Regge Behavior in Inclusive Experiments*

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We discuss the Regge prediction for the behavior of inclusive cross sections near the kinematic boundary, and we compare it with a recent conjecture of Chou and Yang. Some existing experimental data are analyzed and, in particular, we find no evidence yet for diffraction dissociation into high masses (three-Pomeranchon coupling).

The purpose of this note is to discuss the predictions of Regge behavior in inclusive hadronic reactions[1] of the type

$$a+b\to c+\text{anything}, \qquad (1)$$

to compare and contrast them to a recent conjecture of Chou and Yang,[2] to indicate what we believe is the natural way to analyze experimental data, to show some examples of such an analysis, and to point out the significance of future experiments for the multiperipheral model and the understanding of diffraction dissociation and the Pomeranchuk singularity.

For concreteness, let a be the projectile and b the target in the reaction of Eq. (1). We describe the process in the projectile frame, in which a is at rest and b is incident with momentum $-p_b$ along the z axis. We call $p_{\parallel}$ the z (i.e., longitudinal) component of the momentum of the observed particle c, and $p_{\perp}$ the transverse one. For an exclusive two-body process,

$$a+b\to c+d, \qquad (2)$$

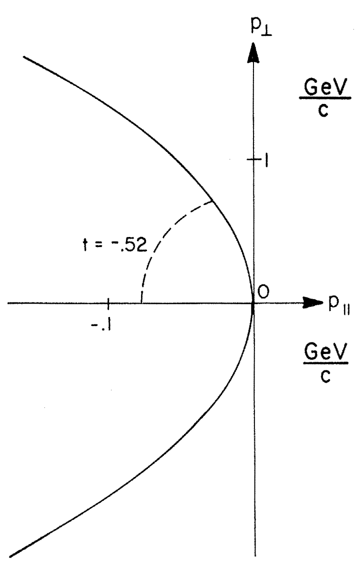

FIG. 1. Asymptotic kinematical boundary for the process $p+p\to p+$anything in the projectile frame. The dashed circle is a constant-t path.

at any given incident energy there is only one degree of freedom, and therefore only a curve in the $p_{\parallel}$, $p_{\perp}$ plane of Fig. 1 is kinematically accessible. The curve is an ellipse, and, in the infinite-energy limit, it approaches the parabola

$$2m_a p_{\parallel}=m_a^2-m_c^2-p_{\perp}^2, \qquad (3)$$

which is shown in Fig. 1. Each point of the curve can be labeled by the invariant momentum transfer between particles a and c:

$$t=(p_c-p_a)^2=m_c^2+m_a^2-2E_c m_a. \qquad (4)$$

The kinematically allowed region for the inclusive reaction (1) at high energy is the interior of the parabola (3). According to the scaling hypothesis of Feynman[1] and of Benecke, Chou, Yang, and Yen,[3] at high energy the inclusive cross section $E_c d\sigma_{ab}^c/d^3p_c$ approaches a nonvanishing, energy-independent limit at any point within this region.[4] This limit is a function of $p_{\parallel}$ and $p_{\perp}$, and we want to study its behavior in the vicinity of the kinematic boundary (3).

The Regge model gives a relation between the behavior near the kinematic limit of the inclusive process (1) and the high-energy behavior of corresponding exclusive processes. In the limit of large c.m.-energy squared, s, large missing-mass squared, s', and large ratio s/s', we can write[5]

$$\frac{d\sigma_{ab}^c}{dp_{\perp}^2dp_{\parallel}}=\frac{1}{E_c}\left(\frac{s}{s'}\right)^{2\alpha(t)-1}|G_{ac}(t)|^2\sigma_{Rb}^{\text{tot}}(s',t). \qquad (5)$$

Here $\alpha(t)$ is the Regge trajectory exchanged (see Fig. 2). It couples to particles a and c with a residue function $G_{ac}(t)$, and controls the high-energy behavior of two-body processes in which this coupling is involved, e.g., $a+c\to c+a$. $\sigma_{Rb}^{\text{tot}}(s',t)$ can be interpreted as a total cross section for Reggeon-particle scattering.

Let us consider a constant-t path in the kinematically allowed region, i.e., a path of constant E_c [see Eq. (4) and Fig. 1]. As we approach the boundary of Eq. (3), $p_{\parallel}$ approaches the value $p_{\parallel}^{\text{boundary}}(t)=(m_a^2-m_c^2+t)/2m_a$. For s' suffi-

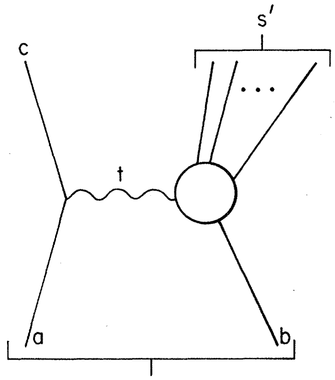

FIG. 2. Dominant diagram near the kinematical boundary in the rest frame of particle a at high energy.

ciently large, we can write

$$s'/s \simeq (p_{\parallel}^{\text{boundary}} - p_{\parallel})/m_a \equiv \delta/m_a. \tag{6}$$

This relation shows that, in the region of large s', we can come arbitrarily close to the boundary, if the energy squared s is large enough. Thus, Eq. (5) is the Regge prediction for the behavior of the asymptotic single-particle spectrum near the kinematical boundary. If we assume σ_{Rb}^{tot} to be asymptotically constant in s',[6] we obtain

$$\frac{E_c d\sigma_{ab}^{c}}{dp_{\perp}^{2}dp_{\parallel}} = |G_{ac}(t)|^2 \beta_{Rb}(t) \left(\frac{s}{s'}\right)^{2\alpha(t)-1}$$

$$\simeq |G_{ac}(t)|^2 \beta_{Rb}(t) (\delta/m_a)^{-2\alpha(t)+1}, \tag{7}$$

(t fixed, s' large, and s/s' large).

We should like to comment on a recent similar prediction by Chou and Yang.[2] These authors relate the behavior of the inclusive cross section near the kinematic limit to the high-energy behavior of the dominant two-body differential cross section (2). When the latter vanishes asymptotically, they call the inclusive process unfavored fragmentation, and conjecture that its spectrum goes to zero on the kinematical boundary. Alternatively, if the two-body process does not vanish asymptotically, they expect the inclusive spectrum not to go to zero as one approaches the boundary (favored fragmentation). We note that, from the Regge point of view, the inclusive spectrum does not vanish whenever $\alpha(t) \geq 0.5$, whereas two-body cross sections require $\alpha(t) = 1$ in order not to vanish asymptotically.

In practice we can hope that Eq. (7) should be valid for missing masses above a few GeV and ratios s/s' larger than, or of the order of, 5 or 10. These constraints do not leave much room

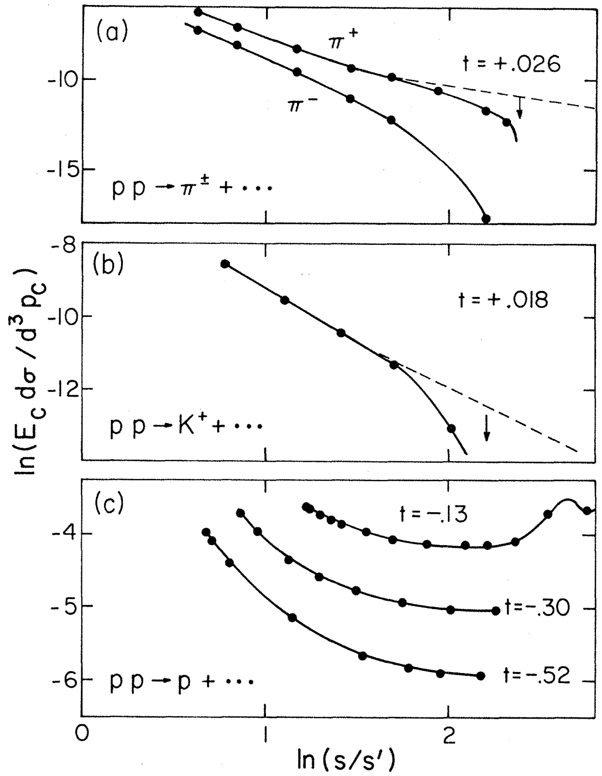

FIG. 3. Inclusive spectra from the experiment of Ref. 9. The experimental points have been interpolated to obtain the cross sections at constant values of t. Cross sections are in barns and momenta in GeV/c. The lines are to guide the eye. The dashed lines correspond to the asymptotic slope predicted by the Regge model: in (a) for nucleon exchange with $\alpha_N(0) = -0.3$, in (b) for Λ exchange with $\alpha_{\Lambda}(0) = -0.75$. The arrows indicate actual kinematic limits at the energy of the experiment. Asymptotically, the kinematic limit in these plots moves to infinity.

to check Eq. (7) with presently available experimental data, but, nevertheless, we analyze below the high-precision single-particle spectra measured by Allaby *et al.*[7] in 19.2-GeV/c pp collisions, to explore the present trends and to provide a background for our expectations for future higher energy experiments.

(a) $p + p \to \pi^+ + $ *anything*.—In this case, the exchange of the nucleon and Δ Regge poles is possible, as in the corresponding two-body process —backward $\pi^+ p$ scattering. It is known that the Δ trajectory is weakly coupled, and that the effect of this exchange, in spite of its higher intercept, is small at present energies.[8] In Fig. 3(a) we show a plot of the inclusive cross section for this process. The dashed line represents the asymptotic slope expected for nucleon exchange. As one can see, it requires a fair amount of imagination to infer that there is a region of the data compatible with this slope. Actually, the energy is not high enough, so that we cannot attain a region for which both s/s' and s' are simultaneously large. For the points for which s/s' is not large, we get substantial background contributions, which, in the multiperipheral model, can be attributed to nonend diagrams—analogous to u-channel exchanges in two-body processes. On the other hand, when s/s' finally becomes respectable, the missing mass s' is too near its kinematic limit, indicated by the arrow in Fig. 3(a), and therefore the approximations made in Eq. (7) are no longer valid.

(b) $p+p\rightarrow\pi^-+anything$.—In this case, we already know that we can only expect a weakly coupled Δ exchange. In practice, Fig. 3(a) shows only the steep behavior corresponding to the disappearance of the background.

(c) $p+p\rightarrow K^+ +anything$.—Here we expect the Σ and Λ trajectories to contribute. The data are presented in Fig. 3(b). The dashed line corresponds to the somewhat more strongly coupled Λ exchange.[9] The same comments made for the π^+ spectrum are relevant here although, in this case, the quantitative agreement seems to be better.

(d) $p+p\rightarrow p+anything$.—This is probably the most interesting case, and we are in a somewhat better position kinematically. The kinematic limit in the missing mass is much further away, and, for values of s' greater than 4 $(\mathrm{GeV})^2$, we are above the region where resonance production is important. (An indication of this is the smoothness of the data of Ref. 7 in this region.) Furthermore for $s'\simeq 4$ $(\mathrm{GeV})^2$, we have $s/s'\simeq 10$ and we may expect that Eq. (7) is relevant. In Fig. 3(c) we present the data at various values of t. For completeness, we have included, for the lowest t value, a portion of the data when resonance production is visible. In the region just above resonance production the spectrum is quite flat, which indicates that the intercept of the exchanged trajectory is approximately equal to 0.5. Thus, it appears that the dominant exchange is not the vacuum trajectory, but rather something like the ρ, ω, P', etc. This, of course, is not what one expects from elastic pp scattering. That lower exchanges are seen in inelastic scattering, however, is not altogether surprising, since it is well known, or, at least, well suspected, that the inelastic Pomeranchon coupling is small. Thus, even though the vacuum trajectory has a higher intercept, it does not yet dominate at the distances from the asymptotic boundary that can be explored at this energy. From the Regge point of view, the present constant trend of this inclusive cross section, as we approach the kinematical boundary, is not an effect related to the dominant mechanism in elastic scattering, but, rather, is due to nonleading exchanges, of $\alpha\simeq 0.5$.

Our analysis of the present experimental situation shows that at 20 GeV/c we are still far from having verified the Regge prediction in inclusive experiments, but it indicates where one should focus one's attention. For the case of nonvacuum quantum-number exchanges, experiments at the new accelerators should be able to test the Regge prediction in its region of validity. For the case of vacuum exchange the question is more subtle, since it involves probing the nature of the Pomeranchuk singularity. Our analysis, based on the treatment of this singularity as a pole, tends to confirm that its coupling to inelastic processes is small, which is in agreement with current multiperipheral estimates.[10] A quantitative measurement of diffraction dissociation into large masses clearly requires high energies, and is most desirable. A Regge analysis of such an experiment, along the lines indicated in this note, would provide information on the high-energy Pomeranchon-particle total cross section.[11] In the language of DeTar *et al.*,[5] such an experiment would measure the three-Pomeranchon vertex. We believe that, in any model of particle production, this measurement would yield one of the most important and still unknown parameters in the regime of ultrahigh energies.[12]

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[1]R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969); see also *High Energy Collisions—Third International Conference*, edited by C. N. Yang (Gordon and Breach, New York, 1970), and Proceedings of the Symposium on High Energy Interactions and Multiparticle Production, Argonne National Laboratory, November, 1970 (to be published).

[2]T. T. Chou and C. N. Yang, Phys. Rev. Lett. 25, 1072 (1970).

[3]J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).

[4]These scaling properties have recently been obtained by Mueller from the Regge analysis of the forward three-particle amplitude. See A. H. Mueller,

Phys. Rev. D 2, 2963 (1970).

[5]This relation can be easily obtained in the multi-Regge model. See G. F. Chew and A. Pignotti, National Accelerator Laboratory Summer Study Report No. E-68-53, 1968 (unpublished), Vol. 3; L. Caneschi and A. Pignotti, Phys. Rev. Lett. 22, 1219 (1969); D. Silverman and C.-I Tan, Phys. Rev. D 2, 233 (1970); A. Pignotti, in Proceedings of the Symposium on High Energy Interactions and Multiparticle Production, Argonne National Laboratory, November, 1970 (to be published). It was also derived by D. Gordon and G. Veneziano and, independently, by M. A. Virasoro in the context of dual models. (See D. Gordon and G. Veneziano, to be published; M. A. Virasoro, to be published.) More recently, DeTar *et al.*, obtained the same relation following an approach analogous to that of Ref. 4 [see C. E. DeTar, C. E. Jones, F. E. Low, J. H. Weis, J. E. Young, and C.-I Tan, Phys. Rev. Lett. 26, 675 (1971)]. A similar expression was obtained by Feynman in Ref. 1. We shall refrain from deriving this expression again here.

[6]This constant behavior is required for the scaling of the inclusive cross section at high energy. If the Pomeranchon intercept is not at 1, the quantity $(E_c d\sigma/d^3p_c)/\sigma_{ab}^{\text{tot}}$ is still expected to scale.

[7]J. V. Allaby *et al.*, Nuclear Physics Division, CERN, Report No. 70-11, 1970 (to be published).

[8]This is the natural explanation for the fact that the backward π^-p elastic cross section is considerably smaller than the corresponding π^+p cross section. For a recent Regge fit of $\pi^\pm p$ backward scattering see L. Durand, III, and H. Lipinsky, Phys. Rev. D 3, 195 (1971).

[9]E. Berger and G. Fox, Phys. Rev. 186, 2120 (1969).

[10]See, for instance, G. F. Chew, T. Rogers, and D. R. Snider, Phys. Rev. D 2, 765 (1970), and references therein.

[11]We understand that A. Kaidalov is trying to extract this cross section from present accelerator data (private communication from M. Baker and G. Kane). We also understand that an analysis similar to ours is being carried out by M.-S. Chen, L.-L. Wang, and T. F. Wong [private communication from L.-L. Wang, Brookhaven National Laboratory; see M.-S. Chen, L.-L. Wang, and T. F. Wong, Phys. Rev. Lett. 26, 280, 598(E) (1971)].

[12]For further discussion of this point see G. G. Chew, Proceedings of the 1971 Coral Gables Conference on Fundamental Interactions at High Energy, University of Miami, Coral Gables, Florida, to be published.

Double Particle Exchange in Hadronic Reactions*

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We show that, contrary to present practice and belief, double-particle-exchange contributions to hadronic scattering amplitudes are not negligible. We introduce a hierarchy of exchange mechanisms: (i) Pomeranchon; (ii) single pole exchange (including pole-Pomeranchon cuts), (iii) double particle exchange, etc. The relative strength of any pair of consecutive terms in this hierarchy of amplitudes is of order $\eta=\eta_0\nu^{-1/2}$, where $\eta_0\sim 0.6$ BeV$^{1/2}$. At $\nu=5$ BeV, double-particle-exchange amplitudes may contribute to 25-50% of the cross section for inelastic processes and should not be ignored.

Phenomenological studies of hadronic reactions at intermediate and high energies usually assume that the *single*-particle-exchange mechanism[1] is dominant at these energies. The absence of important "exotic" exchanges is usually considered as evidence for the smallness of the contributions of *double* particle exchange (or the Regge cuts generated by the exchange of two "ordinary" Regge poles). A typical parametrization of a hadronic amplitude at an energy of a few BeV normally involves a few single-pole terms and perhaps a cut generated by the Pomeranchon and an "ordinary" pole (or some equivalent term such as an absorption correction). Double particle exchange is almost always ignored, even at 3 or 4 BeV.

In this paper we argue that this practice is unjustified and that there is strong evidence for the importance of double particle exchange (DPE) at energies of a few BeV. Several clues indicate that *the ratio between a DPE amplitude and a single-particle-enchange*[1] *amplitude at any given energy is roughly the same as the ratio between the single-particle-exchange term and the Pomeranchon-exchange amplitude at the same energy.* This ratio is typically[2] 0.2-0.3 at $p_L\sim 5$ BeV. We may introduce a hierarchy of contributions for a given energy, with the Pomeranchon-exchange amplitude as the leading term, followed by single particle exchange,[1] double particle exchange, etc. The ratio η between the strengths of any two consecutive terms in this hierarchy is roughly given by[2] $\eta\sim\eta_0\nu^{-1/2}$, where ν is the laboratory energy and $\eta_0\sim 0.6$ BeV$^{1/2}$.