Chiral Algebra, Chiral Breakdown, and ξ Parameter of K_{l3} Decay*

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Using hard-meson current algebra with single meson dominance of currents and the σ commutator, it is deduced that (a) the $0^\pm$ mesons must occur in nonets, and (b) chiral symmetry is broken by only $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ terms (which then implies a small ξ parameter). Recent calculations which obtain a large negative ξ with only $(\underline{3},\underline{3}^*)$ terms are seen to imply a violation of the current-algebra constraints. However, the large experimental ξ can be obtained by including additional nonpole $(\underline{1},\underline{8})\oplus(\underline{8},\underline{1})$ breaking terms.

The nature of the breakdown of chiral symmetry has been the subject of much investigation for for a number of years. Perhaps the simplest and most elegant hypothesis[1] is the postulate that the symmetry-breaking interaction belongs to a $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ representation of chiral SU(3) $\otimes$SU(3). In the first part of this note it will be shown that this assumption can actually be deduced as a consequence of the usual postulates of hard-meson current algebra when single meson dominance is assumed for both the currents and the σ commutators.[2] The analysis is nonperturbative and assumes an arbitrary size for the SU(3) and chiral breakdown terms. The restriction to the $(\underline{3},\underline{3}^*)$ form arises from the conflict of requiring the interaction to satisfy simultaneously the current commutation relations (which are chirally symmetric) and the partial conservation of axial-vector current (PCAC) conditions (which insist on chiral breakdown). Within a fixed dynamics (pole dominance) these two are consistent with only one form of symmetry breaking. Thus one has the remarkable result of the form of a symmetry breaking being deduced from dynamical considerations.

Experimentally, the $(\underline{3},\underline{3}^*)$ chiral breakdown combined with pole dominance appears to be quite successful in the nonstrange channels, but not so successful in the strange ones. Thus the Weinberg prediction[3] that the $\pi\pi$ scattering lengths obey $a^0/a^2 \cong -3.5$ is in good agreement with the experimental result[4] $a^0/a^2 = -(3.2\pm 0.1)$. However, the K_{l3} decay ξ parameter predicted is close to zero,[5,6] while present data now appear to be favoring[7] $\xi \lesssim -0.5$. It is thus becoming a pressing question whether or not the $(\underline{3},\underline{3}^*)$ breakdown can account for the present K_{l3} data. We investigate this problem in the second part of this note, where it is shown that a simple additional nonpole term transforming as a (1, 8) $\oplus$ (8,1) representation can account for a large negative ξ without disturbing the previous successes of current algebra (e.g., pion scattering lengths). Thus current algebra is quite capable of yielding a large negative ξ.

(I) We begin by carefully stating the hard-meson current-algebra assumption. These are (1) the chiral commutation relations

$$\delta(x^0-y^0)[J_a^0(x), J_b^\mu(y)] = ic_{abc}J_c^\mu(x)\delta^4(x-y) + c\text{-number Schwinger terms}, \tag{1}$$

where a, b, and c run over the vector and axial-vector indices for the SU(3)$\otimes$SU(3) algebra. One may, of course, always use the currents as interpolating fields for the octets or nonets of $J^P = 1^\pm$ mesons $v_a^\mu(x)$ of mass m_a and $0^\pm$ mesons $s_a(x)$ of mass μ_a. Thus one may write

$$J_a^\mu(x) = g_{ab}v_b^\mu(x) + F_{ab}\partial^\mu s_b(x). \tag{2}$$

(For the $I=1$, axial channel, $g_{ab}=g_A\delta_{ab}$, $F_{ab} = F_\pi\delta_{ab}$, etc.) (2) The PCAC postulate

$$\partial_\mu J_a^\mu(x) = F_{ab}\mu_b^2 s_b(x). \tag{3}$$

Equation (3) thus includes the partial conservation of vector current relation for the κ meson channel. In addition we assume (3) saturation of intermediate sums by the single-meson $1^\pm$ and $0^\pm$ states, and (4) smoothness, i.e., that meson vertices can be approximated by a linear function of the momentum transfers.[8] Conditions (3) and (4) imply that the dynamics can be characterized by an effective Lagrangian to be used only in the tree and seagull approximation with $v_a^i(x)$ and $s_a(x)$ as canonical variables.[9] We stress that the use of the Lagrangian has no fundamental significance and is merely a device for realizing the conventional hard-meson dynamical assumptions (3) and (4). Thus all the results in this note would follow equally well (though perhaps not as simply) using any of the other hard-meson formalisms (e.g., Ward's identities, etc.).

We now outline the derivation of the following result.[10] Theorem 1: If in addition to assumptions (1)-(4) above, the σ commutators are single-particle dominated, then the $0^{\pm}$ mesons must occur in nonets (not octets) with experimentally correct g-parity assignment. Theorem 2: The conditions of Theorem 1 also imply that chiral symmetry is broken only by a term which transforms as a $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ representation.

We start by noting that the variables $s_{0a}(x)$ canonically conjugate to $s_a(x)$ are obtained from $s_{\mu a} = \partial_\mu s_a - \delta \mathfrak{L}_I / \delta(\partial^\mu s_a)$. One may thus eliminate $\partial^0 s_a$ in Eq. (2) and express $J_a^{\,0}$ in a power series in the canonical variables s_{0a}, s_a whose linear term is just $F_{ab} s_b^{\,0}$. On the other hand, the σ-commutator assumption implies that $[J_a^{\,0}, s_b]$ must be linear in the spin-zero fields. Thus $J_a^{\,0}$ can be at most quadratic in the spin-zero fields and one may write

$$J_a^{\,\mu} = F_{ab} s_b^{\,\mu} + Z_{abc} s_b^{\,\mu} s_c, \tag{4}$$

where terms containing the (irrelevant) dependence on the vector-meson fields have been omitted. The canonical commutation relations then imply that the σ commutators have the form

$$\delta(x^0 - y^0)[J_a^{\,0}(x), s_b(y)] = i[Z_{abc} s_c + F_{ab}]\delta^4(x-y). \tag{5}$$

The parts of Eqs. (1) and (4) which depend only on the spin-zero mesons yield

$$Z_{adc} F_{bc} - F_{ac} Z_{bdc} + c_{abc} F_{cd} = 0, \tag{6a}$$

$$[Z_a, Z_b] = -c_{abc} Z_c, \tag{6b}$$

where Z_a is a matrix with elements Z_{abc}, and appropriate matrix multiplication is implied in Eq. (6b). The only solution of Eqs. (6) are

$$Z_a = S^{-1} A_a S \tag{7a}$$

with S, A_a constrained as follows: (1) S is diagonal with elements constant in each isotopic multiplet except for possible 8-9 mixing of the isotopic singlet states; (2) S obeys [by Eq. (6a)]

$$F_{ac} S_{bc} = A_{abc} b_c$$

$$b_c = 0 \text{ for } c \neq 8, 9; \quad b_8 = \cos\alpha, \quad b_9 = \sin\alpha, \tag{7b}$$

with 8, 9 referring to the isosinglet natural-parity states, and α an arbitrary "mixing angle."

(3) Either (i) $A_{abc} = f_{abc}$ if a are natural-parity components, $A_{abc} = d_{abc}$ if a, b are unnatural parity, and $A_{abc} = -d_{abc}$ if a, c are unnatural parity or (ii) $A_{abc} = c_{abc}$. However, possibility (ii) [which would lead to a $(\underline{1}, \underline{8}) \oplus (\underline{8}, \underline{1})$ chiral breakdown] may be eliminated as it is inconsistent with PCAC in that it would lead to conserved axial currents. Thus by Eq. (7b), (ii) would imply $F_\pi = 0$, for example.[11] The remaining case (i) is actually a solution of Eq. (6b) only if b and c of Z_{abc} run over nonets of natural and unnatural parity states and not just octets. Thus, from Eq. (4) one sees that the $0^{\pm}$ mesons must exist in nonets. Equations (7) thus imply the existence of an 18-plet of scalar and pseudoscalar densities $\omega_a \equiv S_{ab} s_b + b_a$, which form a $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ representation:

$$\delta(x^0 - y^0)[J_a^{\,0}(x), \omega_b(y)] = i A_{abc} \omega_c(x) \delta^4(x-y). \tag{8}$$

We next invoke the PCAC condition, Eq. (3). Inserting Eq. (4) and using the Lagrange equations $\partial_\mu s_a^{\,\mu} = \mu_a^{\,2} s_a - \delta \mathfrak{L}_I / \delta s_a$, where $\mathfrak{L}_I = g_{abc} s_a s_b s_c + \cdots$, one finds from the spin-zero parts

$$6 F_{ad} g_{dbc} = Z_{labc} \mu_b^{\,2} + Z_{lacb} \mu_c^{\,2}. \tag{9}$$

This relation determines various components of g_{abc} but in addition the total symmetry of g_{abc} imposes one further constraint on Z_{abc} which by Eqs. (7) determines the mixing angle α:

$$\tan(\alpha + \alpha_0) = \sqrt{2}\, c_\pi [c_\kappa - c_K \pm \Delta^{1/2}]^{-1},$$

$$\Delta \equiv c_\pi^{\,2} c_K^{\,2} + c_\kappa^{\,2} - 2c_\pi c_K - 2c_\pi c_\kappa - 2c_K c_\kappa, \tag{10}$$

where $\tan\alpha_0 = 1/\sqrt{2}$ and $c_\pi \equiv F_\pi^{\,2} \mu_\pi^{\,2}$, etc.

We can now see that the above analysis implies a $(\underline{3}, \underline{3}^*)$ symmetry breaking. For in the single-meson–dominance case, this hypothesis also requires the existence of an 18-plet of densities $\tilde{\omega}_a$ linear in the s_a, i.e., $\tilde{\omega}_a = (\sqrt{Z})_{ab} s_b + \tilde{b}_a$ obeying the $(\underline{3}, \underline{3}^*)$ condition Eq. (8). (Here Z is the wave-function renormalization matrix of Glashow and Weinberg.[1]) The $\tilde{\omega}_a$ govern the symmetry-breaking interaction $\mathfrak{L}'$ according to $\mathfrak{L}' = \epsilon_a \tilde{\omega}_a$, $\epsilon_{8,9} \neq 0$. Comparing the $(\underline{3}, \underline{3}^*)$ conditions for $\tilde{\omega}_a$ with Eq. (5), we see that they imply that $Z_a = (\sqrt{Z})^{-1} A_a \sqrt{Z}$ and $F_{ac} S_{bc} = A_{abc} \tilde{b}_c$ and hence on comparison with Eqs. (7), $S_{ab} = (\sqrt{Z})_{ab}$ and $\tilde{b}_a = b$. Further, Noether's theorem, when compared with Eq. (3), yields $F_{ab} \mu_b^{\,2} = A_{abc} \epsilon_a S_{cb}$. Using the relation $\tilde{b}_a = b_a$ and Eq. (7b), this result allows one to express $\epsilon_{8,9}$ in terms of α and in addition imposes one further condition that determines α to have precisely the value given in Eq. (10). Thus the current-algebra results Eqs. (7a), (7b), and (10) are isomorphic to the results obtained from the ($\underline{3}$, $\underline{3}$*) symmetry breaking ($\tilde{\omega}_a = \omega_a$, etc.) completing the proof of the theorems.

(II) The above results may be extended by relaxing the single-meson–dominance assumption and assuming that ω_a and $\partial_\mu J_a{}^\mu$ are arbitrary functions of s_a. If in addition, however, one assumes a ($\underline{3}$, $\underline{3}$*) symmetry breaking, one may make a point transformation re-establishing the pole dominance conditions[12] (and hence the theorems of the previous section) and thus again giving rise to a small ξ parameter.[13] One is therefore led to consider the possibility that the pole part of $\mathcal{L}'$ is ($\underline{3}$, $\underline{3}$*), but in addition there is some nonpole ($\underline{1}$, $\underline{8}$) pieces. One can indeed easily construct quadratic ($\underline{1}$, $\underline{8}$) pieces χ_a from the ($\underline{3}$, $\underline{3}$*) ω_a according to

$$\chi_a = \lambda_{abc}\omega_b\omega_c, \tag{11}$$

where $\lambda_{abc} = -d_{abc}$ for a natural parity, $\lambda_{abc} = f_{abc}$ for a, b unnatural parity, and $\lambda_{abc} = -f_{abc}$ for a, c unnatural parity. The simplest model including both ($\underline{3}$, $\underline{3}$*) and ($\underline{1}$, $\underline{8}$) symmetry breaking would then be characterized by $\mathcal{L}' = \epsilon_8\omega_8 + \epsilon_9\omega_9 + \gamma_8\chi_8$. The presence of ($\underline{1}$, $\underline{8}$) pieces in $\mathcal{L}'$ can be thought of as natural from another viewpoint. For, it has been argued[1] that chiral SU(3) ⊗ SU(3) symmetry breaks by first breaking SU(3) while preserving SU(2) ⊗ SU(2), followed by a smaller breakdown of SU(2) ⊗ SU(2). It is easy to see, however, that the first level of symmetry breakdown is achieved by the choice $\epsilon_8 = \sqrt{2}\,\epsilon_9$ and arbitrary γ_8. Indeed, the χ_8 term does not contribute to any of the SU(2) ⊗ SU(2) analyses, and all of the successes in this subspace (e.g., $\pi\pi$ and πN scattering lengths, etc.) are preserved for arbitrary γ_8. Thus there is no a priori reason to set γ_8 to zero, within this framework of symmetry breaking.

The χ_8 term does, of course, effect the strange channels, e.g., the K_{l3} decay. The previous pole-dominance analysis[6] can easily be extended to include the quadratic χ_8 contribution to PCAC. The $f_+(t)$ form factor is unchanged, and we quote here the modified result for ξ:

$$\xi \cong -(m_K{}^2 - m_\pi{}^2)/m_{K^*}{}^2 + G/f_+(0), \tag{12a}$$

$$2F_\pi F_K m_\kappa{}^2 G \equiv c_\kappa - c_K + F_K{}^2(m_\kappa{}^2 - m_\pi{}^2) + F_\kappa{}^2(m_\pi{}^2 - m_K{}^2) + F_\pi{}^2(m_K{}^2 - m_\kappa{}^2) - \sqrt{2}\,c_\pi/x, \tag{12b}$$

where $x \equiv \tan\psi$ is determined by

$$2x(x+\sqrt{2})c_K + 2x(x-\sqrt{2})c_\kappa = (x^2-2)\{c_\pi + \sqrt{3}\gamma_8 x^2(1-\sin^2\psi)\}. \tag{12c}$$

The first term of Eq. (12a) contributes about −0.28 to ξ. For $\gamma_8 = 0$, Eq. (12c) reduces to Eq. (10) with $\psi = \alpha + \alpha_0$. The second term then contributes about +0.18 or +0.29 [depending upon the solution chosen in Eq. (10)]. Thus $\xi \simeq -0.10$ or +0.01 for $\gamma_8 = 0$. This is the usual ($\underline{3}$, $\underline{3}$*) result which is in bad agreement with the present data. However, with $\gamma_8 \neq 0$, the additional ($\underline{1}$, $\underline{8}$) contribution allows one to have a large negative ξ. Thus if we write $x = c_\pi/(\sqrt{2}pc_K)$, then Eqs. (12a) and (12b) yield[14] $\xi \cong -0.60$ for the choice $p = 2$, which is to be compared with the present combined $K_{l3}{}^+$ data result $\xi = -0.65 \pm 0.20$ of the Chounet analysis.[13] [From $f_+(t)$ one has[6] $\lambda_+ \cong 0.026$ consistent with the present world averages of[13] $\lambda_+ = 0.034 \pm 0.006$ from $K_{l3}{}^+$ data, $\lambda_+ = 0.017 \pm 0.008$ from $K_{e3}{}^0$ data.] Equation (12c) for x small (i.e., $p \neq 0$) now determines γ_8 to be $\gamma_8 \cong -2p(c_K/c_\pi) \times [(1+p)c_K - c_\kappa] - (c_K + c_\kappa)$. One may also determine ϵ_8 and ϵ_9 for this model. One finds $\epsilon_8/\epsilon_9 = -\sqrt{2} + O(c_\pi/c_K p)$ and $\epsilon_8/\gamma_8 = -\sqrt{2} + O(c_\pi/c_K p)$. The above results are all insensitive to the precise value of F_κ. We see that due to the smallness of $c_\pi/c_K \approx m_\pi{}^2/m_K{}^2$, the theory arranges ϵ_8/ϵ_9 so that the ($\underline{3}$, $\underline{3}$*) part of the symmetry breaking almost preserves the SU(2) ⊗ SU(2) subgroup [as originally suggested in models without any ($\underline{1}$, $\underline{8}$) breaking[1]] as long as p is not very small, i.e., as long as ξ is not close to zero. [Similarly, while ϵ_8/γ_8 is not theoretically determined and depends upon the value of ξ (through p), one has $\epsilon_8/\gamma_8 \cong -\sqrt{2}$ independent of the precise value of ξ as long as ξ is not close to zero.] Thus all the nice features of models with only ($\underline{3}$, $\underline{3}$*) breaking are maintained.

(III) In the previous section we have shown that hard-meson current algebra can account for a large negative ξ parameter provided one includes some ($\underline{1}$, $\underline{8}$) symmetry breaking, and that such terms do not effect the successes of ($\underline{3}$, $\underline{3}$*) breaking in the nonstrange channels. We conclude with some comments concerning related work on this question. Dashen and Weinstein[15] consider an arbitrary symmetry breaking and yet obtain a small value for ξ. However, they also assume that the symmetry breaking can be treated as a small perturbation. The above analysis shows that $\epsilon_{8,9}$ and γ_8 are not small and so large corrections are neglected in their work. Several authors[8,16] have considered the ($\underline{3}$, $\underline{3}$*) breaking case, but have concluded ξ can indeed become large and negative provided $\epsilon_8 \ll \epsilon_9$. These authors make the a priori reasonable sounding assumption that the off-shell pion matrix element $\langle \kappa | J_\pi | K\rangle$ is essentially constant in the range $0 \lesssim q_\pi^2 \lesssim \mu_\pi^2$, the so-called weak PCAC or pole dominance of the divergence of the axial-vector current condition. However, by direct calculation one may easily show[6] that this matrix element has nongentle pieces precisely when $\epsilon_8 \ll \epsilon_9$, invalidating the argument. (Actually, when ϵ_8 is comparable with ϵ_9, the matrix element is gentle, and then, of course $\xi \approx 0$.) This nongentleness is a particular effect of the strange channel, and arises as a consequence of the current-algebra conditions. On the other hand, current algebra requires that all nonstrange matrix elements are gentle.[9,10] Thus the theory produces no violations of gentleness in the nonstrange channels where the concept has been experimentally verified, and the nongentle behavior in the strange channels arises naturally (not by ad hoc assumption). However, this phenomenon points up the dangers of making an apparently reasonable additional noncurrent-algebra assumption, as the extra condition may actually be inconsistent with the current-algebra postulates already made.

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[8]Some authors [e.g., M. K. Gaillard, CERN Report No. CERN 70-14 (unpublished); R. A. Brandt and G. Preparata, Lett. Nuovo Cimento 4, 80 (1970); R. A. Brandt, M. H. Goldhaber, G. Preparata, and C. A. Orzalesi, Phys. Rev. Lett 24, 1517 (1970)] have suggested that some meson vertices be *constant* in the momentum transfers. Such assumptions appear to us to be *a priori* dangerously restrictive (particularly for processes involving an energy range of ≈500 MeV or more), and in fact turn out to be *generally in conflict with the current algebra constraints*.

[9]R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Lett. 19, 1085 (1967); R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. 175, 1802 (1968).

[10]A detailed derivation is given in R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. D (to be published).

[11]Possibility (ii) also violates the experimentally known G-parity assignments of the spin-zero mesons. Thus the current algebra also yields the correct spin-zero G parities.

[12]A detailed description of the work in this section will be given in a longer paper submitted elsewhere.

[13]If ω_a and $\partial_\mu J_a^\mu$ are made functions of s_a *and* $\partial_\mu s_a$, the point transformation argument no longer holds. However here consistency with the current algebra implies that the symmetry breaking is quite complex. It cannot be pure ($\underline{3}$, $\underline{3}$*) and must include some ($\underline{1}$, $\underline{8}$) ⊕ ($\underline{8}$, $\underline{1}$) and perhaps other representations.

[14]One finds $\lambda_- = 0.024$.

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