one should be careful about the boundary conditions at infinity; but these considerations are beyond the scope of this Letter.

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Reaction $K_L{}^0 p \to K_S{}^0 p$ from 1.3 to 8.0 GeV/c*

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Total and differential cross sections are presented for the reaction $K_L{}^0 p \to K_S{}^0 p$ from 1.3 to 8.0 GeV/c as measured in an exposure of the Stanford Linear Accelerator Center 40-in. hydrogen bubble chamber to a neutral beam. The forward points of $d\sigma(K_L{}^0 p \to K_S{}^0 p)/dt$ together with K^+n and K^-n total cross sections are used to determine the intercept of the effective Regge trajectory, $\alpha(0) = 0.47 \pm 0.09$, and the regeneration phase $\varphi_f = -43° \pm 8°$.

We present experimental result on the reaction

$$K_L{}^0 p \to K_S{}^0 p \qquad (1)$$

covering the momentum interval from 1.3 to 8.0 GeV/c. Previous investigations of Reaction (1) have been reported by Firestone *et al.*[1] in a hydrogen bubble chamber experiment and by Darriulat *et al.*[2] in a transmission regeneration experiment.

In the t channel, the reaction must proceed through exchange of neutral mesons having natural spin and parity and odd charge conjugation. The only known candidates are the members of the vector nonet (ρ, ω, and φ). As pointed out by Gilman,[3] ω exchange is expected to dominate over ρ in the forward direction, and φ exchange is expected to be negligible because of the experimentally small $\varphi \bar{N} N$ coupling. The s and t behavior of the cross section and the phase of the forward amplitude are therefore powerful tools in understanding the properties of ω exchange. A more complete analysis of these exchanges is given in the following Letter.[4]

The results presented for Reaction (1) are based on an analysis of 200 000 photographs from a total exposure of 800 000 photographs of $K_L{}^0 p$ interactions in the Stanford Linear Accelerator Center (SLAC) 40-in. hydrogen bubble chamber. The details of the beam and the $K_L{}^0$ momentum spectrum are given elsewhere.[5] We have checked carefully for possible systematic uncertainties in our determination of the $K_L{}^0$ momentum spectrum and conclude that they are negligible compared to the statistical errors of our data sample. The events were found in a scan of the one-prong-plus-vee topology, measured on conventional film plane machines, and reconstructed and fitted with the TVGP-SQUAW computer programs. The sample consists of 571 events in the momentum interval 1.3 to 8.0 GeV/c of which less than 1% are kinematically ambiguous with other hypotheses.[6] Corrections have been applied for scanning inefficiencies due to the K^0 lifetime and steeply dipping protons.

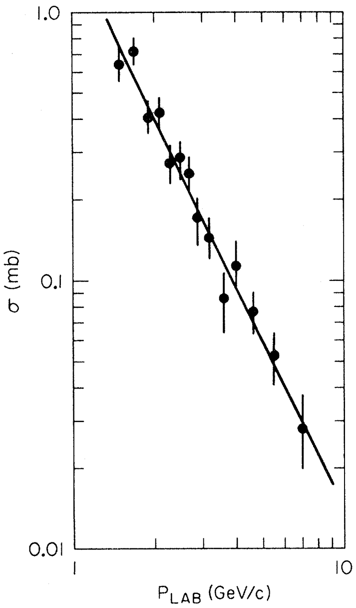

FIG. 1. Cross section for $K_L{}^0p \to K_S{}^0p$ vs p_{1ab}. The solid line corresponds to $\sigma \propto p_{1ab}{}^{-n}$, with $n = 2.1 \pm 0.2$.

The cross section for Reaction (1) as a function of $K_L{}^0$ laboratory momentum is presented in Fig. 1. As indicated by the solid line, the data are well described by the empirical law, $\sigma \sim p_{1ab}{}^{-n}$, with $n = 2.1 \pm 0.2$, a value typical of many inelastic meson-exchange reactions.

The differential cross section, averaged over three momentum intervals, is illustrated in Fig. 2(a). The main features are a sharp forward peak with an average slope of 10 ± 2 GeV^{-2}, a distinct shoulder in the interval $0.3 \leq |t| \leq 0.7$ GeV^2, and a rapid falloff for $|t|$ greater than 1.0 GeV^2. The differential cross section is observed to fall as $p_{1ab}{}^{-1}$ in the forward direction and as $p_{1ab}{}^{-2.5}$ for $|t| \sim 1$ GeV^2.

The intercept of $d\sigma/dt$ at $t = 0$ has been determined in a smooth way over the entire energy interval by a fit to the forward data points using an empirical function of the form

$$d\sigma(s)/dt \propto s^{-m} \exp\{[b + c \ln(s/s_0)]t\}, \tag{2}$$

where m, b, and c are fitted parameters and $s_0 = 1$ GeV^2. In Fig. 2(b) the differential cross section for the forward region is shown in small t intervals together with the results of the fit. The values of the parameters are $m = 1.3 \pm 0.3$, $b = 5.5 \pm 2.0$ GeV^{-2}, and $c = 2.0 \pm 1.5$ GeV^{-2}. The parameter c indicates shrinkage of the forward peak as the slope varies from 8.7 to 11.0 GeV^{-2} over the momentum range from 2.0 to 8.0 GeV/c. The values obtained for $(d\sigma/dt)_{t=0}$ as a function of p_{1ab}

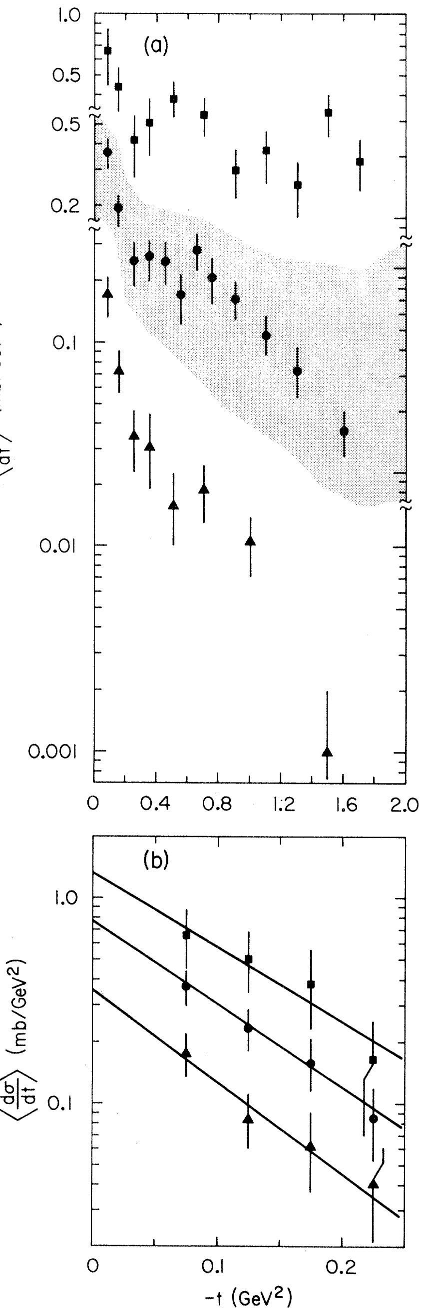

FIG. 2. Differential cross sections for $K_L{}^0p \to K_S{}^0p$. The data are averaged over three momentum intervals: $1.3 \leq p_{1ab} \leq 2.0$ GeV/c (squares); $2.0 \leq p_{1ab} \leq 4.0$ GeV/c (circles); $4.0 \leq p_{1ab} \leq 8.0$ GeV/c (triangles). (a) Full t region. Note the breaks in the ordinate scale as indicated by the shaded area. (b) Small t region. The curves result from a fit used to determine $(d\sigma/dt)_{t=0}$ (see text).

are shown in Fig. 3 by the solid curve with the experimental corridor of uncertainty represented by the two dashed curves. For comparison, the values reported recently by Ref. 2 are also shown

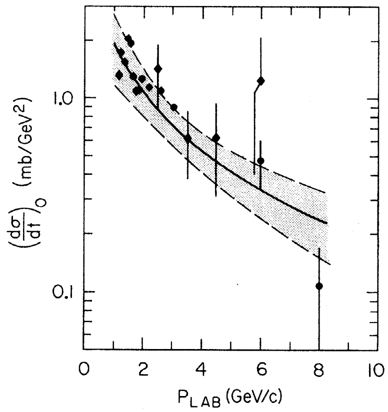

FIG. 3. Forward differential cross section for $K_L{}^0p \to K_S{}^0p$. The data are summarized by the smooth curve and the uncertainties by the dashed curves. For comparison, we show the results of Ref. 2 (diamonds) and the optical points (multiplied by a factor of 2) calculated from the total cross sections of Refs. 7-9 (circles).

and agree well with the present experiment.

The ratio of the real to imaginary part and the phase of $A(K_L{}^0p \to K_S{}^0p)$, the forward amplitude, are given by

$$\left|\frac{\mathrm{Re}A}{\mathrm{Im}A}\right| = \left\{\left[\frac{(d\sigma/dt)_{t=0}}{(d\sigma/dt)_{\mathrm{opt}}}\right] - 1\right\}^{1/2}, \qquad (3)$$

and

$$\varphi = \tan^{-1}(\mathrm{Im}A/\mathrm{Re}A).$$

The optical point is given by

$$(d\sigma/dt)_{\mathrm{opt}} = (\pi/k^2)|\mathrm{Im}A|^2, \qquad (4)$$

where k is the center-of-mass momentum and where, from isospin invariance and the optical theorem,

$$\mathrm{Im}A(K_L{}^0p \to K_S{}^0p) = (k/8\pi)[\sigma_T(K^+n) - \sigma_T(K^-n)]. \qquad (5)$$

In Fig. 3, the values for the optical point computed from total-cross-section data[7-9] and multiplied by a factor of 2 are compared with our measured $(d\sigma/dt)_{t=0°}$. The ratio of the real to imaginary part of A is near unity over the entire energy range as summarized in Table I.

The quadrant ambiguity for φ may be resolved by noting first that the experimental cross sections in Eq. (5) imply that $\mathrm{Im}A < 0$. In addition, Regge theory gives the relation

$$\mathrm{Re}A/\mathrm{Im}A = \tan[\tfrac{1}{2}\pi\alpha(0)], \qquad (6)$$

where $\alpha(0)$ is the effective trajectory intercept. For any reasonable trajectory the intercept should be between 0 and 1, which implies $\mathrm{Re}A/\mathrm{Im}A > 0$. To satisfy these two relations φ must lie in the third quadrant. The values of φ given in Table I are consistent with being constant from 1.3 to 8.0 GeV/c.[10] The weighted average over this energy range is

$$\varphi = -133° \pm 8°.$$

The values of the trajectory intercept calculated from Eq. (6) are also presented in Table I and give an average value

$$\alpha(0) = 0.47 \pm 0.09.$$

If ω exchange is assumed to dominate the forward amplitude, $\alpha(0)$ may be identified with the ω-trajectory intercept. The above value is higher than those obtained in previous phenomenological Regge fits[11] but is *consistent with a linear trajectory of unit slope passing through the physical ω mass.* It should be noted that the ρ contribution has been ignored in determing the intercept because it is expected to be less than a 20% effect.[3]

The regeneration phase for hydrogen, φ_f, is

Table I. Forward amplitude results for $K_L{}^0p \to K_S{}^0p$.

p_{lab}[a] (GeV/c)	$(d\sigma/dt)_0$ (mb/GeV^2)	$(d\sigma/dt)_{\mathrm{opt}}$ (mb/GeV^2)	$\frac{\mathrm{Re}A}{\mathrm{Im}A}$	φ (deg)	$\alpha(0)$
1.3-2.0	1.31 ± 0.40	0.73 ± 0.05[b]	0.89 ± 0.36	-132 ± 14	0.46 ± 0.16
2.0-3.3	0.85 ± 0.20	0.52 ± 0.06[b]	0.79 ± 0.31	-129 ± 13	0.43 ± 0.14
6.0	0.34 ± 0.10	0.24 ± 0.07[c]	0.64 ± 0.43	-123 ± 18	0.36 ± 0.20
8.0	0.24 ± 0.09	0.06 ± 0.03[c]	1.83 ± 1.17	-152 ± 19	0.68 ± 0.21
	Weighted averages		0.82 ± 0.20	-133 ± 8	0.47 ± 0.09

[a]Momentum values chosen to correspond to available $\sigma_T(K^\pm n)$ data.

[b]Calculated from the cross sections of Refs. 8 and 9. We have estimated the overall systematic uncertainty in $\sigma_T(K^\pm n)$ as 2%. This estimate includes a 1% contribution for nuclear screening. Corrections for Fermi motion are not important for this comparison because wide intervals of momentum have been used.

[c]Calculated from the cross sections of Ref. 7.

defined by

$$\varphi_f = \arg[iA(K_L{}^0 p \to K_S{}^0 p)_{t=0}] = \varphi + \tfrac{1}{2}\pi.$$

Thus, we find for an average value

$$\varphi_f = -43° \pm 8°,$$

which may be compared with the experimental result of Ref. 2 for hydrogen (−42° ± 17°), and also with the results for copper[12] (−45.2° ± 7.3°) and for carbon[13] (−37° ± 10°). Within the errors, the regeneration phase for hydrogen is the same as for heavy nuclei in agreement with recent optical model calculations.[14]

In summary, we find the main features of the reaction $K_L{}^0 p \to K_S{}^0 p$ in the momentum range 1.3 to 8.0 GeV/c to be the following: (1) The cross section σ falls as $(p_{\rm lab})^{-n}$, with $n = 2.1 \pm 0.2$. (2) The forward differential cross section $(d\sigma/dt)_{t=0}$ falls as $(p_{\rm lab})^{-m}$, with $m = 1.3 \pm 0.3$. (3) The ratio of the real to imaginary part of the forward amplitude is consistent with unity over the entire energy region, with an average value of 0.82 ± 0.20. (4) The average values of φ, the phase of the forward amplitude, and φ_f, the regeneration phase, are −133° ± 8° and −43° ± 8°, respectively. (5) If the reaction is assumed to be dominated in the forward direction by Reggeized ω exchange, then the average value of the trajectory intercept $\alpha_\omega(0)$ is 0.47 ± 0.09.

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Interpretation of the Reaction $K_L{}^0 p \to K_S{}^0 p$†

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Recent data on $K_L{}^0 p \to K_S{}^0 p$ are interpreted in terms of two distinctly different Regge models, both of which provide good descriptions of the data. The forward differential cross sections for $K_L{}^0 p \to K_S{}^0 p$ and $\pi^- p \to \pi^- p \to \pi^0 n$ are used to determine an f/d ratio for the nonflip coupling of vector mesons to baryons.

The recent data[1-3] on the reaction

$$K_L{}^0 p \to K_S{}^0 p \tag{1}$$

add interesting information to the class of pseudoscalar-meson–baryon inelastic scattering reactions. The behavior of Reaction (1) is in several ways similar to pion charge exchange,

$$\pi^- p \to \pi^0 n. \tag{2}$$