at 12 GeV/c indicates (1) agreement with the model of Rarita and Schwarzschild, (2) equality of K^+n and K^-p charge-exchange differential cross sections, and (3) essentially real forward amplitudes for K^+n charge exchange.

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PARTIAL-WAVE ANALYSIS OF THE 3π DECAY OF THE A_2†

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A new partial-wave analysis is applied to the 3π decay of the A_2^- in the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ at 5 and 7.5 GeV/c. The relative importance of the spins and parities 0^-, 1^+, 1^-, 2^+, and 2^- are obtained as a function of 3π mass. Only the 2^+ spin and parity shows resonant structure. Results are also given on the polarization and momentum-transfer dependence of the A_2 and on the mass dependence of the interference of 1^+ background with 2^+ resonance.

Recently there has been renewed interest in the A_2 meson produced in the reaction $\pi^\pm p \to pA_2^\pm$. At the moment the mass spectra indicate a split peak for π^-p reactions[1] in contrast to a single unsplit peak for the largest π^+ experiment.[2] To understand this splitting as well as the difference in the π^- and π^+ channels it is very desirable to examine the spin and parity of all decay channels of the A_2 and to understand the production mechanism. The continuing theoretical interest in daughter trajectories also makes it desirable to look for peaks in other spins and parities in the region of the A_2 mass. The 3π decay mode is particularly useful since the spins and parities 0^-, 1^+, 2^-, $\cdots$, 1^-, 2^+, $\cdots$ are all allowed whereas for $\eta\pi$ or $K\bar{K}$ decays only 0^+ and 2^+ are expected to be seen.[3] In the past,[4] the experimental information has been limited since most analyses of the 3π final state explored only distributions in the Dalitz plot or only certain angular distributions, and allowed for only one spin and parity above a background. These analyses also omitted possible interference effects between resonance and background.

In this paper we wish to describe a new partial wave analysis of the 3π system in the A_2 region. Our analysis includes all allowed spins and parities up to spin 2 as well as the possibility of a phase-space background, and takes into account interferences between the various spins and parities. The 3π events used in this analysis were obtained from the reaction $\pi^-p\rightarrow p\pi^+\pi^-\pi^-$ observed in bubble chambers: 14 000 such events at 5 GeV/c from exposures in the Lawrence Radiation Laboratory 72-in. bubble chamber and in the Stanford Linear Accelerator Center 82-in. bubble chamber, as well as 10 000 such events at 7.5 GeV/c from an exposure in the Brookhaven National Laboratory 80-in. bubble chamber.

We will summarize the method of analysis, and give an example of the quality of fit obtained. Then an analysis of the 3π final state in 50-MeV intervals throughout the A_2 region will be presented showing the mass dependence of the spin and parity states. Then we will examine several aspects of the production mechanism of the A_2^-. First we examine the polarization of the 2^+ state in both the lower (1.2-1.28 GeV) and the upper (1.32-1.4 GeV) parts of the A_2 region, second we present results on the momentum-transfer dependence of the production cross section, and third we show some preliminary results on the interference between the A_2 and the 1^+ background term.

The present analysis, described in more detail in a forthcoming paper,[5] uses the maximum-likelihood method to fit the data using all the coordinates of each event simultaneously. The probability W of observing an event (for a fixed region of the 3π mass $M_{3\pi}$ and square of momentum transfer to the proton Δ^2) is written as a function of the Dalitz-plot coordinates s_1 and s_2 and the three Euler angles φ, θ, γ, as

$$W(s_1,s_2,\varphi,\theta,\gamma)=\sum\rho_{M1,M2}^{JP1,JP2}\mathfrak{M}_{M1}{}^{JP1}(\mathfrak{M}_{M2}{}^{JP2})^*,$$

where $\rho_{M1,M2}^{JP1,JP2}$ is the density matrix describing the production process and $\mathfrak{M}_M{}^{JP}$ is the amplitude describing the decay of a state of definite spin and parity J^P and z spin component M into three pions. To determine the relative importance of $\rho\pi$, $\epsilon\pi$, or $f\pi$ decay modes for the allowed orbital angular momentum states we further expand the decay amplitude as

$$\mathfrak{M}_M{}^{JP}=\sum_l C_l{}^{JP}\mathfrak{M}_M{}^{JP,l},$$

where $C_l{}^{JP}$ is the amplitude for the $\rho\pi$, $\epsilon\pi$, or $f\pi$ decay in a state of definite orbital angular momentum l, and $\mathfrak{M}_M{}^{JP,l}$ describes the decay of that state. Unnatural-parity states 0^-, 1^+, 2^-, $\cdots$, can decay into both $\epsilon\pi$ and $\rho\pi$, while natural-parity states 1^-, 2^+, $\cdots$, cannot decay into an $\epsilon\pi$ intermediate state. We have included $f\pi$ decay only for the 2^+ state; for 2^+ we included $f\pi$ P wave as well as $\rho\pi$ D wave.

In the analysis we use the method of maximum likelihood to fit the experimental data. The likelihood is maximized as a function of the unknown parameters $\rho_{M1,M2}^{JP1,JP2}$ and $C_l{}^{JP}$. In maximizing the likelihood, events in the N^{*++} region (1.16 to 1.32 GeV) were excluded both from the experimental data and the theoretical expression.

We have examined the goodness of the fit obtained by the maximum-likelihood method by generating events using Monte Carlo techniques. The Monte Carlo events were generated according to the probability distributions $W(s_1,s_2,\varphi,\theta,\gamma)$ evaluated for the parameters $\rho_{M1,M2}^{JP1,JP2}$, $C_l{}^{JP}$ determined by the fit. The solid curves in Fig. 1 are the predictions of the fit for two experimental distributions. Figure 1(a) shows the projection of the Dalitz plot for events lying in the ρ band (0.665-0.865 GeV). A large peak is seen for values of $M_{\pi^+\pi^-}$ corresponding to the mass of the ρ; this peak comes from large contributions in the 2^+ and 1^+ states. Figure 1(b) shows the distribution in $\cos\theta$, where θ is the angle between the incident π^- and the outgoing π^+ measured in the A_2 rest frame. In this plot, the forward and backward peaks come primarily from the 1^+ contribution. The agreement between the fit and the data is good for both distributions.

In order to determine the dependence of the spin and parity as a function of the 3π mass, we have divided the mass region from 1.1 to 1.5

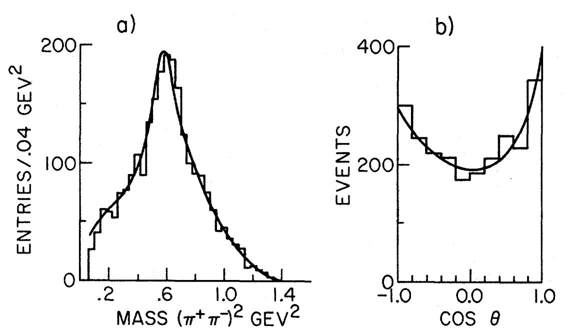

FIG. 1. Comparison of the maximum-likelihood fit (curves) with the data using events with 3π mass between 1.2 and 1.4 GeV and Δ^2 between 0.05 and 0.65 (GeV/c)^2. (a) Projection of the ρ band. (b) π^+ polar angle as explained in text.

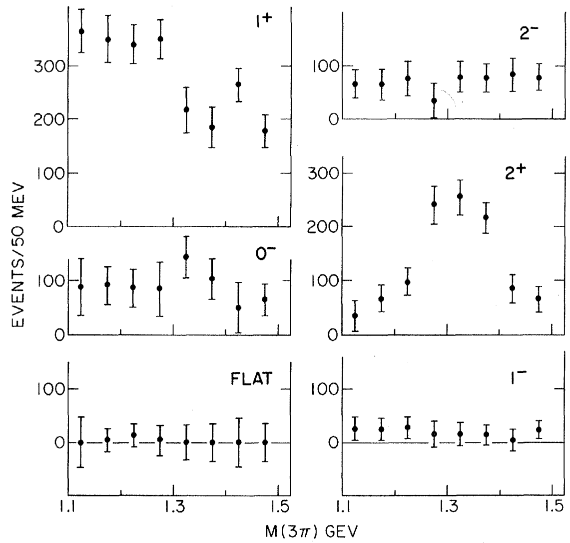

FIG. 2. Number of events for spins and parities as a function of 3π mass using events with Δ^2 between 0.05 and 0.65 $(\text{GeV}/c)^2$.

GeV into 50-MeV bins and in each bin determined the density-matrix elements as well as the orbital angular momentum coefficients. The number of events corresponding to each spin and parity can be determined from the diagonal matrix elements and the results are shown in Fig. 2. For these fits we have included events with square of momentum transfer between 0.05 and 0.65 $(\text{GeV}/c)^2$. The effect of cutting out events with smaller square of momentum transfer is simply to reduce the number of 0^- and 1^+ events throughout the whole mass region. From the figure it is clear that only the $J^P=2^+$ state shows structure, all other J^P states varying smoothly with the 3π mass; therefore in the present sample there is no evidence for any daughters. We also note that the contribution from the phase-space or flat term is completely negligible. This result is in contrast to some previous analyses of the A_2 in which appreciable amounts of flat background have been found. The 2^+ contribution is at a maximum slightly above 1.3 GeV. A maximum-likelihood fit to the region 1.2 to 1.4 assuming a simple Breit-Wigner dependence yields a value of 1.305 ± 0.003 GeV for the peak.

The orbital coefficients C_l^{JP} in the A_2 region are in good agreement with those found in the A_1 region for spins and parities 0^- and 1^+.[5] For spin and parity 2^+, we have also included the possibility of $f\pi$ decay and found the contribution quite small, less then 10% between 1.2 and 1.4 GeV. This is to be expected since the A_2 is below $f\pi$ threshold and only the low-mass tail of the f can contribute.

Table I. Polarization of the lower and upper parts of A_2.

	A_2^L 1.2–1.28 GeV	A_2^H 1.32–1.4 GeV
ρ_{00}	0.04 ± 0.07	0.08 ± 0.05
ρ_{11}	0.46 ± 0.05	0.45 ± 0.03
ρ_{1-1}	0.36 ± 0.07	0.34 ± 0.05
ρ_{22}	0.02 ± 0.05	0.01 ± 0.03

In order to obtain information on the production mechanism, as well as to examine possible differences between the upper and lower A_2, we have determined the polarization in the mass regions 1.2 to 1.28 and 1.32 to 1.4. The results are shown in Table I, and indicate clearly that the polarizations are the same for both mass regions. They are consistent with polarization found in the other decay channels of the A_2.[2,6]

The dependence of the various spin and parity components on square of momentum transfer has been examined in the A_2 region and the results are shown in Fig. 3. For these fits we have taken the mass region between 1.2 and 1.4 GeV and used bins of $\Delta^2-\Delta_{\min}^2$, where $\Delta_{\min}^2$ is the smallest kinematically allowed square of momentum transfer. In Fig. 3 it is seen that the dominant background terms 0^-, 1^+ are strongly peripheral whereas the 2^+ contribution is small in the forward direction, has a broad maximum and then decreases again for larger Δ^2.

The present analysis allows us to determine the magnitude and phase of the interference between two different spins and parities. In our data we

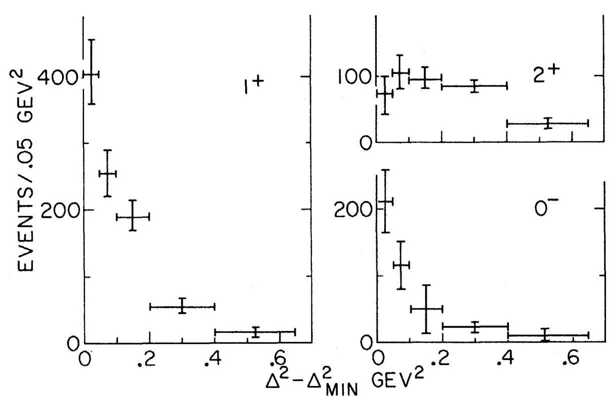

FIG. 3. Momentum-transfer square distributions for spins and parities 0^-, 1^+, and 2^+ for events with 3π mass between 1.2 and 1.4 GeV.

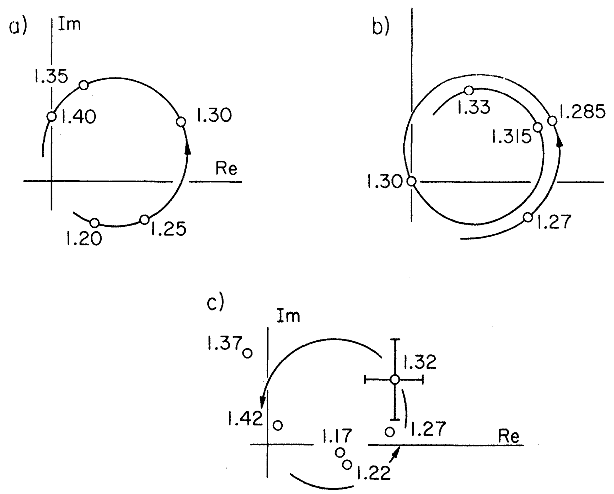

FIG. 4. Variation with 3π mass of the interference between 1^+ and 2^+. (a) Simple Breit-Wigner model. (b) Dipole model. (c) Results of the maximum-likelihood fits to the data in 50-MeV intervals using events with Δ^2 between 0.05 and 0.65 $(\text{GeV}/c)^2$.

find the interference between the 2^+ resonant term and the 1^+ background term to be important. Figure 4 shows the variation of this interference term with 3π mass for two simple models as well as the results from this experiment. The two models assume that the background 1^+ amplitude is constant through the mass region. The 2^+ resonant amplitude is assumed to have a Breit-Wigner form in the first model and a dipole form in the second model. In the figure, the phase near resonance is chosen arbitrarily to give agreement with the experimental data. The two models are quite similar except in the immediate vicinity of 1.3 GeV. Near 1.3 GeV, the dipole amplitude, using the CERN dipole parameters,[1] sweeps rapidly around a circle between 1.285 and 1.315 vanishing at 1.3. Unfortunately we have insufficient data and resolution to make a clear determination of the behavior in the central region at this time. We see, however, that the data follow the general trend of either model in the regions below and above the central region.

We conclude that the partial-wave analysis has shown that the resonant 3π effect near 1.3 GeV is $J^P = 2^+$ and no other allowed spins and parities up to spin 2 show any significant structure. The polarization is the same in the upper and lower halves of the A_2. Although the present data do not allow a distinction between dipole and simple resonance behavior, the phase variation of the resonant amplitude near the resonant energy has been determined.

In the future we hope to compare these results with similar analyses in the π^+ reactions. The 1^+ is presumably produced by isoscalar exchange. If this is true, then the phase of the interference term should change (not change) if the 2^+ is produced by isovector (isoscalar) exchange. The present analysis allows a determination of the total number of 2^+ events with a minimum of reliance on assumption about the background. A reliable comparison with $A_2{}^+$ and $A_2{}^0$ production cross sections should further elucidate the important exchange processes.

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