the same in the two cases. In terms of p_0, the shift in the former case would be by -0.3 GeV/c, and by zero in the latter case. The present data are insufficient to make a clear distinction between these two possibilities. The comparison does indicate a way, at least in principle, for distinguishing a dibaryon resonance from other possible mechanisms for obtaining a bump in the cross section. In this connection, it should be noted that the $\Delta(2420)$ resonance does not have a more prominent 2π than 1π decay mode.

A special effort was made in this experiment to verify the report by Oostens et al.[9] of the observation of a narrow δ^+ in $p+p\rightarrow d+x^+$ at 3.8 GeV/c and 0° in the lab. The possible δ^+ cross sections consistent with our data were obtained by making fits as described above, adding resonances of isospin 1, $\delta(962)$, $A_1(1070)$, and $B(1235)$ with widths <5, 95, and 102 MeV.[3] The δ^+ was treated like the π^+. The A_1 and B were included, where the spectra permitted, to enhance any δ^+ effect. The insert in Fig. 1 shows part of the fit for the 12.3-GeV/c data which gives an indication of a possible δ^+. The δ^+ cross sections obtained for momenta 4.4, 4.7, 5.0, 5.4, 5.6, 6.4, 7.1, 8.4, 9.5, 10.5, and 12.3 GeV/c were 6.4 ± 12.2, 7.6 ± 13.5, 6.9 ± 10.3, 16.1 ± 5.7, -2.4 ± 11.3, 10.0 ± 13.0, 12.1 ± 11.8, -2.4 ± 6.6, -40.7 ± 28.5, -9.8 ± 8.3, and 26.4 ± 8.4 nb/sr, respectively. Thus, although some positive evidence for the δ^+ appears in our data at 5.4 and 12.3 GeV/c, it is not supported by the weight of evidence at other values of momentum. In particular, the expected increase in cross section for lower values of momentum is not seen. The possible δ^+ cross sections, below 5.0 GeV/c, taking into account the fact that our measurements were not at 0°, are less than $\frac{1}{10}$ that expected from the data of Oostens et al.[9] Our results are based on the assumption of a δ^+ width small compared with the experimental resolution, which varies from 0.016 to 0.085 GeV^2 (full width at half-maximum), almost linearly with momentum over our momentum range. They fail to corroborate a narrow δ in this reaction, in agreement with the result of similar measurements of Abolins et al.[10] Wider versions of the $\delta(962)$ in combination with the $A_1(1070)$ and other higher resonances could reproduce most of the $d\pi\rho$ phase space as we used it. Thus, our results speak less clearly if the resonances are wide. Although our measurements are comparable in sensitivity with those of Abolins et al.[10] a wide $\pi_N(980)$ resonance as reported by them is not evident and would be hard to distinguish from a $d\pi\rho$ threshold effect.

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Dual, Crossing-Symmetric Amplitude with Mandelstam Analyticity*

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We present a class of scattering amplitudes which are dual and crossing symmetric, have Regge asymptotic behavior and second-sheet poles for resonances in all channels, and satisfy a Mandelstam representation with correct double-spectral-function boundaries. The Regge trajectories and residues are essentially arbitrary.

In the following we describe a class of scattering amplitudes. They possess the set of properties connected with crossing symmetry and duality that has recently led to interesting algebraic and symmetry-type results,[1] and they simultaneously possess some dynamical properties related to unitarity. In particular, all resonances occur as second-sheet poles, the threshold behavior is correct, and the amplitudes can be written in the form of Mandelstam double dispersion relations with correct double-spectral-function boundaries.

Before we present the detailed results we would like to note several implications of this work. First, it finally gives us a scattering amplitude that is Regge behaved, is crossing symmetric, and has a Mandelstam representation. To find such an amplitude has been a basic problem in particle physics. It may lead to new insight into the meaning and uniqueness of bootstrap ideas to have an amplitude that satisfies all the conditions that are usually postulated except unitarity. This amplitude has still an essentially arbitrary trajectory and residue; they will be related by unitarity.

Second, we show that a dual, crossing-symmetric, Regge-behaved, etc., amplitude can be constructed; it has all of the conditions that are put on the Veneziano amplitude, but is is not unique at all. Thus many speculations on the meaning of the Veneziano model and on duality must be reconsidered. Fermion channels may be included in our model by $\sqrt{u}$ dependence in the residue and trajectory.

Third, the present amplitude is suitable for phenomenology without modifying any of its good properties; e.g., it already has second-sheet poles. Thus any relevance to experimental data can be tested unambiguously.

We proceed by exhibiting the amplitude and its properties, with proofs briefly indicated. Consider, for equal-mass scalar-meson scattering [for simplicity assume a process with exotic u channel; or else add $M(s,u)$ and $M(u,t)$ to $M(s,t)$ just as in the Veneziano model],

$$M(s,t)=\int_0^1 dx\, x^{-\alpha(tx')-1}(1-x)^{-\alpha(sx)-1}f(sx)f(tx'),\quad x'\equiv 1-x, \tag{1}$$

where α represents a (complex) Regge trajectory, $f(y)$ and $\alpha(y)$ are real for $y<4m^2$ and analytic at $y=0$, and have threshold branch points for $y=4m^2$. $f(y)$ satisfies the condition that as y becomes infinite anywhere in the physical sheet f vanishes faster than any inverse power of y. The condition is sufficient to guarantee the convergence of all integrals of interest. For example,[2] one might use $\alpha(y)=\alpha_0+\alpha'y+\gamma(4m^2-y)^{1/2}$, and $f(y)=\exp[-\beta(4m^2-y)^{1/4}]$. For trajectories with a positive intercept f will also contain a factor proportional to $\alpha(y)$, removing the spin zero pole.

The properties of $M(s,t)$ include the following:

(1) Regge-pole asymptotic behavior in all channels. This is easily shown by a change of variables $\mu=-sx$, which gives $M=(-s)^{\alpha(t)}G(t)$ in the limit as $|s|\to\infty$, with G a convergent integral, $G(t)=f(t)\times\int_0^\infty \mu^{-\alpha(t)-1}f(-\mu)d\mu$.

(2) Crossing symmetry. After putting $s\leftrightarrow t$, change variables $x\leftrightarrow 1-x$.

(3) Second-sheet resonance poles in all channels. The poles arise from the end-point singularity near $x=0$, and are of the form $1/(n-\alpha)$. Since $\mathrm{Im}\alpha\neq 0$ they are explicitly second-sheet poles.