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DEPENDENCE OF "ANOMALOUS" CONDUCTIVITY OF PLASMA ON THE TURBULENT SPECTRUM

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The scaling laws for "anomalous" conductivity seen in a turbulent plasma are shown to depend on the type of electrostatic fluctuation spectrum present, which in turn depends on the relative drift velocity between ions and electrons.

In the last few years there have been several reports of experimentally measured values of plasma conductivity which are much smaller than the so-called "classical" value based on binary Coulomb collisions. Most such reports relate to plasma in which there is a suprathermal level of electrostatic fluctuation, arising from some instability excited either deliberately (as in turbulent heating experiments[1-3] or collisionless shocks[4]) or inadvertently (as in toroidal containment systems[5] or theta-pinches[6]). Since a wide range of experimental parameters have been involved, and the theory of turbulent plasma is still in an exploratory stage, some confusion has arisen regarding the scaling laws which govern the conductivity and their relation to known plasma instabilities, etc. In particular, for the class of experiment in which the turbulence arises as the result of applying a strong electric field to a weakly collisional plasma, three separate (though related) mechanisms have been proposed to explain the observed effects based on the excitation of different forms of electrostatic instability. It is the purpose of this paper to show that certainly two, and probably all three, of these situations can occur for different plasma conditions even in the same apparatus, the scaling laws (i.e., variation of conductivity with density, applied field, ion mass, etc.) being different for each.

Very briefly, the three exciting mechanisms referred to above are these:

(1) The excitation of ion-sound waves by induced Cherenkov emission from the drifting electrons when the drift velocity $v_d \geq c_s = (T_e/M)^{1/2}$, the sound speed.[7-9] (T_e is the electron temperature in energy units, M the ion mass.) The frequency spectrum which develops as a result of nonlinear effects has been discussed for this case by Kadomtsev[8] and Tsytovich,[9,10] and, as would be expected, lies in the frequency range $<\omega_{pi}$, the ion plasma frequency.

(2) The development of a hydrodynamic instability as predicted by Budker[11] and Buneman[12] resulting from electron-ion counterstreaming. In a warm plasma this is expected when $v_d \gtrsim v_e = (2T_e/m)^{1/2}$, the electron thermal speed, and is characterized by fastest initial growth at frequencies around $\omega^* = \frac{1}{2}(m/M)^{1/3}\omega_{pe} = \frac{1}{2}(M/m)^{1/6} \times \omega_{pi}$.

(3) Various forms of beam-plasma instability resulting from the formation of a secondary "runaway" beam of electrons which interacts with the background electrons. In the laboratory frame these could have frequencies $\omega \approx \omega_{pe}$ if $\omega_{pe} \gg \omega_{ce}$ (the gyrofrequency) or $0 < \omega < \omega_{pe}$ if $\omega_{pe} < \omega_{ce}$.[13]

We have studied the spectrum of short-wavelength potential fluctuations in a toroidal apparatus, already well described,[14] using calibrated high-impedance floating double probes (spacing $\leq$1 mm) with frequency response up to 2 GHz. For comparison, the ion plasma frequency in these experiments was 100-700 MHz, and the Debye distance of order 10^{-3}-10^{-2} cm. The technique used was to record the potential difference V between the probe electrodes directly on an oscilloscope (Tektronix model 519), and to obtain the power spectra $\langle V^2(\omega)\rangle$ (Fig. 1) by computing numerically the Fourier transform of

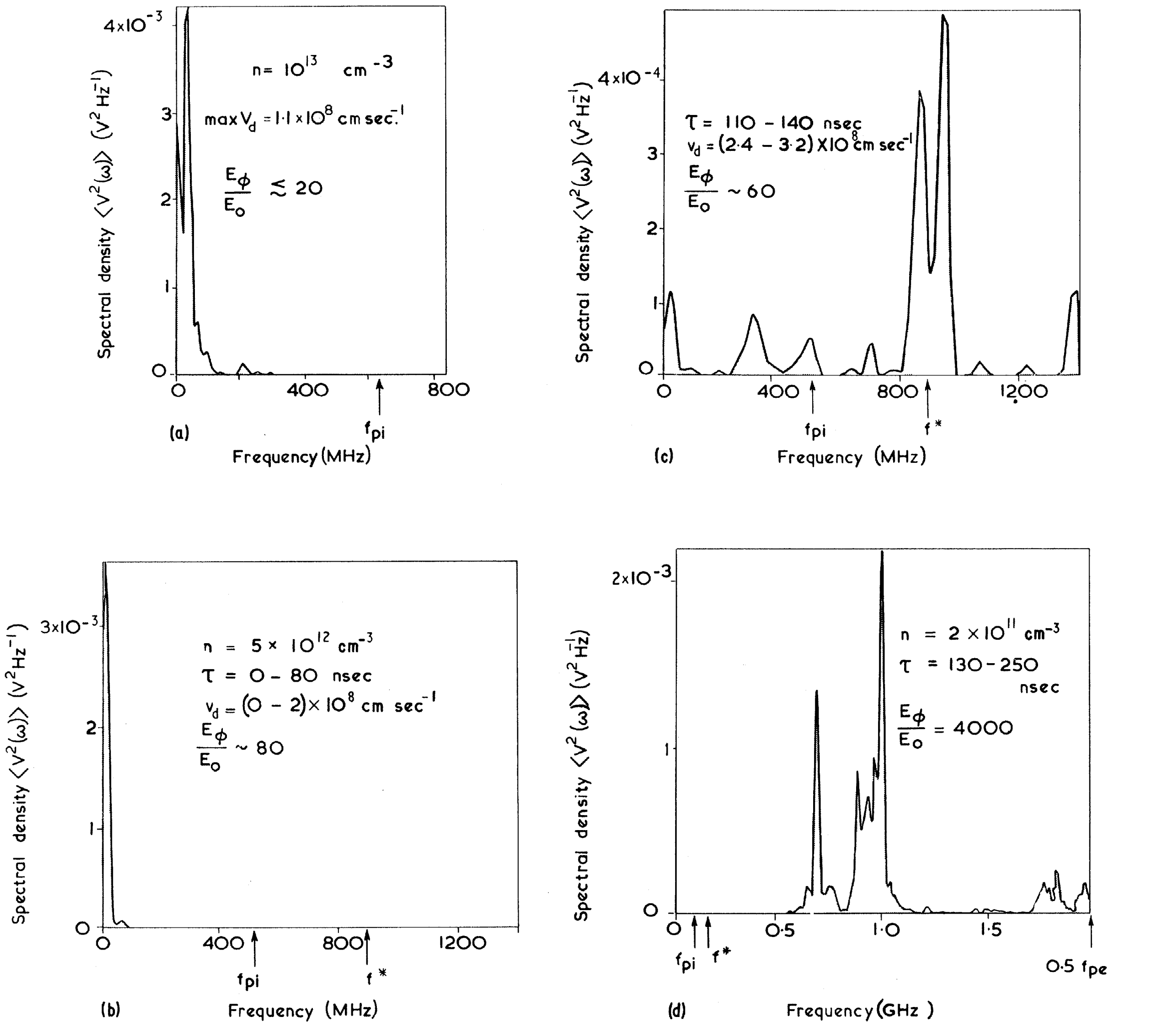

FIG. 1. Typical spectra seen in various regimes: (a) throughout regime A, (b) early in regime B ($v_d \leq 2\times10^8$ cm sec^{-1}), (c) same pulse as (b), but later in time ($v_d \geq 3\times10^8$ cm sec^{-1}), and (d) late in regime C.

the autocorrelation function of the signal, correcting for the known frequency sensitivity of the overall system. The time development of the spectrum could be obtained by analyzing various time segments τ within one oscillogram, assuming the spectrum to be stationary for each.

By varying the initial hydrogen plasma density n and the applied longitudinal electric field E_φ over a wide range (10^{11}-10^{13} cm^{-3} and 100-500 V cm^{-1}, respectively), we could change the critical-field parameter E_φ/E_0 [where $E_0 \approx 2\times10^{-12}n/T_e$ V cm^{-1} is the critical field for runaway[15]] in the range 10-10^4, and the measured maximum drift velocity during the current pulse (lasting between 300 and 500 nsec) between 10^8 and 2×10^9 cm sec^{-1}, compared with initial values $c_s \approx 10^7$, $v_e \approx 2\times10^8$ cm sec^{-1}.

We have found that we can identify three main regimes according to the type of spectrum seen and its temporal behavior during the pulse.

Regime A [Fig. 1(a)]. –Only frequencies $\omega < \omega_{pi}$ are seen throughout the pulse, and the fluctuations appear to be roughly localized in directions parallel to the electron current. The general shape of the spectrum appears to be established in 10-20 plasma periods. This occurs when $E_\varphi/E_0 \leq 40$ and $v_d \leq (2$-$3)\times10^8$ cm sec^{-1}. A suprathermal level of unpolarized microwave emission is observed at frequencies of order ω_{pe}.

Regime B. –When $E_\varphi/E_0 \gtrsim 40$, a spectrum similar to that above is seen [Fig. 1(b)] until the current increases sufficiently for $v_d \leq 2\times10^8$ cm sec^{-1}, at which point a short (20-40 nsec), very intense burst of signal with $\omega \sim \omega^*$ appears; this higher-frequency spectrum persists at

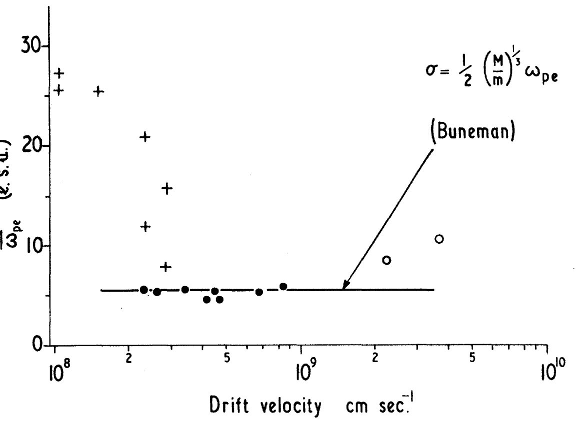

FIG. 2. Variation of normalized conductivity σ/ω_{pe} with drift velocity v_d: crosses, regime A; solid circles, regime B; open circles, regime C.

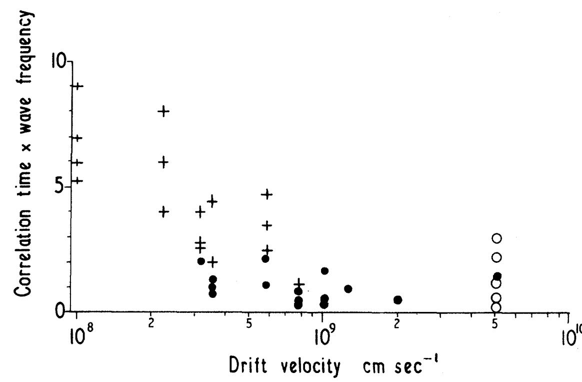

FIG. 3. Variation of wave correlations with drift velocity $v_d\,\tau_{\text{corr}}$: crosses, regime A; solid circles, regime B: open circles, regime C.

smaller amplitude until the current decreases [Fig. 1(c)]. The microwave emission is typically one order of magnitude more intense in this regime than in regime A. The observed signal does not depend on probe orientation with respect to the current flow.

Regime C [Fig. 1(d)]. —At small n and large E_φ ($\gtrsim 10^3 E_0$) the spectrum is at first similar to that in regime B, changing later in the pulse to one containing much higher frequencies $\lesssim \omega_{pe}$.

If we now plot (Fig. 2) the normalized conductivity σ/ω_{pe} against the measured drift velocity v_d, using different symbols for each measured point according to the class of spectrum observed at the instant of measurement (i.e., at peak current), we can see clearly that the dependence is quite different for each regime. [Notice that to compare results from different experiments using the parameter E_φ/E_0 can be somewhat misleading; in this experiment the current (and hence v_d) depends on the circuit inductance for the larger values of σ.]

The solid line shown in Fig. 2 is the semiempirical constant value $\sigma = \frac{1}{2}(M/m)^{1/3}\omega_{pe}$ found in the earlier work of Buneman[12] and ourselves[2] for large E_φ. The experimental values of σ for regime A vary with v_d as expected for anomalous conductivity during ion-sound turbulence, viz., $\sigma \propto (c_s/v_d)\omega_{pe}$.[10,16] [Notice that c_s (which was not measured) was not constant for each datum point.] It is interesting that Fig. 2 shows a critical range of drift velocities [$v_d \approx (2\text{-}3)\times 10^8$ cm sec^{-1}] within which the instability apparently develops into either of the two main types of turbulent spectrum. From the computed autocorrelation function for various time segments we can make a rough estimate of the correlation time of the fluctuations. Figure 3 shows a plot of the number of correlated wave periods (i.e., of the strongest frequency component present) observed compared with the drift velocity for the three regimes, and demonstrates a similar trend to that of the normalized conductivity.

By making an ensemble average of several shots in regime A under identical conditions we can derive from $\langle V^2(\omega)\rangle$ the corresponding spectrum of plasma potential fluctuations $I(\omega)$ (to do this we assign the various maxima to spatial probe resonances). Over more than three decades of intensity the spectrum derived agrees in shape with that predicted theoretically for current-driven ion-sound turbulence, which has the general form[8] $I(\omega)\propto\omega^{-1}\ln(\omega_{pi}/\omega)$. Thus we feel safe in identifying regime A with the presence of ion-sound turbulence.

Regime B, because of the observed frequency spectra, the higher drift velocities required, the measured independence of σ on v_d, and the scaling with $M^{1/2}$ (both of σ^2 and ω^*[17]), we must associate with the Buneman hydrodynamic instability, although there is as yet no theoretical treatment of its nonlinear development.

Finally, regime C, which, we should emphasize, is seen only intermittently and under rather poorly defined initial plasma conditions, we tentatively ascribe to the third, beam-plasma instability, which may arise as a result of axial inhomogeneities of plasma density and thus locally stronger accelerating electric fields which produce a double-humped electron distribution. Such a situation has been observed in linear discharges, e.g., by Karchevskii, Averiv, and Bezmel'nitsyn,[18] who suggested that a similar effect could possibly occur in a toroidal system if the plasma were nonuniform.

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NEW RELAXATION EFFECT IN THE SOUND ATTENUATION IN LIQUID ^{4}He UNDER PRESSURE*

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Observation of the first-sound attenuation in liquid ^{4}He under pressure has revealed a new relaxation mechanism which is not predicted by any present theory.

In previous work we have shown that certain aspects of the theory of sound attenuation in liquid ^{4}He are not in agreement with experiment at very low temperature.[1] In the collisionless regime ($\omega\tau_{pp} \gg 1$, where τ_{pp} is the wide-angle phonon-phonon relaxation time) the calculated attenuation due to a three-phonon process is given by[2]

$$\alpha(\omega, T) = \frac{\pi^2}{60}\frac{(u+1)^2}{\rho}\frac{k_B{}^4}{\hbar^3 c^6}\omega T^4 \times[\tan^{-1}(2\omega\tau_{pp}) - \tan^{-1}(3\gamma\bar{p}^2\omega\tau_{pp})], \quad (1)$$

where ρ and c are the density and sound velocity, respectively, and $u \equiv (\rho/c)\partial c/\partial\rho$. The dispersion constant, γ, usually thought to be positive, is defined through the relation $\epsilon = cp(1-\gamma p^2)$, where ϵ and p are the energy and momentum, respectively, of an elementary excitation and $\bar{p} \equiv 3k_BT/c$ is the average thermal phonon momentum. The measured attenuation was always found to be greater than that predicted by Eq. (1). A recent precise determination of u has eliminated the possibility that an uncertainty in that parameter might account for the discrepancy.[3] The change of the sound velocity with temperature also disagreed with theory although in this case the measured change was smaller than the predicted value.[2] Moreover, this change was observed to be smaller at higher frequencies, i.e., opposite to that predicted by theory.[4] The calculation of