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DOMINANT COLLIDING-BEAM CROSS SECTIONS AT HIGH ENERGIES*

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We report on our calculation of the energy and angular dependence of the cross sections for the production of various particles by two-photon annihilation processes in e^+e^- and e^-e^- colliding beams. For beam energy E of more than 1 GeV, these cross sections $[\sigma \propto \alpha^4(\ln E)^3]$ become increasingly more important than the usual one-photon cross sections $[\sigma \propto \alpha^2(E)^{-2}]$ for hadron production.

Although the study of hadron production by electron-positron collision is of intense experimental and theoretical interest,[1] it is not generally appreciated that the most frequent events at energies $E_{\text{c.m.}} = (s_{\text{c.m.}}/4)^{1/2}$ above 1 GeV occur via a two-photon annihilation process.[2,3] In fact two-photon cross sections for processes of the type

$$e + e \to e + e + N,$$

where $N = \pi^+\pi^-$, $\pi^+\pi^-\pi^0$, π^0, η^0, etc., are rather large and increase logarithmically at high energies. For example

$$\sigma_{ee \to ee\pi^+\pi^-} \sim \frac{8\alpha^4}{3\pi} \frac{1}{m_\pi^2} \left(\ln \frac{E}{m_e}\right)^2 \left(\ln \frac{E}{m_\pi}\right). \quad (1)$$

In contrast, the usual one-photon cross sections for hadron production inevitably decrease with energy as E^{-2} or faster. We wish to emphasize that, whereas the study of one-photon processes require the e^+e^- colliding beam, the two-photon processes can be investigated by both e^+e^- and e^-e^- colliding beams.

In our calculation we have adopted the Weizsäcker-Williams equivalent-photon approach since it yields transparent and simple results in terms of cross sections for photon-photon annihilation—a process of immense theoretical interest itself.

If the scattered leptons are not observed, then to leading order in $\ln(E/m_e)$ each lepton (e^+ or e^-) is equivalent to a beam of real transversely polarized photons (the energy k) with a spectrum[4]

$$\frac{\alpha}{\pi} \frac{E^2 + (E-k)^2}{E^2} \ln\left(\frac{E}{m_e}\right) \frac{dk}{k}. \quad (2)$$

This result is derived by neglecting the dependence of the production-process matrix element on the direction of the photon [which is roughly within $(m_e/E)^{1/2}$ of the incident lepton direction] and by assuming that the longitudinal current matrix element is not anomalously large compared with the transverse components.[5]

In this manner we can relate the differential cross section for $e + e \to e + e + N$ to the differential cross section for two oppositely directed real photons annihilating into a neutral $C = +1$ state N:

$$d\sigma^N = \left(\frac{2\alpha}{\pi}\right)^2 \left(\ln \frac{E}{m_e}\right)^2 \int \frac{dk_1}{k_1} \frac{dk_2}{k_2} \frac{[E^2 + (E-k_1)^2][E^2 + (E-k_2)^2]}{4E^4} d\sigma_{\gamma\gamma}^N(k_1, k_2). \quad (3)$$

(See Fig. 1 for notation and geometry.) The total cross section is given by

$$\sigma^N = 2\left(\frac{\alpha}{\pi}\right)^2\left(\ln\frac{E}{m_e}\right)^2\int_{s_{\rm th}}^{4E^2}\frac{ds}{s}f((s/4E^2)^{1/2})\sigma_{\gamma\gamma}{}^N(s), \tag{4}$$

where

$$f(x) = (2+x^2)^2\ln x^{-1} - (1-x^2)(3+x^2), \tag{5}$$

and $\sigma_{\gamma\gamma}{}^N(s)$ is the two-photon annihilation cross section for two photons of c.m. energy squared s. These and the following formulas give the leading logarithmic behavior of the order-α^4 cross sections. The correction terms, which are of order $\ln(E/m_e)$ (~7.6 for $E\sim 1$ GeV) smaller, can be computed from the complete Feynman amplitudes.

A total cross section of the type (4) was first derived by Low[6] for the colliding-beam production of the π^0. In this case $\sigma_{\gamma\gamma}\propto\alpha^2\delta(s-m_{\pi^0}{}^2)$, and the colliding-beam cross section is simply

$$\sigma(e+e\to e+e+\pi^0) = (4\alpha)^2\left(\ln\frac{E}{m_e}\right)^2\frac{1}{m_\pi{}^3\tau}f\left(\frac{m_\pi}{2E}\right), \tag{6}$$

where τ is the lifetime of π^0.

In Fig. 2 the total cross sections for the colliding-beam production of $\mu^+\mu^-$, $\pi^+\pi^-$, π^0, and η^0 are given. The two-photon annihilation cross section for muon-pair production in order α^4 exceeds the lowest-order QED cross section for $E_{\rm c.m.} > 1$ GeV. The two-photon cross section for pion-pair production exceeds the usual one-photon order-α^2 cross section for $E_{\rm c.m.} > 1.25$ GeV if one treats the pions as pointlike. Since the two-photon total cross section is dominated by pair production near threshold ($s\sim 4m_\pi{}^2$), this rate is not likely to be strongly modified by hadronic corrections. In fact, it is obviously very interesting to search for a resonance enhancement in the cross section for $e+e\to e+e+\pi^++\pi^-$ in order to study $C=+1$ resonances in the $\pi^+\pi^-$ system such as the σ or ϵ[7] and to understand in general an analytic continuation of the hadron Compton amplitude from forward scattering to the threshold region.

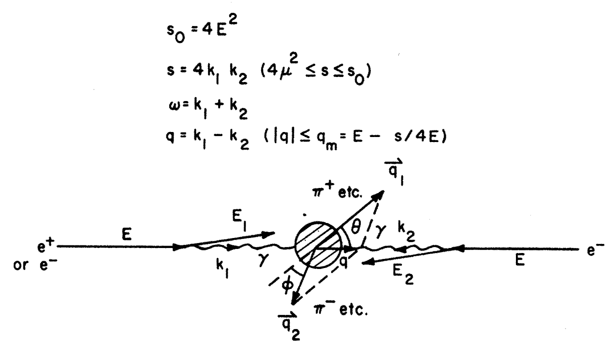

FIG. 1. Notation and geometry for the pair production of various particles by two-photon annihilation.

Differential cross sections.—Various differential cross sections can be readily determined from (3). A very convenient and simple result can be obtained for $d\sigma/dsd\Omega$ for pion-pair production, assuming pointlike pions, $\vec{q}_{1,2}{}^2 \gg m_\pi{}^2$ and $k_{1,2} \ll E$. Then

$$\frac{d\sigma}{dsd\Omega_1} = \left(\frac{2\alpha^2}{\pi}\right)^2\left(\ln\frac{E}{m_e}\right)^2\frac{1}{s^2}\frac{\zeta}{1-\zeta^2\cos^2\theta}, \tag{7}$$

where

$$\zeta = \frac{1-s/4E^2}{1+s/4E^2}, \tag{8}$$

which shows the dominance of the low-s region. The last factor of (7) from the Lorentz transformation changes the isotropic distribution in the pion-pair c.m. system (which originates from the dominant "seagull" graph) to an elongated distribution at small s. In Fig. 3(a) we show the results for $d\sigma/d\Omega$ obtained by numerical integration of Eq. (3) without going through the approximate formula (7), and compare them with the

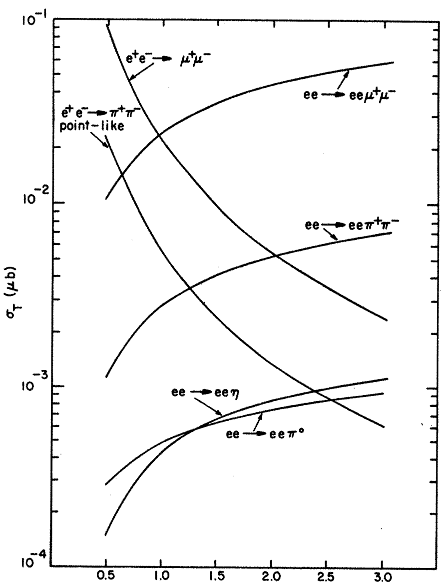

FIG. 2. The total cross sections for the colliding-beam production of various particles.

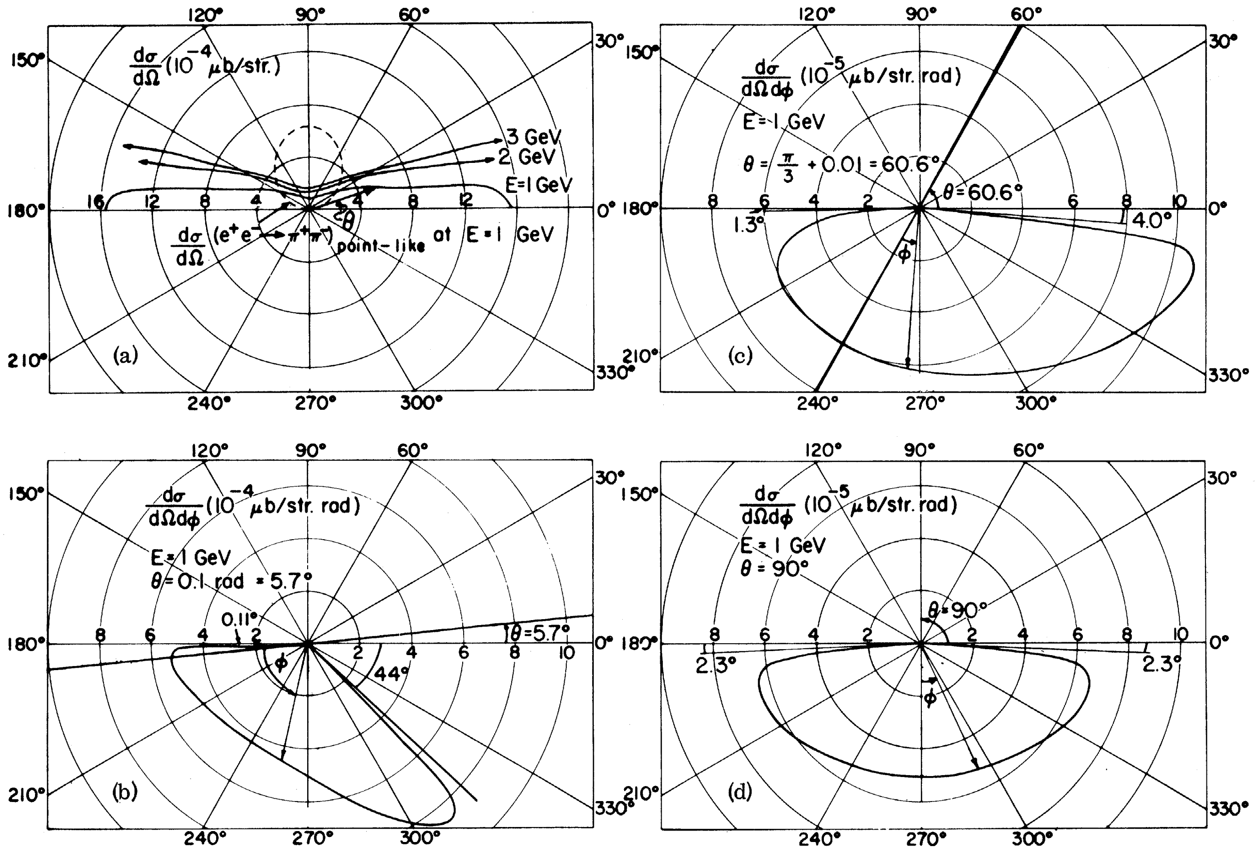

FIG. 3. The differential cross sections of pion-pair production via two-photon annihilation. The length of an arrow drawn from the origin to the curve at an angle θ gives the differential cross section $d\sigma/d\Omega$ at the production angle θ in (a). In (b)-(d), the length of a similar arrow gives the differential cross section $d\sigma/d\Omega\varphi$ for fixed θ as a function of the angle φ between the π^+ and π^-. E is the energy of an electron in the initial beam.

angular distribution of the one-photon pointlike pion cross section.

In Figs. 3(b)-3(d) the differential pion-pair cross section is shown for $E = 1$ GeV, where φ is the angle between the π^+ and π^- in the production plane ($\varphi = 0$ for pairs emerging in opposite directions). The signature for pion-pair production through two-photon annihilation is two coplanar but quite noncollinear tracks. In the above, only the "seagull" contribution to $\sigma(\gamma\gamma \to \pi^+\pi^-)$ is retained; the neglected terms are of order m_π^2/s smaller and contribute only for $\cos\theta_{1,2} \sim \pm 1$ because of the pole structure.

The differential cross sections for muon-pair production are very similar. In this case the cross section $\sigma(\gamma\gamma \to \mu^+\mu^-)$ contains an angle-independent piece exactly twice as large as one for spin-0 pairs. The remainder of the cross section is strongly peaked in the beam direction where other neglected amplitudes for $ee \to ee\mu^+\mu^-$ must be taken into account.

Two-photon annihilation as a background of e^+e^- annihilation processes.—Although the angular distribution for the hadrons are maximized in forward and backward directions, an appreciable fraction of the events are found in the transverse directions, as is shown in Fig. 3. The pion pairs are approximately coplanar with the beam but do not appear collinear in the laboratory system due to the Lorentz transformation from the photon-photon c.m. frame. Nevertheless, these processes will provide a serious background to multipion production via one photon at high energies. The background problem can be minimized, however, if energy measurements of particles in the final state are made to require a minimum s, removing the dominant low-s component of the cross section in Eq. (7).

Other higher-order colliding-beam cross sections.—The other process in order α^4, e.g., conversion of a bremsstrahlung photon into a pion pair, has at least one less power of $\ln(E/m_e)$ ($\sim$7.6) and produces noncoplanar pion pairs predominantly in the same direction along the beam.

It should be noted that the order-α^3 cross section for a pion pair plus a hard photon decreases at least as E^{-2}.

Details of our results will be published elsewhere.

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SPECTRAL FUNCTIONS FROM ELECTRON-POSITRON ANNIHILATION

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It is shown, on the basis of partial conservation of axial-vector current and current-algebra notions, how spectral-function information on the weak axial-vector current and isovector electromagnetic current can be extracted from soft-pion production in e^+-e^- annihilations.

The great interest which attaches to measurement of the total inelastic cross section for electron-positron annihilation stems from the fact that this quantity gives precise information about the absorptive part of the photon propagator. This is expressed by the well-known formula[1]

$$\sigma(k^2) = (16\pi^3\alpha^2/k^2)\Pi(k^2), \tag{1}$$

where $\Pi(k^2)$ is defined by

$$(2\pi)^3\textstyle\sum_n\langle 0|j_\mu{}^{\rm em}|n\rangle\langle n|j_\nu{}^{\rm em}|0\rangle\delta(p_n-k) = (k^2\delta_{\mu\nu}-k_\mu k_\nu)\Pi(k^2). \tag{2}$$

Here $j_\mu{}^{\rm em}$ is the electromagnetic current, and the summation is over a complete set of hadron states $|n\rangle$. Of course Eq. (1) implies the neglect of higher-order electromagnetic corrections. To this leading approximation (which will be used throughout in this note) we may picture the total cross section as characterizing the decay of a massive photon, with four-momentum k ($k^2>0$), into all hadron channels:

$$e^+ + e^- \to \text{“}\gamma\text{”} \to \text{“everything.”} \tag{3}$$

The electromagnetic current is composed of an isovector part and an isoscalar part; correspondingly,

$$\Pi(k^2) = \Pi^{(1)}(k^2) + \Pi^{(0)}(k^2). \tag{4}$$

Regrettably, the pair-annihilation reactions do