point in the matrix element in (10) by x and subsequently change x into $-x$, the expression (10) becomes

$$-i\int \bar{v}(p,-\mu)\langle CP(b)|\{j(0),\bar{\psi}(x)\}_+|0\rangle\theta(x_0)(-\gamma\overleftarrow{\partial}+m)u(q,-\lambda)e^{iqx}dx. \tag{11}$$

The expression (11) differs from the last expression in (5) only in $\theta(x_0)$ being replaced by $-\theta(-x_0)$ in the latter. Therefore, if the continuation (5) satisfies the usual analyticity, the difference between these two expressions vanishes as long as q_0 is real, which is assumed throughout the above proof, and also $q^2+m^2=0$ where the absorptive part of (5) vanishes. We thus see that the vertex in question satisfies (4) in the sense of (5).

Finally, we remark that the first equality implied in (5) can be put in the form

$$\bar{v}(p,\lambda)\langle b|j(0)|q,\mu\rangle=[\langle q,\mu|\bar{j}(0)|P(b)\rangle v(p,\lambda)]^*, \tag{12}$$

where the matrix elements stand for their respective continuations with respect to the baryons in the sense of (5). We make use of (9) to derive (12). In contrast to (4), the equality (12) holds only in the unphysical region where the aforementioned decay into the pair of baryons is forbidden kinematically. The significance of (12) becomes clear if we recall that the absorptive part of the forward baryon-antibaryon scattering amplitude in the unphysical region contains the product of the matrix elements that appear in (12). In other words, (12) implies that the sign of the contribution from a boson to the above absorptive part is opposite to the parity of this boson. Thus, the contributions from the pseudoscalar and vector mesons have the sign of the absorptive part in the physical region, whereas the scalar meson makes a contribution of the opposite sign.

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[1]See, for example, A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. 42, 87 (1970).

Stress-Tensor–Current Commutation Relations, the Pion-Mass Form Factors, and Tensor- and Scalar-Meson Dominance

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The low-energy behavior of the pion-mass form factors is derived using stress-tensor–current commutation relations. The results indicate significant deviations from tensor-meson and scalar-meson pole dominance of the stress tensor. The stress-tensor–ϵ-meson coupling and the scale dimension of the pion field are obtained in terms of $g_{A_1\epsilon\pi}$. Numerical estimates are given.

The matter distribution within a particle may be characterized by its mass form factors, which we define in terms of the matrix element of the stress tensor $\theta^{\lambda\sigma}$ between single-particle states.[1] In this note, we derive the pion-mass form factors at low energies, and the pion field dimension, by solving the Ward identities for the $\theta^{\lambda\sigma}\varphi\varphi$ and $\theta^{\lambda\sigma}\mathfrak{A}^{\mu}\varphi$ vertices, where φ is the pion field and $\mathfrak{A}^{\mu}$ the axial-vector current.

We define the mass form factors of a 0^- meson as follows[2]:

$$\langle p_2|\theta^{\lambda\sigma}(0)|p_1\rangle=\tfrac{1}{2}[k^\lambda k^\sigma G_1+(q^2\eta^{\lambda\sigma}-q^\lambda q^\sigma)G_2+2\mu^2\eta^{\lambda\sigma}G_3+(q^\lambda k^\sigma+k^\lambda q^\sigma)G_4], \tag{1}$$

$$=R^{\lambda\sigma}\mathcal{G}_T+\tfrac{2}{3}\mu^2S^{\lambda\sigma}\mathcal{G}_S+\mathcal{V}^{\lambda\sigma}G_3+W^{\lambda\sigma}G_4, \tag{2}$$

where $q=p_1-p_2$, $k=p_1+p_2$, μ is the meson mass, and

$$R^{\lambda\sigma}=\tfrac{1}{2}(k^\lambda k^\sigma-\tfrac{1}{3}k^2S^{\lambda\sigma});\quad S^{\lambda\sigma}=(\eta^{\lambda\sigma}-q^\lambda q^\sigma/q^2);\quad \mathcal{V}^{\lambda\sigma}=\tfrac{1}{3}\mu^2(-\eta^{\lambda\sigma}+4q^\lambda q^\alpha/q^2);$$

$$W^{\lambda\sigma}=[q^\lambda k^\sigma+k^\lambda q^\sigma-\tfrac{2}{3}(q\cdot k)S^{\lambda\sigma}]/2.$$

For an on-mass-shell meson, $G_3=G_4=0$ and $\mathcal{G}_T(0)=\mathcal{G}_S(0)=G_1(0)=1$. We term $\mathcal{G}_S$ and $\mathcal{G}_T$ the scalar and tensor mass form factors. We use the model-independent equal-time commutator (ETC) relation of $\theta^{\lambda\sigma}$ with $\mathfrak{A}^\mu$ and $\partial_\mu \mathfrak{A}^\mu$ derived by Jackiw,[3] and the divergence relations $\partial_\lambda \theta^{\lambda\sigma}=0$ and $\partial_\mu \mathfrak{A}_+{}^\mu = C_\pi \varphi_{\pi^+}$, where φ is the pion field.

We define the noncovariant three-point function $\tilde{F}^{\lambda\sigma}$ by

$$\tilde{F}^{\lambda\sigma} = -(\mu^2-p_1^2)(\mu^2-p_2^2)\int dz\, dx \exp[i(q\cdot z - p_1\cdot x)]\langle 0|T[\theta^{\lambda\sigma}(z)\varphi_{\pi^+}{}^\dagger(x)\varphi_{\pi^+}(0)]|0\rangle, \tag{3}$$

and the function $\tilde{F}^{\mu;\lambda\sigma}$ by replacing $\varphi_{\pi^+}{}^\dagger$ in (3) by $\mathfrak{A}_-{}^\mu$. We construct the corresponding covariant functions $F^{\lambda\sigma}$ and $F^{\mu;\lambda\sigma}$, using the methods of Gross and Jackiw.[4] We identify $F^{\lambda\sigma}$ with the covariant matrix element $\langle \pi^+(p_2)|\theta^{\lambda\sigma}(0)|\pi^+(p_1)\rangle$; this defines the mass form factors $\mathcal{G}_S$ and $\mathcal{G}_T$.

Using the ETC $[\theta^{\lambda\sigma}, \mathfrak{A}^\mu]$ and

$$[\mathfrak{A}_-{}^0(x), \varphi_+(y)]\delta(x_0-y_0) = i\sqrt{2}\,\sigma(x)\delta(x-y), \tag{4}$$

where $\sigma(x)$ is a scalar-isoscalar operator, we obtain the Ward identities

$$q_\lambda F^{\lambda 0} = p_1{}^0(p_1{}^2-\mu^2) + p_2{}^0(p_1{}^2-\mu^2); \tag{5a}$$

$$iq_\lambda F^{0;\lambda 0} = -C_\pi\{-q\cdot p_2 + q_0 p_{10} - \mu^{-2}(\vec{p}_1{}^2 p_2{}^2 + p_1{}^2 \vec{p}_1\cdot\vec{p}_2)\}; \tag{5b}$$

$$ip_{1\mu}F^{\mu;00} = C_\pi F^{00} + C_\pi(\mu^2-p_1^2) + i\sqrt{2}\Delta_{\theta\sigma}{}^{00}(q)(p_1{}^2-\mu^2)(p_2{}^2-\mu^2). \tag{5c}$$

Here, $\Delta_{\theta\sigma}{}^{\lambda\rho}$ is the two-point function involving $\theta^{\lambda\sigma}$ and σ.

We solve these for $F^{\lambda\sigma}$ up to the fourth order in q, p_1, p_2, and for $F^{\mu;\lambda\sigma}$ to the third order in the momenta, in terms of two constants. We express one of these in terms of $g_{A_1\epsilon\pi}$ by relating $F^{\mu;\lambda\sigma}$ to the $A_1\epsilon\pi$ vertex, and obtain the following low-momentum solutions for $\mathcal{G}_S$ and $\mathcal{G}_T$, in terms of one parameter[5] h:

$$\mathcal{G}_T(q^2, p_1^2, p_2^2) = G_1(q^2, p_1^2, p_2^2) = 1 + hq^2 + \cdots, \tag{6a}$$

$$2\mu^2\mathcal{G}_S(q^2, p_1^2, p_2^2) = 4\mu^2 - p_1^2 - p_2^2 + q^2(1-3\alpha) + 3(q^2)^2(r - \alpha/m_\epsilon^2) - q^2(p_1^2+p_2^2)(h + 3r - 3\alpha/\mu^2), \tag{6b}$$

where we have taken

$$\Delta_{\theta\sigma}{}^{\lambda\rho}(q) \approx i\beta(q^2\eta^{\lambda\rho} - q^\lambda q^\rho)/(m_\epsilon^2 - q^2)$$

and

$$f_\pi \equiv C_\pi/\mu^2, \quad \alpha \equiv \sqrt{2}\beta\mu^2 m_\epsilon^{-2} f_\pi^{-1}, \quad r \equiv 2\lambda_A g_{A_1\epsilon\pi}\beta/f_\pi m_\epsilon^2 m_{A1}^2.$$

We have assumed that only one low-lying scalar meson, the ϵ at about 700 MeV, couples strongly to $\pi+\pi$. The solution (6) leads to the following results:

(a) For pions with zero external mass, the scalar mass form factor has the low-energy form

$$\mathcal{G}_S(q^2, 0, 0) \approx 2 + q^2(1-3\alpha)/2\mu^2 + (q^2)^2(1-\alpha)/2\mu^2 m_\epsilon^2 + \cdots, \tag{7a}$$

$$\approx 2 + q^2/2\mu^2 + (q^2)^2/2\mu^2 m_\epsilon^2 + \cdots, \tag{7b}$$

where we have estimated α (or β) as discussed in (c) below.

The result (7) shows that at least for $p^2=0$ pions, the scalar mass form factor is quite different from the pion-charge form factor. The rms scalar mass radius is much larger than the charge radius $r_{\text{ch}}^2(\pi) \approx 6/m_\rho^2$, and seems to be determined by the pion Compton wavelength rather than by scalar-meson intermediate states. (7) suggests that it is inconsistent to assume a simple unsubtracted, pole-dominant form for $\mathcal{G}_S$ at small q^2.

Recently, Kleinert and Weisz[6] have obtained $\hat{\Gamma}'(0,0,0) \approx 1 - \mu^2 d/m_\sigma^2$, where $\hat{\Gamma}$ is the (noncovariant) function obtained by replacing $\theta^{\lambda\sigma}$ in $\tilde{F}^{\lambda\sigma}$ by $\theta_\sigma{}^\sigma$, and d is the scale dimension of the pion field φ. Note that their work refers to the noncovariant function involving $\langle 0|T(\theta_\sigma{}^\sigma\varphi\varphi)|0\rangle$ and hence involves d, which occurs in the singular term in the ETC $[\theta^{\lambda\sigma}, \varphi]$. In our results for the covariant $F^{\lambda\sigma}$, this singular term drops out. With the assumptions made here and in Ref. 6, $2\mu^2\mathcal{G}_S'(0,0,0) = \hat{\Gamma}'(0,0,0)$, and $d = 3\sqrt{2}\beta/f_\pi$, which is equivalent to the relation noted in Ref. 6.

(b) For *physical* pions, we obtain the following sum rule for the scalar and tensor (rms) mass radii:

$$\tfrac{1}{6}(r_S{}^\pi)^2 + \tfrac{1}{6}(r_T{}^\pi)^2 = (1-3\alpha)/2\mu^2 - 3r \approx 1/2\mu^2, \tag{8}$$

where $\mathcal{G}_S' \equiv \partial \mathcal{G}_S/\partial q^2$ and

$$(r_S^\pi)^2 \equiv 6\mathcal{G}_S'(0, \mu^2, \mu^2)/\mathcal{G}_S(0, \mu^2, \mu^2),$$

etc. (8) shows that it is inconsistent to assume pole dominance (by the ϵ and f poles) for both mass form factors, $\mathcal{G}_S$ and $\mathcal{G}_T$.

(c) To determine the remaining parameter h in $\mathcal{G}_S$ and $\mathcal{G}_T$, we assume maximal smoothness of $\mathcal{G}_S$ (which has an ϵ-meson pole) in q^2, p_1^2, and p_2^2, in the sense that $\mathcal{G}_S$ is well approximated by $(q^2-m_\epsilon^2)^{-1} \times(a+bq^2+cp_1^2+cp_2^2)$ for $0 \lesssim q^2 \lesssim m_\epsilon^2$, $0 \lesssim p_1^2$, and $p_2^2 \lesssim \mu^2$, correct to terms of the second order in q^2, p_1^2, and p_2^2. This determines h and β, and $d=3\sqrt{2}\beta/f_\pi$, giving

$$\beta=\sqrt{2}f_\pi m_{A_1}^2/6\lambda_A g_{A_1\epsilon\pi}; \quad h\equiv\tfrac{1}{6}(r_T^\pi)^2=m_{A_1}^2/m_\epsilon^2\lambda_A g_{A_1\epsilon\pi}; \quad d=hm_\epsilon^2=m_{A_1}^2/\lambda_A g_{A_1\epsilon\pi}. \tag{9}$$

The sum rule (8) now gives $\frac{1}{6}(r_S^\pi)^2=(1/2\mu^2-1/m_\epsilon^2-\frac{3}{2}h)$. Note that β and d in (9) do not involve the ϵ mass explicitly.

To obtain numerical estimates, we need λ_A and $g_{A_1\epsilon\pi}$. We assume $\lambda_A \approx \lambda_\rho \approx f_\pi m_\rho$, using the Weinberg sum rules and the Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin result.[7] For $g_{A_1\epsilon\pi}$, we first note that (9) and the experimental limit $\Gamma(A_1\to\epsilon\pi)<\Gamma(A_1\to 3\pi)\approx 95\pm 35$ MeV give $d>{\sim}0.6$. In a model with chiral SU(2) $\otimes$ SU(2) sum rules saturated by $0^\pm$ and $1^\pm$ mesons, Gilman and Harari[8] have obtained $|g_{A_1\epsilon\pi}|\approx m_{A_1}/f_\pi$. In this model, we obtain the estimates

$$\beta\approx\tfrac{1}{3}f_\pi\approx\tfrac{1}{3}, \quad d\approx\sqrt{2}, \quad h\equiv\tfrac{1}{6}(r_T^\pi)^2\approx\sqrt{2}m_\epsilon^{-2}\approx\sqrt{2}m_\rho^{-2}, \quad \tfrac{1}{6}(r_S^\pi)^2\approx 0.4/\mu^2. \tag{10}$$

The estimates (10) are characterized by a large scalar mass radius for the pion, and a tensor mass radius of the same order as the pion charge radius. The validity of the results (9) depends mainly on the smoothness assumption for $\mathcal{G}_S$, while the numerical estimates (10) depend, in addition, on the Gilman-Harari model being a good approximation. The results indicate that tensor-meson and scalar-meson pole dominance of the matrix elements of the traceless part and the trace, respectively, of the stress tensor are not adequate approximations.

We note that taking $d\approx 1$ in (9) would correspond to a universality limit in which $\mathcal{G}_T$ is (approximately) degenerate with the pion-charge form factor, with $\frac{1}{6}(r_T^\pi)^2\approx m_\epsilon^{-2}\approx m_\rho^{-2}$, as may be obtained if both are dominated by a universal, exchange-degenerate ρ-f_0 trajectory. This would require $\lambda_A\approx m_{A_1}^2/g_{A_1\epsilon\pi}$. Assuming $\lambda_A\approx\lambda_\rho\approx f_\pi m_\rho$, this would imply $g_{A_1\epsilon\pi}\approx g_{\rho\pi\pi}m_{A_1}^2/m_\rho^2$ and $\Gamma(A_1\to\epsilon\pi)\approx 110$ MeV. An experimental estimate of either $g_{A_1\epsilon\pi}$ or $g_{A_1\rho\pi}$ will enable a direct estimate of d and of the deviation from the universality limit.

(d) We have found that simple resonance-dominated unitarization schemes (with finite width) are quite inadequate, at least for $\mathcal{G}_S$. Possible mechanisms for a large $\mathcal{G}_S'(0)$ would be (1) very large inelasticity in high-energy, S-wave $\pi\pi$ scattering, (2) a zero in $\mathcal{G}_S(t)$ for Ret small and negative, and (3) an S-wave $\pi\pi$ bound state. In the absence of evidence for a 2π bound state, the most reasonable mechanism seems to be a zero in $\mathcal{G}_S(t)$ for Re$t\lesssim 0$. This could arise from a zero in the $\pi\pi$ S-wave amplitude $A_0^0(t)$ for Re$t\lesssim 0$.[9] An interesting consequence would be the following. A zero for t real and negative would imply a zero in the appropriate (physical) amplitude for pion scattering by a gravitational field, while a pair of complex conjugate zeros, with Re$t\lesssim 0$, would give a dip in the amplitude. Unitarization schemes and Veneziano-type models for $\mathcal{G}_S$, $\mathcal{G}_T$, and $F^{\mu;\lambda\sigma}$ will be discussed in a separate paper.

(e) The solution for $F^{\mu;\lambda\sigma}$ gives the result that in the decays $f\to A_1+\pi$ and $f'\to A_1+\pi$, the F wave is suppressed compared to the D wave. Approximating the parts of $F^{\lambda\sigma}$ and $F^{\mu;\lambda\sigma}$ traceless in λ and σ by a constant plus an f pole plus an f' pole,[10] we obtain the sum rule

$$g_{fA_1\pi}+(m_f/m_{f'})^2 g_{f'A_1\pi}\tan\theta_T=f_\pi m_{A_1}^2 g_{f\pi\pi}/\lambda_A\sqrt{2}, \tag{11}$$

where θ_T is the f-f' mixing angle. If the $f'A_1\pi$ coupling is small, this gives $g_{fA_1\pi}\approx f_\pi m_{A_1}^2 g_{f\pi\pi}/\lambda_A\sqrt{2}$.

We have also solved for the three-point function with $\mathcal{A}^\mu$, $\mathcal{A}^\nu$, and $\theta^{\lambda\sigma}$. This involves additional parameters and does not give further restrictions on $\mathcal{G}_S$ and $\mathcal{G}_T$.

The extension of this work to the 0^- octet and other details will be discussed elsewhere.

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[1]See, for example, H. Pagels, Phys. Rev. 144, 1250 (1966).

[2]We normalize states covariantly: $\langle p'|p\rangle=(2\pi)^3 2p_0\delta(\vec{p}-\vec{p}')$. $\eta^{\lambda\sigma}$ is the metric $(1,-1,-1,-1)$.

[3]R. Jackiw, Phys. Rev. 175, 2058 (1968); D. Gross and R. Jackiw, Phys. Rev. 163, 1688 (1967).

[4]D. Gross and R. Jackiw, Nucl. Phys. B14, 269 (1969).

[5]We define $\langle A_1^-(p)|\mathfrak{a}_-^{\mu}(0)|0\rangle = \sqrt{2}\lambda_A\epsilon^{\mu}(A_1)$, and similarly for λ_ρ. Also, $\langle\epsilon(q),\pi^-(p)|A_1^-(k)\rangle = (2\pi)^4\delta(k-p-q)g_{A_1\epsilon\pi} \times (q-p)\cdot\epsilon(A_1)$.

[6]H. Kleinert and P. H. Weisz, CERN Report No. Th. 1234, 1970 (to be published). See also H. A. Kastrup, Nucl. Phys. B15, 179 (1970). Related results for $\hat{\Gamma}$ have been discussed by R. Jackiw, to be published.

[7]S. Weinberg, Phys. Rev. Lett. 18, 507 (1967); K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).

[8]F. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968).

[9]A zero in A_0^0 below threshold has been discussed by Y. Fujii and K. Hayashi, Progr. Theor. Phys. 39, 126 (1968); H. Yabuki, Progr. Theor. Phys. 39, 118 (1968); J. R. Fulco and D. Y. Wong, Phys. Rev. Lett. 19, 1399 (1967); K. Kang and D. J. Land, Phys. Rev. Lett. 18, 503 (1967); and references quoted in the above.

[10]For a discussion of such an assumption and its consequences, see, for instance, K. Raman, Phys. Rev. D 2, 1577 (1970), and to be published.

Production of Electron Pairs from a Zero-Mass State

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Results are presented for the production of electron pairs by the collision of high-energy bremsstrahlung and high-intensity laser beams. With Stanford linear accelerator bremsstrahlung parameters, it is found that the laser must be focused to about 10^{18} W/cm^2 to produce pairs. Approximately 25 laser photons participate in each event, and the required laser intensity is outside the known radius of convergence of perturbation theory for this process. The electron mass shift may also be determined.

The physical process investigated here is one in which electron pairs are produced in the collision of two photon beams. One beam is taken to be as energetic as possible and the other to be as intense as possible—i.e., we consider the collision of a bremsstrahlung beam from the Stanford Linear Accelerator Center (SLAC) accelerator with a focused beam from a pulsed laser.[1,2] This process is particularly interesting for a number of reasons. One reason is that it represents the generation of massive particles from an initial state with no mass whatever. An experimental demonstration of the creation of matter would have considerable philosophical significance.[3] Although the crossed-channel processes of Compton scattering and pair annihilation have long been familiar in the laboratory, photon-photon pair creation has never been observed. Another interesting aspect of the process lies in the very high order of the interaction. We show below that energy conservation requires that about 25 laser photons contribute to the production of each electron pair. The fact that pair production can actually occur with such a high-order interaction is a consequence of the inherently nonperturbative nature of a process which takes place at such a high intensity of the laser field. We show that the required intensity is well beyond the upper limit of validity of perturbation theory. All the above features would be demonstrated by a simple observation of the photon-photon production of pairs. If the momentum spectrum of the pairs was also observed, then it would be possible to measure the controversial intense-field mass shift of the electron.[4]

We denote by ω the energy of a single laser photon, and $\tilde{\omega}$ is the energy of a bremsstrahlung photon. (We set $\hbar = c = 1$.) If the photon beams are collinear, then the product $\omega\tilde{\omega}$ is invariant under Lorentz transformations along the beam direction. The energy threshold for pair production may be specified[5] in terms of $\omega\tilde{\omega}$ as

$$N\omega\tilde{\omega} \geq m^2(1+z). \tag{1}$$

N is the number of laser photons that participate, m is the electron mass, and z is the intensity parameter of the laser field. The parameter z can be written as $e^2a^2/2m^2$ (where a is the amplitude of the vector potential of the laser field), or as $2\rho\lambda\bar{\lambda}_C r_0$ (where ρ is the density of laser photons, λ the laser wavelength, $\bar{\lambda}_C$ the electron Compton wavelength $1/m$, and r_0 the classical electron radius $e^2/4\pi m$). The appearance of m^2z on the right-hand side of Eq. (1) in addition