PINCH-EFFECT OSCILLATIONS IN AN UNSTABLE TOKAMAK PLASMA

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If Tokamak toroidal discharges are operated with $q < 3$, relaxation oscillations are observed to occur involving alternate contraction and expansion of the plasma. It is shown that the detailed properties determined for these oscillations on the Canberra experiment can be explained on the basis of the newly discovered trapped-particle pinch effect.

In the early experiments with the Tokamak toroidal plasmas it was discovered that discharges with the parameter q less than about 3 or 4 exhibit relaxation oscillations involving periodic changes in the minor and major radii of the plasma column and negative spikes on the voltage oscillograms[1,2] ($q \equiv aB_\varphi/RB_\theta$, where a and R are the minor and major radii of the plasma; θ and φ are the angular coordinates going the short and long way round the torus, respectively). These relaxation oscillations have been the subject of a detailed study on the Canberra Tokamak-type experiment.[3] From the results,[4] the main properties of these oscillations can be summarized as follows:

(1) The oscillations occur when T_e exceeds a critical value. For the parameters of the Canberra experiment, namely, $a = 8$ cm, $R = 40$ cm, $B_\varphi = 6.5$ kG, $I_\varphi = 22$ kA, and $n \sim 10^{13}$ cm^{-3}, the critical T_e is about 30 eV.

(2) The radial profiles for the toroidal current density (j_φ) show a marked discontinuity at a certain radius, the value of j_φ being large within this radius and from $\frac{1}{2}$ to $\frac{1}{5}$ of this value outside.

(3) During the contraction phase of an oscillation, the current discontinuity moves towards the magnetic axis with a radial velocity of the order of $-cE_\varphi/B_\theta$, with the internal j_φ increasing to maintain an approximately constant total current. (E_φ is the toroidal electric field obtained from the measured volts per turn.) There is also an increase in plasma density at the center implying that plasma is being transported inwards with a comparable velocity.

(4) During the contraction there is a higher frequency oscillation superimposed on the contraction with frequency of the order 2×10^4 sec^{-1}.

(5) Towards the end of the contraction the magnetic surfaces have an elliptic shape with the major axis approximately parallel to $\theta = \frac{1}{4}\pi$. (θ is measured from the equitorial plane, which is hereafter assumed to be horizontal. j_φ and B_φ are assumed to be positive.)

(6) After the current channel has contracted by a significant fraction of the tube radius, the direction of motion reverses and a rapid expansion of both the plasma and the current channel occurs. A large negative spike occurs simultaneously on the voltage-per-turn oscillogram.

(7) The mean radius of the plasma (R) increases during the contraction phase and decreases (rapidly) during the expansion. The plasma column also oscillates vertically, the direction of movement being downwards as R_0 increases.

(8) The electron temperature rises from 30 eV to between 100 and 200 eV during the contraction. It falls rapidly during the expansion.

In an earlier paper[5] it was shown that the tendency for the plasma to contract with velocity of order $-cE_\varphi/B_\theta$ could be explained by the properties of the trapped particles. From the conservation of canonical angular momentum it was shown that all trapped particles (trapped magnetically or electrostatically) drift towards the magnetic axis with the radial velocity $-cE_\varphi/B_\theta$. In this paper it will be shown that the other detailed properties of the relaxation oscillations have simple explanations in terms of the trapped particles.

There is general agreement[6-8] that trapped-particle effects become important in collisional transport processes when the particle collision frequency (ν) satisfies

$$\nu < B_\theta v_T/Br, \tag{1}$$

where r is the minor radius and v_T is the thermal velocity. (Since the diffusion is ambipolar,[7] we consider only the electrons.) Substituting for ν this becomes

$$T_e^2 > 6.7\times10^{-13} nqR, \tag{2}$$

where T_e is in eV. From the parameters given above for the Canberra experiment, the critical value for T_e is found to be 28 eV. Hence the observed relaxation oscillations occur only under those conditions when trapped-particle effects are important.

In the treatments of particle transport for conditions when the inequality (1) is satisfied, the effects of a toroidal electric field E_φ and of the associated current j_φ have been omitted. Thus, the zero-order distribution functions have been taken as stationary Maxwellians. If, instead, the zero-order electron distribution function is taken to be of the form

$$f_0(v_{\parallel}) \sim \exp[-(v_{\parallel}-v_0)^2/v_T^{\,2}],$$

where $v_0 = -j_{\parallel}/ne \simeq -j_\varphi/ne$, the effects of both E_φ and the friction between electrons and ions will be included to a first approximation. Making this substitution in the analysis of Galeev and Sagdeev[6] for the intermediate collision frequency range yields the modified radial particle flux

$$\langle nv\rangle = -\frac{D}{kT_e}\left[j_\varphi B_\theta + k(T_e+T_i)\frac{dn}{dr}\right], \qquad (3)$$

where D $(=\pi^{1/2} q\rho_e ckT_e/2R_0 eB)$ is the Galeev-Sagdeev diffusion coefficient with the numerical correction found by Rutherford[7] and by Stringer,[8] and where temperature gradients have been assumed to be zero. ρ_e is the electron Larmor radius. The range of collision frequencies for which this diffusion formula is valid is

$$(r/R)^{3/2} B_\theta V_T/Br < \nu < B_\theta V_T/Br. \qquad (4)$$

The upper limit has already been given in (1). The lower limit corresponds to the collision frequency for scattering between trapped and untrapped velocity directions $(R\nu/r)$ being too small to create a local Maxwellian distribution in the time scale of the other processes, namely, the bounce time for trapped particles.

The term $j_\varphi B_\theta$ in (3) is new and indicates that the diffusion will be radially inwards until $j_\varphi B_\theta + k(T_e+T_i)(dn/dr)$ is zero. This is analogous to the pinch effect in the absence of B_φ, except that the velocities involved are different. Assuming that dn/dr is small, the velocity of constriction is

$$v = -Dj_\varphi B_\theta/nkT_e \simeq -\alpha\pi^{1/2}(r/R)^2(cE_\varphi/B_\theta), \qquad (5)$$

where $\alpha = B_\theta v_T/Br\nu$. At the critical collision frequency given by equality in (1), α is unity and it increases inversely as ν decreases. Since T_e is observed to rise rapidly during the contraction phase making α large, the velocity in (5) could possibly explain the observed pinching of the plasma (there is no accurate experimental measurement of the velocity), but it will not explain the sharp discontinuity in j_φ.

Considering, therefore, the edge of the plasma, the part of $j_{\parallel}$ resulting from the nonzero divergence of $j_\perp$, namely, $j_{\parallel\beta} = -2r(dp/dr)\cos\theta/RB_\theta$, requires an additional electric field $E_{\parallel\beta} = \eta j_{\parallel\beta}$ to maintain this current, where η is the resistivity. The ratio of this electric field for the edge of the plasma $(r=a)$ to the applied electric field is

$$\frac{E_{\parallel\beta}}{E_\varphi} = \frac{\eta_a j_{\parallel\beta}}{\langle\eta\rangle\langle j_\varphi\rangle} = \left(\frac{\eta_a}{\langle\eta\rangle}\right)\left(\frac{a}{R}\right)\left(\frac{a}{B_{\theta a}^{\,2}}\frac{dp}{dr}\right)\cos\theta. \qquad (6)$$

Hence, if $a(dp/dr)/B_{\theta a}^{\,2} > R\langle\eta\rangle/a\eta_a$, the ratio in (6) can exceed unity and hence electrostatic trapping of particles will occur. At the edge of the plasma this inequality can be expected.

The significance of electrostatic trapping by a potential V compared with magnetic trapping is that there will be a group of low-energy electrons $(mv_T^2/2 < eV)$ which will be trapped irrespective of their velocity direction. For these particles to become passing particles by collisions, the appropriate collision frequency is ν_E, their energy-exchange time. From Spitzer,[9] for low-energy particles $\nu_E \simeq 2\nu$. Hence, the assumption of a local Maxwellian velocity distribution becomes invalid for the edge of the plasma if

$$\nu < B_\theta v_T/2Br, \qquad (7)$$

which is only a factor 2 less than conditions (1). In particular, because of the uncertainty in the experimental parameters, equality in condition (7) could correspond to the critical temperature for relaxation oscillations as equally as condition (1). When condition (7) is satisfied, the region of positive potential will have a shortage of low-energy electrons due to the trapped-particle pinch effect carrying these electrons inwards. This local shortage of electrons cannot be neutralized by passing electrons since these will have a lower density where V is positive. Hence, the shortage of low-energy electrons will increase the positive potential, leading to more trapped electrons. This amplification process for the number of trapped particles is believed to be the cause of the sharp edge to the plasma column. In the edge, the electrostatic potential will cause a large proportion of the particles to be trapped. These particles, and hence the edge, will move inwards with velocity $-cE_\varphi/B_\theta$, since the increased potential will cause the particle bounce times to be less than their collision times. A plasma with lower density will remain outside the edge which is deficient in trapped particles, and has a reduced number of passing particles.

The assumption that the ratio in Eq. (6) is greater than unity is equivalent to assuming that the contribution to $j_{\|}$ due to the trapped particles $(j_{\|T})$ is comparable with the contribution from the passing particles. Hence, the sharp edge of the trapped-particle density profile will generate a corresponding sharp fall in j_φ. (When electrostatic trapping is involved, the simple $\cos\theta$ variation of $j_{\|T}$ is no longer valid. Ions will be trapped on the opposite side of the minor diameter to the trapped electrons and if "banana" orbits extend beyond 180° in θ, the sets of trapped particles will overlap.)

Another effect of the enhanced electrostatic potential will be to excite an ion oscillation. Ions will be accelerated along the magnetic lines of force away from the region of positive potential. Because of the inertia of the ions there will be an oscillation. If the potential is of the order kT_e/e the period of the oscillation will be the transit time for an ion with this energy to rotate 2π in θ moving along $\vec{B}$. Thus the oscillation frequency is $B_\theta(kT_e/M)^{1/2}/2\pi rB$. Taking a mean value of T_e for the initial part of the contraction to be 50 eV, this frequency is 1.5×10^4 sec^{-1}, in good agreement with the observed high-frequency oscillation.

In the main body of the plasma, magnetic trapping of electrons will be dominant. This trapping will occur for high-energy electrons with sufficiently small $v_{\|}$. Their maximum density will occur where the minimum in the potential $[\mu|B| - \int(E_\varphi B_\varphi/B)dx_{\|}]$ occurs, which must lie between $\theta=0$ and $\theta=\frac{1}{2}\pi$. Because of the trapped-particle pinch effect, their density will exceed that predicted by a Maxwellian velocity distribution. This will lead to a local negative potential which will cause electrostatic trapping of low-energy electrons diametrically opposite. Assuming no mass rotation of the plasma in the θ direction, only a smaller proportion of the ions will be trapped. This is because of the effect of the radial electric field[10] $[E_r \simeq (dp_i/dr)/ne]$. The maxima in the density of trapped electrons will therefore occur in the two quadrants $0>\theta>-\frac{1}{2}\pi$ and $\pi>\theta>\frac{1}{2}\pi$ and hence maximum contributions to $j_{\|}$ will occur in these two regions. Assuming that the pressure of the magnetically trapped electrons exceeds that of the electrostatically trapped electrons, the former will produce the larger $j_{\|}$. Hence $j_{\|}$ will have components proportional to $\cos(\theta-\delta)$ and $\cos2(\theta-\delta)$, where $0<\delta<\frac{1}{2}\pi$. The first harmonic term will cause a shift of the magnetic axis and the magnetic surfaces towards the quadrant 0 to $-\frac{1}{2}\pi$ and this will explain the downward movement of the plasma column. The second harmonic term will cause elliptic-shaped magnetic surfaces with major axis in the quadrant 0 to $+\frac{1}{2}\pi$, as observed.

The most likely cause of the rapid-expansion phase is a magnetohydrodynamic instability. Even if the rotational transform were to remain below 2π during the contraction, a study of the necessary and sufficient conditions for stability[11] shows that the increasing pressure gradient and its increasing separation from the conducting wall will lead to instability. However, in the Canberra experiments, the rotational transform increases to well above 2π ($\iota\sim4\pi$) and since this makes the average field curvature unstable, an interchange instability is likely. Once expansion has commenced the trapped particles could play a part in the expansion. The changing B_θ (decreasing inductance) will cause E_φ to become negative over the outer regions of the plasma, and this will cause the motion of the trapped particles to be outwards.

Lastly, during contraction the electron temperature will rise (a) because of the Ohmic heating combined with the improved insulation from the walls, (b) because of the compression of the plasma, and (c) because heat conduction will be radially inwards until $-d\ln T/dr$ is comparable with $-d\ln n/dr$ as a result of a form of the Ettingshausen effect associated with the trapped-particle pinch effect.[12] The increase in the major radius during contraction and decrease during expansion follow directly from Shafranov's equilibrium formula for Δ in terms of the minor radius of the plasma and the plasma pressure.[13] (Δ is the displacement of the center of the plasma cross section from the tube center.)

It is concluded, therefore, that the pinch-effect property of trapped particles, combined with the multiplication process for trapped particles associated with electrostatic trapping at the edge of the plasma, leads to comparatively simple explanations for all the observed properties of the relaxation oscillations.

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TIGHT BINDING AND TRANSITION-METAL SUPERCONDUCTIVITY*

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The phonon-induced d-d coupling, calculated in tight-binding approximation, is shown to account for the order of magnitude of T_c in transition metals. The model relates the coupling constant to the cohesive energy rather than to the melting temperature as proposed by Matthias.

Recently it was suggested[1] that the superconductivity in transition metals is essentially due to phonon-induced d-p coupling. In the present work, we estimate the coupling term for a pure d band in tight-binding approximation and show that it can, alone, explain the order of magnitude of the superconducting transition temperature T_c, observed in transition metals and related alloys. This term is not incompatible with the general symmetry arguments presented in Ref. 1.

For the sake of simplicity, we choose to study a nondegenerate band of tightly bound electrons, described in the undeformed lattice (one atom per unit cell, six nearest neighbors) by the Hubbard Hamiltonian. When the lattice is deformed by displacing the ion on the site j from the position $\vec{R}_j$ to $\vec{R}_j+\vec{u}_j$, we assume, as is usual in the study of one-dimensional systems[2,3] and in some recent three-dimensional computations,[4-6] that the tightly bound d function follows the displaced ion without notable deformation. For small displacements the quasiorthogonality relation of well-localized d functions remains valid. Therefore, we can introduce the operator $a_j^\dagger$ creating the electron in the state $|d(\vec{r}-\vec{R}_j-\vec{u}_j)\rangle$, such that $(a_i^\dagger, a_j)_+ \simeq \delta_{ij}$, and extend the Hubbard Hamiltonian to describe the deformed state:

$$H=\sum_{j,\alpha,\delta_\alpha,\sigma} J_{j,\delta_\alpha} a_{j,\sigma}^\dagger a_{j+\delta_\alpha,\sigma} + U\sum_j n_{j\uparrow}n_{j\downarrow}. \tag{1}$$

$J_{j,\delta_\alpha}=J(\vec{a}_{\delta_\alpha}+\vec{u}_{j+\delta_\alpha}-\vec{u}_j)$ is the overlap integral between the site j and one of two nearby sites δ_α lying on the lattice axis $\vec{a}_\alpha$:

$$J(\vec{a}_{\delta_\alpha}+\vec{u}_{j+\delta_\alpha}-\vec{u}_j)=\int d(\vec{r}+\vec{a}_{\delta_\alpha}+\vec{u}_{j+\delta_\alpha}-\vec{u}_j)V(\vec{r})d(\vec{r})d^3\vec{r}. \tag{2}$$

Here $\vec{a}_{\delta_\alpha}=\vec{R}_{j+\delta_\alpha}-\vec{R}_j$ and $V(\vec{r})$ is the self-consistent potential attached to the site in the undeformed lattice and carried rigidly by the displaced ion. This is a supplementary approximation of our calculation. It is consistent with a rigid displacement of the d function if $\langle n_{j\sigma}\rangle$ does not change with respect to its value Q in undeformed state. It can be shown[7] that even if one works with a long-range Hamiltonian instead of (1), but with an almost half-filled band, the self-consistency corrections arising from $\delta\langle n_{j\sigma}\rangle$ can be neglected with respect to the rigid-ion part of electron-phonon coupling constant. This latter is obtained on expanding the overlap integral (2) in the Hamiltonian (1) to the first order in $\vec{u}_{j+\delta_\alpha}-\vec{u}_j$:

$$J_{j\delta_\alpha}=J(\vec{a}_\alpha)+\left.\frac{\partial J(\vec{R})}{\partial\vec{R}}\right|_{\vec{R}=\vec{a}_{\delta_\alpha}}(\vec{u}_{j+\delta_\alpha}-\vec{u}_j). \tag{3}$$

Usually the symmetry of the d function with respect to the equilibrium lattice is such that the gradi-