Scaling and Factorization in $K^+ + p \to \pi^- +$ Anything*

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We report, with good statistics, the π^- momentum distributions from the reaction $K^+ + p \to \pi^- +$anything at 11.8 GeV/c. We compare the shape of the distribution in longitudinal momentum of the π^- with that of π^- mesons produced in the reaction $\pi^- + p \to \pi^- +$ anything at 25 GeV/c and $\pi^+ + p \to \pi^- +$anything at 18.5 GeV/c in terms of the scaling variable $x = 2p_L/\sqrt{s}$. The backward distributions are found to be independent of the nature of the projectile. The forward distributions are different in shape from the backward, but are remarkably similar for K^+p and π^+p interactions. The distribution in longitudinal momentum in the center-of-mass system is not independent of the value of the transverse momentum.

We report on the momentum distributions of the negative pions from the "inclusive" reaction[1]

$$K^+ + p \to \pi^- + \text{anything} \tag{1}$$

at 11.8 GeV/c. We know of no published data reporting on an inclusive reaction with incident kaons and having good statistics at both small and large values of the longitudinal momentum of the negative pion. We report here on 30 000 pions from Reaction (1), a 9.5% subsample of the data available.

The events discussed here were produced in the Lawrence Radiation Laboratory–Stanford Linear Accelerator Center 82-in. hydrogen bubble chamber. They correspond to 2.8 ± 0.2 events/μb. We examined all four-, six-, and eight-prong events. Two-prong events cannot produce a negative particle, and events with ten or more prongs constitute only ~0.5% of the total cross section. Therefore we do not feel that these omissions introduce a significant bias. No kinematic fitting was done on any events, and events both with and without associated "vees" were used. The measured momentum of each negative track was used, and every negative track was assumed to be a pion. The K^- and $\bar{p}$ contamination introduced by this procedure is probably about 1%.[2]

The K^+p reaction is particularly suitable to a study of *produced* particles, such as the π^- in Reaction (1), since the production of negative particles (K^-,$\bar{p}$) other than pions is small, making the purity of the sample good; and neither of the incident particles (K^+,p) can be mistaken for produced pions, so all distributions are free of "leading-particle" effects.[3]

We should like, with these data, to examine "scaling" behavior and also factorization, by which we mean here the independence of the longitudinal and transverse momentum distributions. Factorization can be examined with the present data alone, but scaling requires data at a second energy. No suitable K^+p data exist in published form, and we shall have to resort to comparison with πp data.

The c.m. system cross section for Reaction (1) may be written[4]

$$d\sigma = (d^3p/E) f(p_T, p_L, s), \tag{2}$$

where $d\sigma$ is the differential cross section for production of a negative pion of momentum $\vec{p}$ and energy E in the c.m. system. The total energy in the c.m. system is $\sqrt{s}$, p_T and p_L are the transverse and longitudinal momenta of the pion, respectively, and f is some unspecified function of p_T, p_L, and s.

Various authors have suggested[5] that $f(p_T, p_L, s)$ in one reference frame or another scales with s or becomes independent of s. In particular, Feynman suggests[1] that the form $f(p_T, 2p_L/\sqrt{s})$ may hold, where $2p_L/\sqrt{s}$ is Feynman's x variable. To test this possibility, we may rewrite (1) in the form

$$(2E/\pi\sqrt{s})(d^2\sigma/dx\,dp_T^2) = f(p_T, x) \tag{3}$$

and compare the left-hand side at two or more values of s, with p_T fixed. As we have said, no published K^+p data suitable for comparison exist. We may, however, be able to compare in a meaningful way the *backward* π^- mesons from this reaction with the backward negative pions from π^-p collisions. If the backward pions are in some way associated mostly with the proton, then the *shapes* of their momentum distributions may not depend upon the nature of the incident particle, although the total backward cross sections may.

With this possibility in mind, we have compared the K^+p data with a π^- momentum distribu-

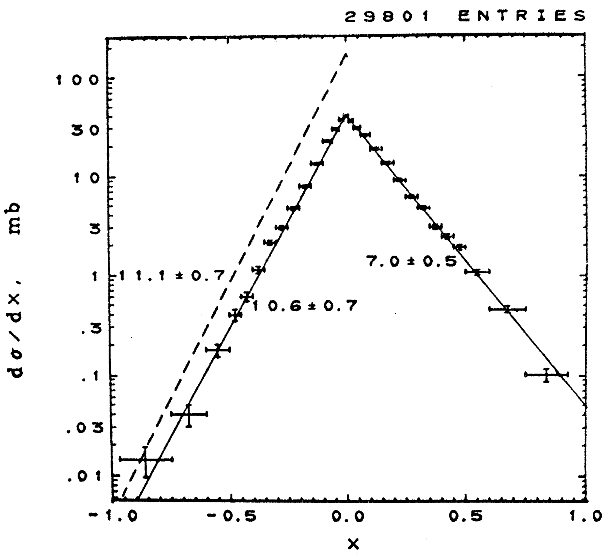

FIG. 1. The dashed lines represent the 25-GeV/c $\pi^- p$ data of Ref. 6. The crosses are the 11.8-GeV/c K^+p data and the solid line is a fit to those data. The parameter x is defined by $x = 2p_L/\sqrt{s}$.

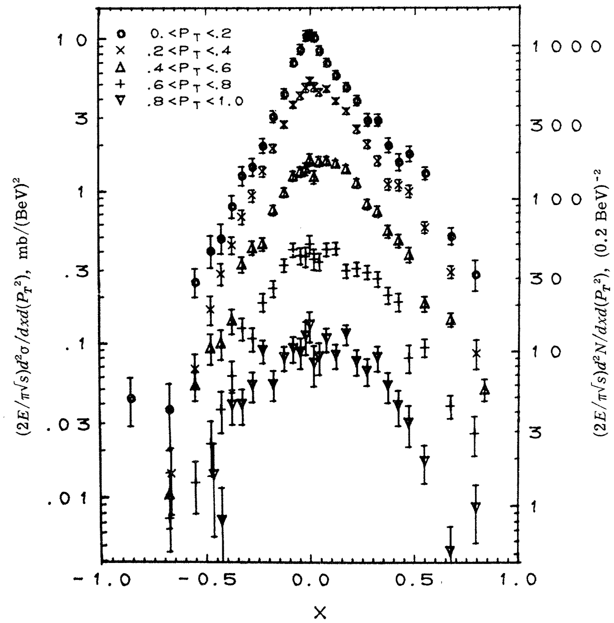

FIG. 2. The double differential cross section in the center of mass is plotted versus x for various intervals in p_T after having been multiplied by the energy of the π^- and other constants so that it represents $f(x, p_T)$.

tion observed in the reaction

$$\pi^- + p \to \pi^- + \text{anything} \tag{4}$$

at 25 GeV/c, studied by Elbert, Erwin, and Walker.[6] These authors presented the longitudinal momentum spectrum in the c.m. system in the form $d\sigma/dp_L$, without the multiplicative factor E and after integrating over p_T. Their distribution was exponential, and they obtained a good fit using $d\sigma/dp_L \propto \exp[(3.2 \pm 0.2)p_L]$ for the backward part of the spectrum. Therefore we compare this result with our K^+p data. For convenience, we first make the trivial conversion of their $d\sigma/dp_L$ distribution to $d\sigma/dx$ by using $d\sigma/dx = \frac{1}{2}s^{1/2} d\sigma/dp_L$. Figure 1 then shows, as a dashed line, the fit obtained by Elbert, Erwin, and Walker[6] for the longitudinal momentum distribution of the produced pions in the 25-GeV/c $\pi^- p$ reaction. The data points shown in Fig. 1 are those obtained in the K^+ experiment reported here. We see that the slope of the K^+p data at 11.8 GeV/c is quite consistent with the slope obtained from the 25-GeV/c $\pi^- p$ data, considering only backward pions. The absolute values of the backward cross sections from the π^--induced reactions are about three times as large as those from the K^+-induced reactions. This ratio is even larger than that of the total $\pi^- p$ and K^+p cross sections. We note that the π^- selection necessarily excludes some part of the total cross section. In the K^+ data, for example, charge conservation completely eliminates the two-prong final states when a π^- is selected. These states account for almost one half the total cross section. The π^- selection in the 25-GeV/c $\pi^- p$ data excludes a smaller fraction of that total cross section. The values of $\sqrt{s}$ are 4.8 and 6.9 GeV for the K^+p and $\pi^- p$ reactions, respectively. It is remarkable that the shapes are so similar even down to small (negative) values of x, and this similarity may mean that $p_L/\sqrt{s}$ scaling is indeed operating at these energies, at least with respect to $d\sigma/dx$, if not $f(p_T, x)$ alone.

The forward π^- distribution from our data may not safely be compared with that from the 25-GeV/c $\pi^- p$ data because of the possibility of confusion with leading π^- mesons in the latter.[7] We may, however, use data from the reaction $\pi^+ + p \to \pi^- + \text{anything}$ at 18.5 GeV/c.[8] The slope of the distribution in x for π^- mesons from this reaction is quoted in Ref. 8 to be 7.0 and 10.25 in the forward and backward directions, respectively. Here, not only do we have a similar distribution in the backward direction, but also the slope of the *forward* π^--production distribution is remarkably similar to that from the K^+p data. The forward-produced particles might be expected to depend on the nature of the projectile; yet we see that they have very similar shapes. Thus the forward π^- production appears to be the same for these two pseudoscalar-meson projectiles.

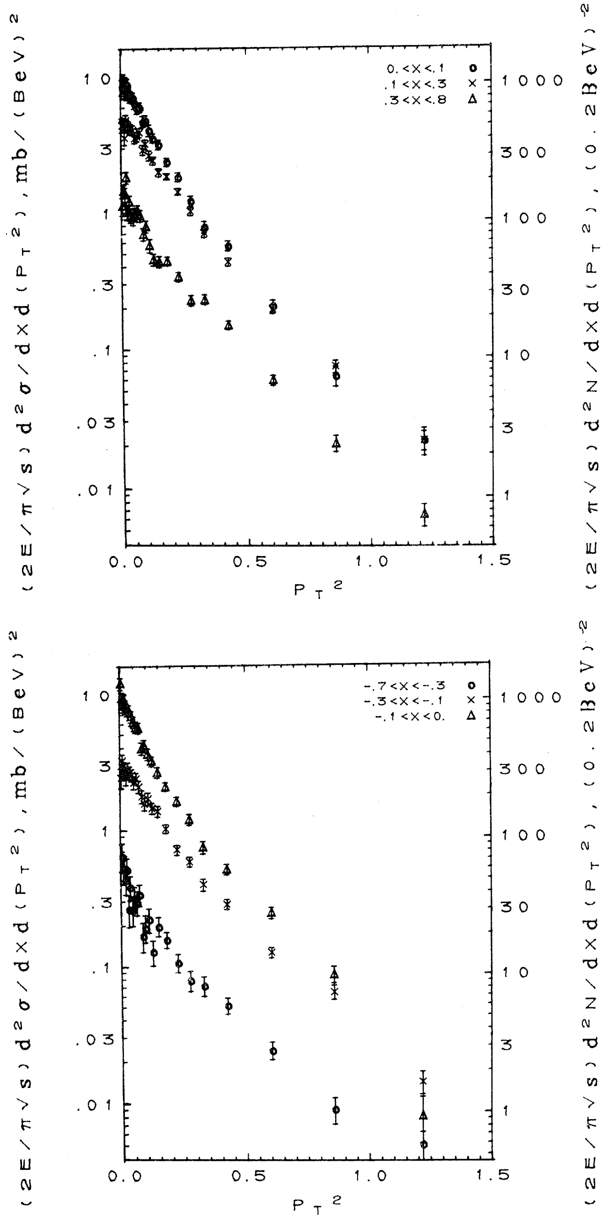

FIG. 3. The same quantity as Fig. 2 plotted here versus p_T^2 for various intervals in x, as labeled in the figure.

Even apart from scaling, knowledge of the function $f(p_T, p_L, s)$ is of interest. We have therefore examined, at our fixed s, the quantity given by the left-hand side of Eq. (3). (At fixed s, it matters not whether p_L or x is used.) We display in Fig. 2 the values of this double differential cross section as a function of x for various fixed intervals in p_T. We see in Fig. 2 that these distributions in $Ed^2\sigma/dxd(p_T^2)$ are not very well represented by a simple exponential in x, nor even by a sum of two or three exponentials. In particular, although the distributions at larger values of p_T are rounded near $x=0$, the distributions with small values of p_T show sharp peaking near $x=0$. Thus, although the distributions in x have the same *gross* behavior for any value of p_T, namely, a rapid decrease as $|x|$ increases, they do not exhibit true independence of p_T and x. Therefore $f(p_T, p_L, s)$ does not factor into $F(p_T) \times G(p_L, s)$. The distributions in p_T^2 for fixed intervals in x are shown in Fig. 3.

The picture that emerges from these data is one of a very sharply peaked distribution in x when $p_T \sim 0$, becoming "rounded" near $x=0$ when the value of p_T deviates much from zero. We note that within the accuracy of the present data, the peak in Fig. 2 occurs at $x=0$, and this $x=0$ value of $Ed^2\sigma/dxd(p_T^2)$ is the same whether approached from the left or the right. That is, there is no discontinuity in the *value* of $Ed^2\sigma/dxd(p_T^2)$ at $x=0$. The derivative, however, more nearly becomes discontinuous as p_T decreases. Within our statistical accuracy, we have not been able to observe any "flat top" in the distribution in x when p_T is small. Any such flat portion must be confined to $|x| \lesssim 0.03$.

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[1]R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).

[2]See, for example, Lawrence Radiation Laboratory Particle Data Group, Report No. UCRL 20 000 K^+N, 1969 (unpublished).

[3]By "leading particle" we are referring to the following effect generally found in high-energy collisions: If one examines, in the c.m. system, the longitudinal-momentum distribution of a secondary particle that is identical to one of the incident particles, one observes that there are many more such particles moving in the same direction as their incident counterpart than there are moving in the opposite direction. The distribution in longitudinal momentum is (very roughly) constant in the forward direction for larger values, while in the backward direction it falls off rapidly. For those secondary particles that are distinct from the incident particles, the forward distribution generally falls off more

rapidly. The production mechanisms are presumably different in the two cases.

[4]See, for example, R. Hagedorn, *Relativistic Kinematics* (Benjamin, New York, 1964), p. 92.

[5]D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962); K. Wilson, Acta Phys. Austr. 17, 37 (1963); R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969); J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).

[6]J. W. Elbert, A. R. Erwin, and W. D. Walker, to be published. Strange particles have been eliminated from their data, but not from the K^+p data at 11.8 GeV/c.

[7]The leading particle contribution to the π^- momentum spectrum in the 25-GeV/c π^-p data, as estimated from the value of $d\sigma/dp_L$ in the flat portion of the distribution, is about 10% of the value at the peak ($p_L = 0$). If we therefore ignore this contribution and obtain the slope for the region $p_L < 1.0$ GeV/c, we estimate it to be 6.6 when converted to a distribution in x. This value is rather close to the value 7.0 that we obtain for the forward π^- mesons in our K^+p data.

[8]N. N. Biswas, N. M. Cason, M. S. Farber, V. P. Kenney, J. D. Poirier, J. T. Powers, O. R. Sander, and W. D. Shephard, to be published.

Quantum Numbers of Nonstrange, Neutral Bosons*

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It is shown that the trilinear (or Yukawa-type) vertex of a nonstrange, neutral boson with a spin-$\frac{1}{2}$ baryon vanishes unless the boson has the quantum numbers of the quark-antiquark ($q\bar{q}$) system, as long as this vertex satisfies the usual analyticity conditions and C and P are good quantum numbers. Thus, the quantum numbers of the observed nonstrange bosons may be explained without the quark.

The fact that all the observed nonstrange bosons have the quantum numbers of the $q\bar{q}$ system[1] may appear to indicate that the quark actually exists. The purpose of this note is, however, to point out that the quark may not be needed to explain this fact. We show, in particular, that the trilinear vertex of a nonstrange, neutral boson with a spin-$\frac{1}{2}$ baryon vanishes unless the boson has the quantum numbers of the $q\bar{q}$ system. In other words, if a boson has the quantum numbers other than those of the $q\bar{q}$ system, such a boson must interact with the spin-$\frac{1}{2}$ baryon only as a pair or more and, therefore, would be difficult to observe.

We begin by discussing the decay of a nonstrange, neutral boson in its rest system into a spin-$\frac{1}{2}$ baryon with momentum q and helicity μ and its antiparticle with momentum $p = (-\vec{q}, iq_0)$ and helicity λ. This decay matrix element transforms under C and P as

$$\langle b|p,\lambda;q,\mu\rangle = \langle b|P^{-1}P|p,\lambda;q,\mu\rangle = -\langle P(b)|q,-\lambda;p,-\mu\rangle = \langle CP(b)|p,-\mu;q,-\lambda\rangle, \tag{1}$$

where $|P(b)\rangle$ and $|CP(b)\rangle$ are, respectively, the P and CP transforms of the boson state $|b\rangle$, and where the first set of momentum and helicity in the baryon-antibaryon state stands for the antibaryon. The transformations (1) imply that the decay matrix element vanishes unless the parity of the boson is opposite to the orbital parity of the baryon-antibaryon state. They also imply that the decay matrix element vanishes unless the CP parity of the boson is even or odd, according as this pair of baryons assumes a spin-triplet or a spin-singlet configuration. The latter implication of (1) can be seen by rewriting the CP transformation in (1) as, when $\mu = -\lambda$,

$$\langle b|p,\lambda;q,-\lambda\rangle = \langle CP(b)|p,\lambda;q,-\lambda\rangle, \tag{2}$$

or, when $\mu = \lambda$,

$$\langle b|p,\lambda;q,\lambda\rangle \pm \langle b|p,-\lambda;q,-\lambda\rangle = \pm[\langle CP(b)|p,\lambda;q,\lambda\rangle \pm \langle CP(b)|p,-\lambda;q,-\lambda\rangle], \tag{3}$$

with either the upper or the lower signs taken everywhere. In other words, the transformations (1) imply that the above decay is forbidden unless the boson has the quantum numbers of the $q\bar{q}$ system. These transformations can also be written as

$$\bar{v}(p,\lambda)\langle b|j(0)|q,\mu\rangle = -\bar{v}(q,-\lambda)\langle P(b)|j(0)|p,-\mu\rangle = \bar{v}(p,-\mu)\langle CP(b)|j(0)|q,-\lambda\rangle, \tag{4}$$

if antibaryons are reduced everywhere in (1).