these experiments are weak, the consistency between them indicates that we should take them seriously. Obviously extensive experimental study in this area is needed.

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Extraction of $\Lambda K^- p$ and $\Sigma K^- p$ Coupling Constants from $K^\pm$-p Forward Amplitudes

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The difficulty about the unphysical part of the cut of $K^\pm$-p forward amplitudes is avoided by conformally mapping the unphysical and low-energy K^--p regions on the cut onto the boundary of an ellipse and the whole cut plane to the inside of the ellipse. A discrepancy function constructed on the physical region using experimental information on the $K^\pm$-p forward amplitudes is stably extrapolated to estimate $(g_\Lambda^2+0.8g_\Sigma^2)/4\pi$ as 14_{-3}^{+4}. The signs of the real part of K^--p forward amplitudes at high energies are found to be positive.

Recent works on estimating the coupling constant g_Λ^2 by using the $K^\pm$-p forward dispersion relation,[1] since the result by Kim was published,[2] have raised serious doubts on the validity of the usual method used to extrapolate results of K^--p multichannel analysis to the unphysical region between the K^--p and π-Λ thresholds. Depending on the model of parametrization of the K matrix, various results have been found which are inconsistent among themselves. Here we report on our analysis using the new method of extrapolation introduced by Ciulli[3] and Cutkosky and Deo,[3] which enables us to get rid of the whole unphysical region and make a reliable model-independent estimation of $(g_\Lambda^2+8g_\Sigma^2)/4\pi$, which turns out to be 14_{-3}^{+4}.

We conformally map, for fixed $t=0$, the laboratory energy plane ω onto the inside of a unifocal ellipse in the z plane as shown in Fig. 1, where A, B, and C correspond to the lowest energies at which reliable data on the real parts of the $K^{\pm}$-p forward amplitudes and K^--p total cross sections are available. We choose for A and B the values corresponding to Martin and Perrin's[4] choice of 140 and 617 MeV/c, so that the two works can be directly compared. The point C is chosen at the energy corresponding to 366 MeV/c, which covers the recent precise measurements of K^--p total cross sections by the University of Arizona group.[5] The semimajor axis of the unifocal ellipse is about 5 and the poles are at about $z=-2$. This gives good convergence in the interpolation and extrapolation.

We then construct our discrepancy function, $D(z)$, in the z plane in the following way:

$$D(z)=(z-z_\infty)f_R(z)-\frac{1}{\pi}\mathrm{P}\left[\int_{z_{K^+p}}^{z_\infty}\frac{A^+(z')(z'-z_\infty)}{z'-z}dz'+\int_{z_\infty}^{z_C}\frac{-A^-(z')(z'-z_\infty)}{z'-z}dz'\right], \tag{1}$$

where $f_R(z)$ is the real part of the forward amplitude and the A's are the absorptive parts. The factor $z-z_\infty$ is put in to eliminate the asymptotic divergence of the absorptive parts $A^{\pm}(z)=(K/4\pi)\times\sigma^{\pm}(z)$ with increasing laboratory momentum k, which goes like $1/(z-z_\infty)$. The discrepancy function $D(z)$ constructed in this way is analytic inside the whole ellipse except for the pole terms. Discrete data points for $D(z)$ can be obtained in the region $(-1,1)$, and a stable extrapolation can be made to the poles to estimate the residues.

The advantages of the method are the following:

(1) Because of the finite path of integration in Eq. (1), the difficulty of subtraction in the usual dispersion integrals is removed. The integral converges even if Pomeranchuk's theorem is violated; in fact Froissart's bound is sufficient. Furthermore, the high-energy regions for $K^{\pm}$-p scattering are greatly squeezed in the z plane, relative to the low-energy regions, so that with the present Serpukhov data[6] no more Regge extrapolation is needed for the total cross-section data at very high energies.

(2) Because the boundary is so far away, the effect of the unphysical and low-energy K^--p regions on the result is found, in practice, to be reduced to a very small amount.

(3) The poles are pulled so much closer to the K^+-p region that the coupling constant is mainly determined by the K^+-p data which happen to be more precise and reliable than the K^--p data. We found that the present K^+-p data actually provide us with more information about the poles than do the K^--p data.

(4) The K^+-p and K^--p asymptotic regions are forced to meet at z_∞, so this provides an easy test of the consistency of the real parts of the $K^{\pm}$-p forward amplitudes, $f_R^{\pm}$, at high energies by checking whether they make $D(z)$ smooth at z_∞. As we shall see, this resolves the sign ambiguity on f_R^- at high energies.

(5) This is the correct extrapolation method to use, as Ciulli and Cutkosky and Deo have repeatedly emphasized in their pioneering works on problems of extrapolating scattering amplitudes.

Having obtained the data $D(z)$ in $(-1,1)$, we have fitted it by pole terms plus a background which is expanded in polynomials. We find that we only need a straight line for the background. The pole residues are $(g_\Lambda^2+0.8g_\Sigma^2)/4\pi=14_{-3}^{+4}$. As in all previous work, the Λ pole and Σ pole are indistinguishable. We used 97 experimental data on $f_R^{\pm}$, which are from the compilation by Dum-

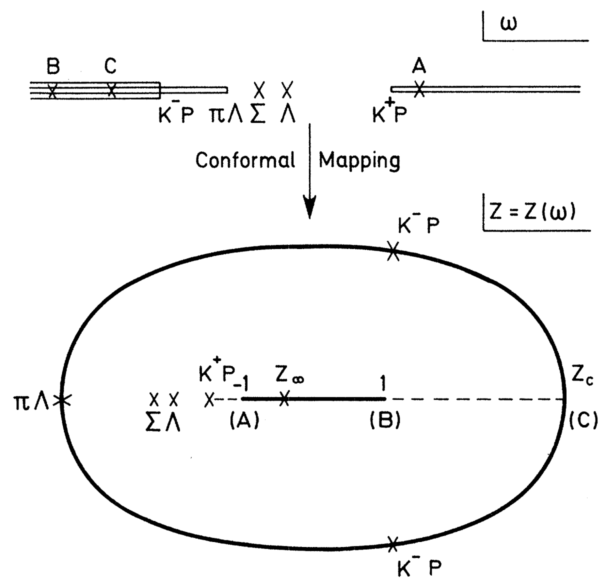

FIG. 1. Conformal mapping from the laboratory energy plane, ω, to the z plane. Points A and B are at the lowest energies where reliable real parts of $K^{\pm}$-p forward amplitudes are available, and C at the lowest energy where K^--p total cross-section data are available.

brais, Dumbrais, and Queen[7] (hereafter, DDQ) and got a χ^2 value of 75. Our fitting curve is compared with the input in Fig. 2 where, as one can see, the fitting is in general good. We tested the background effect by removing the resonance poles, $\Sigma^*(1385)$ and $\Lambda^*(1405)$, which lie outside but close to the ellipse. This was done simply by multiplying $D(z)$ by $(z-z_R)(z-z_R^*)$, where z_R and z_R^* are the complex conjugate positions of a resonance pole, Σ^* or Λ^*. The effect was that the modified discrepancy function became smoother and easier to fit. The best value of $g_{\Lambda,\Sigma}{}^2$ stayed about the same but the uncertainty became smaller. We also examined the effect of the weak logarithmic singularity at a point z_c on the boundary, which arises from the principal-value integral in Eq. (1). This effect turned out to be negligible, as expected. Our error estimate includes a truncation error in addition to the usual statistical one, which is estimated by a method based on the idea suggested by Cutkosky[3] but constructed in a different way from his.

To make a more conservative error estimate than the usual χ^2 method, Cutkosky introduced a probabilistic measure of goodness of convergence, based on the theoretical asymptotic convergence rate of the interpolating series. We regard this as a smoothness condition on the interpolating series in the physical region and interpret the probabilistic measure as an *a priori* probability density for the coefficients $C_1, C_2, \cdots, C_N$ in the interpolating function expanded in polynomials which are orthonormalized with respect to the experimental errors in the least squares sense. That is,

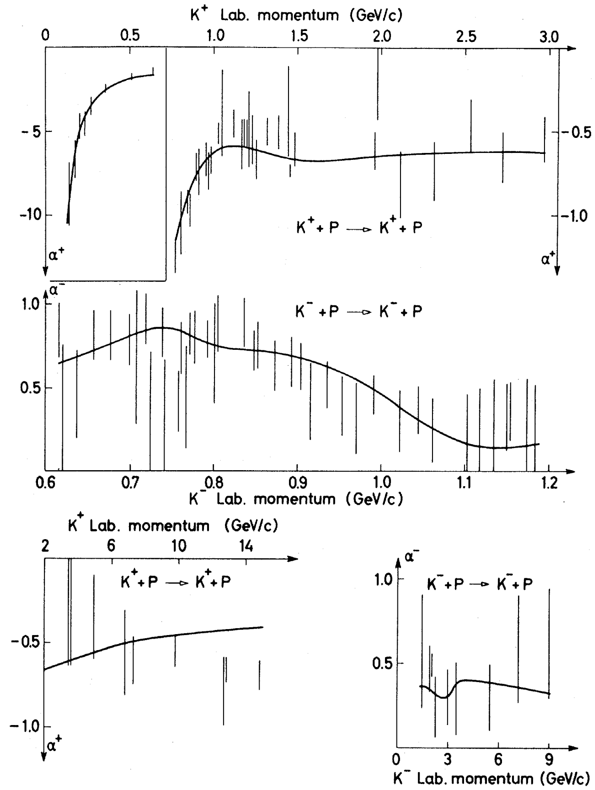

FIG. 2. Comparison of our fitted values to the experimental ones of α, the ratio of the real part of the forward scattering amplitude to the imaginary part. The experimental values are taken from DDQ's compilation (see Ref. 7).

$$P(C_1, C_2, \cdots, C_N|\lambda) \propto \prod_1^N \exp(-\lambda R_n C_n{}^2/2)(\lambda R_n)^{1/2} dC_n, \tag{2}$$

where $R_n = R^{2(n-1)}$ characterizes the theoretical asymptotic convergence rate and R is equal to the sum of the semimajor and semiminor axes. (In our case $R = 10$.) The scaling factor λ determines the effective truncation. The posterior probability after the data have been neasured is then

$$P'(C_1, C_2, \cdots, C_N|\lambda) \propto \prod_1^N \exp[-(d_n - C_n)^2/2][\exp(-\lambda R_n C_n{}^2/2)(\lambda R_n)^{1/2}]dC_n, \tag{3}$$

where the d_n's are the coefficients projected out from the observed data. The problem of estimating $C_1, C_2, \cdots, C_N$ is solved by maximizing the posterior probability P', which can be rewritten as

$$\left\{\prod_1^N \exp\left[-\tfrac{1}{2}(1+\lambda R_n)\left(C_n - \frac{d_n}{1+\lambda R_n}\right)^2\right]\right\}\left\{\prod_1^N \exp\left[-\frac{1}{2}\left(1-\frac{1}{1+\lambda R_n}\right)d_n{}^2\right]\left(1-\frac{1}{1+\lambda R_n}\right)^{1/2}\right\}. \tag{4}$$

If λ is assumed to have a uniform *a priori* distribution, its *a posteriori* distribution is given by

$$P_\lambda \propto d\lambda \left\{\prod_1^N \exp\left[-\frac{1}{2}\left(1-\frac{1}{1+\lambda R_n}\right)d_n{}^2\right]\left(1-\frac{1}{1+\lambda R_n}\right)^{1/2}\right\}. \tag{5}$$

By the maximum-likelihood principle the best estimates of C_n will be $\overline{C}_n = d_n/(1+\overline{\lambda}R_n)$, where $\overline{\lambda}$ is the appropriate scaling determined by maximizing P_λ or minimizing

$$-2\ln P_\lambda = \sum_1^N \left[d_n{}^2\left(1-\frac{1}{1+\lambda R_n}\right) - \ln\left(1-\frac{1}{1+\lambda R_n}\right)\right]. \tag{6}$$

The truncation error can be estimated by varying λ from $\bar{\lambda}-\Delta\lambda$ to $\bar{\lambda}+\Delta\lambda$, where $\Delta\lambda$ corresponds to one standard deviation or $-2\ln P_\lambda$ increased by one. In our case, d_n is a function of the coupling constants, so P_λ will be further maximized with respect to the coupling constants. This simple method worked very nicely in our case.

On closely related recent works, we have the following comments:

(1) A very similar attempt was carried out by DDQ in one of their preprints[8] which reached us after we started this work. Unfortunately, the mapping they used is not the optimized one. Their interpolating series did not have a domain of convergence big enough for a stable extrapolation to the poles, although they could fit the data in the physical region.

(2) Martin and Perrin[4] recently reanalyzed the $K^{\pm}$-p differential-cross-section data at 64 energies and found values of f_R^- considerably different in magnitude from those in DDQ's compilation and of negative sign at high energies. We started using the same input as Martin and Perrin's and found that values of $D(z)$ in the K^+-p and K^--p regions could not be joined smoothly at z_∞, with the high-energy f_R^- of negative signs. If we changed those signs, we found a coupling constant of about 9, while the corresponding 64 data in DDQ's compilation gave a coupling constant of 14.

(3) Cutkosky and Deo have extrapolated K^+-p differential-cross-section data in the cut t plane, for fixed s, to extract the coupling constant.[9] Their result is in complete agreement with ours. It would increase the sensitivity very much if the two approaches could be combined.

More precise analyses of the K^--p differential-cross-section data are needed to get better values for f_R^-. To this, of course, the same mapping method should be applied.

Extension of this work is going on and details will be published elsewhere.

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