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Production of Electron Pairs from a Zero-Mass State

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Results are presented for the production of electron pairs by the collision of high-energy bremsstrahlung and high-intensity laser beams. With Stanford linear accelerator bremsstrahlung parameters, it is found that the laser must be focused to about 10^{18} W/cm^2 to produce pairs. Approximately 25 laser photons participate in each event, and the required laser intensity is outside the known radius of convergence of perturbation theory for this process. The electron mass shift may also be determined.

The physical process investigated here is one in which electron pairs are produced in the collision of two photon beams. One beam is taken to be as energetic as possible and the other to be as intense as possible–i.e., we consider the collision of a bremsstrahlung beam from the Stanford Linear Accelerator Center (SLAC) accelerator with a focused beam from a pulsed laser.[1,2] This process is particularly interesting for a number of reasons. One reason is that it represents the generation of massive particles from an initial state with no mass whatever. An experimental demonstration of the creation of matter would have considerable philosophical significance.[3] Although the crossed-channel processes of Compton scattering and pair annihilation have long been familiar in the laboratory, photon-photon pair creation has never been observed. Another interesting aspect of the process lies in the very high order of the interaction. We show below that energy conservation requires that about 25 laser photons contribute to the production of each electron pair. The fact that pair production can actually occur with such a high-order interaction is a consequence of the inherently nonperturbative nature of a process which takes place at such a high intensity of the laser field. We show that the required intensity is well beyond the upper limit of validity of perturbation theory. All the above features would be demonstrated by a simple observation of the photon-photon production of pairs. If the momentum spectrum of the pairs was also observed, then it would be possible to measure the controversial intense-field mass shift of the electron.[4]

We denote by ω the energy of a single laser photon, and $\tilde{\omega}$ is the energy of a bremsstrahlung photon. (We set $\hbar = c = 1$.) If the photon beams are collinear, then the product $\omega\tilde{\omega}$ is invariant under Lorentz transformations along the beam direction. The energy threshold for pair production may be specified[5] in terms of $\omega\tilde{\omega}$ as

$$N\omega\tilde{\omega} \geq m^2(1+z). \tag{1}$$

N is the number of laser photons that participate, m is the electron mass, and z is the intensity parameter of the laser field. The parameter z can be written as $e^2a^2/2m^2$ (where a is the amplitude of the vector potential of the laser field), or as $2\rho\lambda\lambda\!\!\!^{-}_C r_0$ (where ρ is the density of laser photons, λ the laser wavelength, $\lambda\!\!\!^{-}_C$ the electron Compton wavelength $1/m$, and r_0 the classical electron radius $e^2/4\pi m$). The appearance of m^2z on the right-hand side of Eq. (1) in addition to m^2 can be understood on the basis that the charged particles of the electron pair have an interaction energy with the intense laser field as soon as they are created, and this extra energy must be supplied by the photons which create the pair. The bremsstrahlung intensity is too small to contribute to this effect.

As an end result of the calculation we shall find that z must be of order unity. With $\omega = 1.17$ eV (corresponding to radiation from a 1.06-μm Nd-glass laser) and $\tilde{\omega} = 18$ GeV, we find that the smallest N which satisfies Eq. (1) is $N_{\rm min} \simeq 25$.

The transition probability for the production of electron pairs by an intense photon beam in interaction with a low-intensity photon beam is known.[5] The general results are rather complicated, but in view of the large value for $N_{\rm min}$, we shall use an approximation valid for $N_{\rm min} \gg 1$. The result given in Ref. 5 for linearly polarized photon beams with perpendicular relative polarization is

$$W_\perp = \tfrac{1}{2}(3\pi^3)^{-1/2} m^4 \tilde{z} u^{-3} \exp(-u^2), \quad (2)$$

where $\tilde{z}$ is the intensity parameter for the bremsstrahlung beam, and u is a dimensionless parameter most conveniently defined in terms of its inverse square:

$$u^{-2} = \tfrac{3}{4}(\omega/m)(\tilde{\omega}/m)(2z)^{1/2}. \quad (3)$$

For parallel relative polarization of the beams,

$$W_\parallel = \tfrac{1}{2} W_\perp.$$

An important aspect of Eq. (2) is that it is independent of $N_{\rm min}$ and gives the sum over all processes with $N_{\rm min}$. If z is expressed in terms of laser energy flux P, then

$$u^{-2} = \tfrac{3}{4}(\tilde{\omega}/m) m^{-2} (2e^2 P \times 10^7)^{1/2} \quad (4)$$

with P in units of watts per square centimeter. In this form, the ω dependence of u has been absorbed entirely into P. Since Eq. (2) does not contain ω explicitly, we see the convenient fact that if the laser intensity is expressed in terms of P, Eq. (2) will give transition probabilities valid for any type of laser, i.e., for any ω for which $N_{\rm min} \gg 1$ is true.

We shall take the laser beam to be linearly polarized, and the bremsstrahlung beam to be randomly polarized. We then average over bremsstrahlung polarization orientations to obtain

$$\overline{W} = (2/\pi)(W_\parallel + W_\perp) = (3/\pi) W_\perp. \quad (5)$$

We assume the laser to be monochromatic. We shall approximate the bremsstrahlung spectrum by a rectangle, i.e., we take the bremsstrahlung energy distribution to be constant from $\tilde{\omega} = 0$ to $\tilde{\omega} = \tilde{\omega}_{\rm max} \equiv \tilde{\omega}_0$. We then integrate the $\overline{W}$ of Eq. (5) over the bremsstrahlung spectrum to obtain

$$W_{\rm tot} = 4(3\pi^3)^{-1/2}(e^2/4\pi)\tilde{\rho} m (m/\tilde{\omega}_0) u_0^{-3} \exp(-u_0^2)[1 - 2u_0^2 + 2\pi^{1/2} u_0^3 \exp(u_0^2)\,\mathrm{erfc}(u_0)], \quad (6)$$

where $\tilde{\rho}$ is the density of bremsstrahlung photons contained in the entire spectrum, and u_0 is u evaluated at $\tilde{\omega} = \tilde{\omega}_0$. To find the yield of electron pairs, we take the transition probability of Eq. (6) and multiply by the focal volume of the laser beam and the time duration of the laser pulse. The corresponding volume and time quantities for the bremsstrahlung pulse are very much larger, so it is the laser pulse parameters which determine the interaction volume and duration. We shall assume a laser focal volume of 5×10^{-6} cm^3, and a laser pulse length of 20 psec. The total yield of electron pairs per laser pulse is thus given by

$$Y = 10^{-16} W_{\rm tot},$$

where $W_{\rm tot}$ is expressed in units of events per cubic centimeter per second.

We calculate Y for three sets of bremsstrahlung beam parameters.[6] The usual present operating conditions for the SLAC machine give $\tilde{\omega}_0 = 18$ GeV, a beam diameter of 2 mm, and a pulse length of 1.6 μsec, with an electron beam current of 55 mA. With a 20% radiator, one can calculate[7] a bremsstrahlung output of 3×10^{12} photons/pulse cm^2. Under these circumstances only a very small fraction of each bremsstrahlung pulse interacts with the laser pulse, so that it is advantageous to shorten the SLAC electron beam pulse even at the cost of reducing the total charge contained in a pulse. We thus also present results for a 1.6-nsec pulse, which can be achieved with a reduction in the number of electrons per pulse by a factor of 5. As a less immediate possibility, we also consider a doubling of the SLAC accelerator energy to 36 GeV. A beam-current reduction by a factor of 10 is estimated in a study[8] which considers this energy doubling. With the given parameters inserted into Eq. (6), we get

$$Y = (4.17\times 10^5)u_0^{-3}\exp(-u_0^2)\times[1-2u_0^2+2\pi^{1/2}u_0^3\exp(u_0^2)\operatorname{erfc}(u_0)] \quad (7)$$

for an 18-GeV beam with 1.6-μsec pulse length, where, from Eq. (4),

$$u^2 = (1.82\times 10^{10})P^{-1/2}.$$

For an 18-GeV, 1.6-nsec pulse, Eq. (7) is to be multiplied by 200. For a 36-GeV, 1.6-nsec pulse, Eq. (7) is to be multiplied by 10, and the relation between u and P changed to $u^2 = (9.12\times 10^9)P^{-1/2}$.

Results are presented in Fig. 1. The curve of electron pairs per laser pulse rises so sharply as a function of laser intensity that one may think in terms of an effective intensity "threshold" for pair production, even though there is no conservation condition which forbids the process below "threshold." If we arbitrarily define this "threshold" as the intensity at which one electron pair is created for each laser pulse, then a reduction of intensity to $\frac{1}{10}$ this value leads to a pair-production probability of less than 10^{-12} for the 18-GeV, 1.6-nsec example. The intensity "thresholds" are 5.3×10^{18} W/cm^2 for the 18-GeV, 1.6-μsec pulse; 2.2×10^{18} W/cm^2 for the 18-GeV, 1.6-nsec pulse; and 8.8×10^{17} W/cm^2 for the 36-GeV, 1.6-nsec pulse. If problems arise from background pairs generated by the bremsstrahlung radiation on residual gas atoms in the experimental region, the signal-to-noise ratio can be sharply improved by slight increase in intensity.

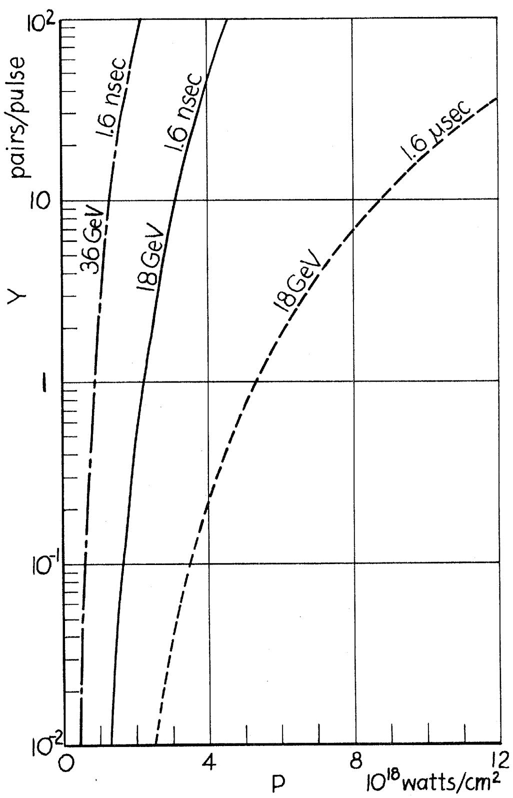

FIG. 1. Number of electron pairs produced, Y, for each coincident laser and bremsstrahlung pulse, as a function of the laser intensity P at the focus. The different curves give results for different bremsstrahlung conditions. Numbers on the curves give the maximum bremsstrahlung energy $\tilde{\omega}_0$ and the bremsstrahlung pulse length. The bremsstrahlung pulse length serves only as an index of the bremsstrahlung intensity since the laser pulse is more confined in space and time and determines the interaction volume and duration. The curves are universal for all laser wavelengths.

The critical problem for the feasibility of an experiment is the availability of the required laser intensity. A recent issue of a trade magazine[9] lists a commercially available laser which produces a peak power of 2×10^{13} W, with a beam divergence of 1.5 mrad, from a laser rod of 51 mm diam. If the output is focused with an $f/1$ lens to a focal spot limited by the beam divergence, then about 4×10^{17} W/cm^2 can be achieved. Although this intensity falls short by about a factor of 10, two considerations should be kept in mind. One is the rather obvious matter of the very rapid upward trend in laser power achieved, which has not yet reached a plateau. Another consideration arises from the strongly nonlinear behavior of the pair-production probability. Any spatial or temporal nonuniformity in the laser output will produce "hot spots" which can be very productive of pairs. For example, investigations designed to reveal short time-scale structure have found[10] intense subpicosecond spikes contained within a 20-psec laser pulse. The integrated pair-production output from a succession of such spikes would be much higher than the output from a smoothly varying 20-psec pulse of the same total energy. Usual methods for measuring laser output do not resolve picosecond or subpicosecond structure.

The pair-production probability results presented above have interesting implications for perturbation theory. The lowest-order photon-photon pair-production cross section, calculated in 1934 by Breit and Wheeler,[11] is so small that it would be very difficult to observe pairs in the collision of two relatively intense bremsstrahlung beams, each with $\tilde{\omega} > m$.[1] The very high-order process we calculate here might be expected to have a probability smaller than the Breit-Wheeler process by a factor of the order of $(e^2/4\pi)^{25}$, or about 10^{-50}. Since this is clearly not the case when the intensity is very high, we see that qualitative perturbation-theoretic notions fail at high intensity. A quantitative test of perturbation theory is also available. The intensity threshold for the 18-GeV, 1.6-nsec pulse case corresponds closely to a value $z=1$ for the intensity parameter of the laser field. A detailed investigation of the radius of convergence of perturbation theory has been done[12] for the explicit problem calculated here. Both upper and lower bounds were found for the radius of convergence of a perturbation expansion in the intense field. The expansion parameter is exactly the quantity z we have defined above, and the upper limit of convergence is given by

$$z < \frac{[m^2/\omega\tilde{\omega}]}{m^2/\omega\tilde{\omega}} - 1, \tag{8}$$

where the square bracket denotes the smallest integer containing the quantity within the bracket. Since it is true for any positive x that $[x] \leq x+1$, we may simplify Eq. (8) to

$$z < \omega\tilde{\omega}/m^2.$$

For $\omega = 1.17$ eV and $\tilde{\omega} = 18$ GeV, we have $\omega\tilde{\omega}/m^2 = 0.081$. This upper bound on the convergence of perturbation theory is far smaller than the value $z=1$ we calculated for successful realization of pair production.

We have concluded here that an extremely high-order process can be competitive with—and even dominate—the lowest order (Breit-Wheeler) process. A similar conclusion has been reached[13] by quite different methods in the very different physical context of atomic transitions in intense electromagnetic fields. One can ascribe this behavior to the large Bose factor associated with a large number of integer-spin particles in the same state, as in the case of an intense laser beam—a result of the "gregarious" nature of bosons. As we have already pointed out, the perturbation expansion parameter is not just the coupling constant $e^2/4\pi$, but it is proportional also to the photon density.

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