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ISOBARIC ANALOG RESONANCE WIDTHS AND THE DISTRIBUTION OF COULOMB MIXING STRENGTH IN NUCLEI

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Experimental data on the microgiant spreading widths of analog resonances indicate that these widths decrease slightly with increasing mass number A for $A=37$-50 and then increase again for $A>60$. A calculation is presented in which this remarkable property can be understood in terms of a model in which the distribution of Coulomb mixing strength in a nucleus has two peaks with the analog state lying between these two peaks and moving with respect to them with changes in A and T_ζ.

With the increasing number of analog-state resonance experiments, systematic trends of various properties associated with these resonances have emerged. Specifically, the spreading widths of analog-state resonances have been extracted fairly accurately for a number of nuclei (see Table I). An interesting experimental feature of these widths is that they have a minimal value as a function of A for nuclei with $A \approx 50$. The significance of this remarkable feature has motivated this Letter. In particular, we report on a calculation which predicts a minimum in this width at the experimentally observed value.

The spreading width of an analog state describes its average decay into the dense spectrum of normal (T-lower) isospin states which surround it, and therefore proceeds through charge-dependent forces. In the presence of such forces the strength of the analog state, measured, for example, by its proton-emission width, will be shared among nearby levels of the compound nuclear system. Moreover, the averaged proton strength distribution has a single peak of a near Lorentzian shape, with a width given by

$$\Gamma_a^{\downarrow} = 2\pi\langle|V|^2\rangle\rho(E_a). \quad (1)$$

Here, $\langle|V|^2\rangle$ denotes the mean square isospin-nonconserving matrix element between the analog state $\psi_a(T=T_\zeta, T_\zeta-1)$ and the T-lower states $\psi_\lambda(T=T_\zeta-1, T_\zeta-1)$, while $\rho(E_a)$ represents the density of these latter states at the analog energy E_a.

To obtain $\Gamma_a^{\downarrow}$, we first calculate the sum of the squares of the transition-matrix elements[1] from the analog to all normal isospin states ψ_λ. We restrict ourselves, for the moment, to the isovector part C_V of the Coulomb force and write for this strength the expression

$$M_V(T_\zeta-1) = \sum_\lambda |\langle\psi_a|C_V|\psi_\lambda\rangle|^2.$$

This expression can then be simplified (by use of closure) to

$$M_V(T_\zeta-1) = \langle\psi_a|C_V P_{T_\zeta-1} C_V|\psi_a\rangle,$$

where $P_{T_\zeta-1}$ projects onto states with $T=T_\zeta-1$. Then, using uncorrelated statistical-model wave functions in which the density $\rho(r)$ of each nucleon is taken to be the same, this expression can easily be evaluated and reads[2]

$$M_V(T_\zeta-1)=\frac{2T_\zeta-1}{T_\zeta(2T_\zeta+1)}\frac{(A-2)(A+2T_\zeta+2)}{16}[(A-4)I+J]. \quad (2)$$

The I and J symbols are given by the expectation values $I=e^4\langle r_{12}^{-1}r_{13}^{-1}\rangle-\overline{C}^2$ and $J=e^4\langle r_{12}^{-2}\rangle-\overline{C}^2$, where $\overline{C}$ is the average Coulomb interaction of a pair of particles: $\overline{C}=e^2\langle r_{12}^{-1}\rangle$. Next, the I term can be simplified to $I=\langle\varphi^2(r)\rangle-\overline{C}^2$, where $\varphi(r)$ is the potential arising from the charge density $\rho(r)$. If we assume the nucleus to be spherical and uniformly charged and to have a sharp cutoff radius r_0, the value of $I=0.017(e^2/r_0)^2$ and $J=0.81(e^2/r_0)^2$, with $\overline{C}=6e^2/5r_0$. Furthermore, in Ref. 2, I was found to be sensitive to the density of nuclear matter $\rho(r)$ while J was insensitive to it. We therefore calculated the appropriate generalization of I using harmonic-oscillator wave functions. This generalization for closed-shell nuclei can be reduced to

$$I=\frac{4^3}{A^3}\sum_{\substack{nlm\\ n'l'm',n''l''m''}}\langle nlm|\varphi_{n'l'}(r)\varphi_{n''l''}(r)|nlm\rangle-\left(\frac{4^2}{A^2}\sum_{\substack{nlm\\ n'l'm'}}\langle nlm|\varphi_{n'l'}(r)|nlm\rangle\right)^2, \quad (3)$$

where $\varphi_{nl}(r)$ is the potential arising from the harmonic-oscillator radial density. The values of I ob-

Table I. Experimental values and theoretical estimates of the spreading width. The total theoretical estimate for $\Gamma_a{\downarrow}$ is the sum of $\Gamma_{a,c}{\downarrow}$ and $\Gamma_{a,m}{\downarrow}$.

Analog	l, J^π	$\Gamma_a{\downarrow}$, expt (keV)	$\Gamma_{a,c}{\downarrow}$ (keV)	$\Gamma_{a,m}{\downarrow}$ (keV)
^{37}Cl + p	$1, 3^-$	<50[a]	30	4
^{40}Ar + p	$1, \frac{3}{2}^-$	20[b]	10	4
^{42}Ca + p	$1, \frac{1}{2}^-$	7[c]	0	6
^{44}Ca + p	$1, \frac{3}{2}^-$	8[c]	0	5
^{48}Ca + p	$1, \frac{3}{2}^-$	2[d]	0	3
^{50}Ti + p	$1, \frac{1}{2}^-$	12[e]	0	5
^{51}V + p	1	<14[f]	0	6
^{62}Ni + p	$1, \frac{3}{2}^-$	~10[g]	0	10
^{64}Ni + p	$1, \frac{3}{2}^-$	~15[g]	0	7
^{70}Zn + p		<20[h]	5	8
^{90}Zr + p	$0, \frac{1}{2}^+$	37[i]	0	18
^{92}Mo + p	$0, \frac{1}{2}^+$	29[j]	0	22
^{138}Ba + p	$3, \frac{7}{2}^-$	≲40[k]	11	27
^{207}Pb + p	$1, 0^+$	≲80[l]	14	50

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Table II. Values for the single-particle excitation coefficients I, I_0, M, N, and F.

Nucleus	$I/(e^2/r_0)^2$	$I_0/(e^2/r_0)^2$	$M/(e^2/r_0)^2$	$N/(e^2/r_0)^2$	$F/(e^2/r_0)^2$
He^4	0.075	0.060	0.060	0	0
O^{16}	0.050	0.0433	0.0400	0	0.0033
Ca^{40}	0.045	0.0375	0.0315	0.0015	0.0045

tained are given in Table II.

Having obtained the total transition strength to the normal-isospin states, we now investigate the nature of the states which exhaust this sum rule. To facilitate this evaluation, we write the exact potential $\varphi_{nl}(r)$ as $\varphi_{nl}=\varphi_0+\Delta\varphi_{nl}$, where $\varphi_0=(3e^2/2r_0^3)(r_0^2-\frac{1}{3}r^2)$, and calculate how much of I is accounted for by the substitution of φ_0 for φ_{nl}. I can then be written as $I=I_0+\Delta I$ with I_0 given by

$$I_0=[\sum_{nlm}4A^{-1}\langle nlm|\varphi_0|n+1lm\rangle^2]+[\sum_{nlm}4A^{-1}\langle nlm|\varphi_0|n-1lm\rangle^2]+[\sum_{nlm}4A^{-1}\langle nlm|\varphi_0|nlm\rangle^2 -(\sum_{nlm}4A^{-1}\langle nlm|\varphi_0|nlm\rangle)^2]=[M]+[N]+[F], \tag{4}$$

and with ΔI being the remainder arising from the terms in $\Delta\varphi_{nl}$. The first term, M, corresponds to a monopole excitation of the nucleus. The second term, N, corresponds to a contraction of the nucleus and will not be present if the exclusion principle, which we discuss below, is properly accounted for. The fluctuation term, F, involves mixing into states in which a nucleon does not change its orbit. This term can be simplified to

$$F=\{\sum_{\alpha,\beta=nl}N_\alpha N_\beta[\Delta_c(\alpha)-\Delta_c(\beta)]^2\}2^{-1}(\sum_\alpha N_\alpha)^{-2}, \tag{5}$$

where N_α is the number of nucleons in level α and $\Delta_c(\alpha)=\langle\alpha|\varphi_0|\alpha\rangle$. The F term therefore contains the mixing to the configuration states of Lane and Soper.[3] Table II also lists the values of I_0, M, N, and F. From this table we first note that most of I is accounted for by I_0 and, thus, the replacement of the exact potential by φ_0 is an excellent approximation. Secondly, we note that the I_0 term is accounted for predominantly by transitions of the type $nl\to n+1,l$. Hence, the Coulomb force primarily mixes into an analog state those states which are obtained by an excitation of a particle-hole pair (in which a particle is excited from the orbit nl to the orbit $n+1,l$). Moreover, since the analog state has $T=T_\zeta$, $T_3=T_\zeta-1$, this particle-hole pair must couple to $t=1$ so that the normal isospin $T=T_\zeta-1$ can be reached. It should also be noted that it is just this type of excitation which is the primary source of isospin impurity in ground state configurations.[4] For ground states, the monopole particle-hole excitation is coupled to $t=1$ which is then coupled with the isospin of the neutron excess, T_ζ, to give a resultant $T=T_\zeta+1$. For the situation discussed in this Letter, the $t=1$ monopole excitation in the parent is coupled to $T=T_\zeta$; the monopole impurity in the adjacent isobar then results from an isospin mixing to the "antianalog" state corresponding to the analog of this monopole excitation.

Thus, the I part of the sum rule is nearly exhausted by mixing into two types of states: into the configuration states and into a $t=1$ monopole excitation of the core, with the latter mixing being more important. The J term of Eq. (2), which represents two-particle scattering, is approximately $20/A$ times smaller in the harmonic oscillator model. Also, a calculation of the mean energy of the J term[2] shows that it represents the very distant states in $C_V\psi_a$ which do not concern us here.

Returning to the sum M_V, we now consider the effect of the exclusion principle. First, the replacement of A by $2Z$ in Eq. (2) amounts to considering the full Coulomb force instead of the vector part of it. Second, if we replace the exact potential in I by the harmonic potential φ_0 and write $Z\varphi_0=\varphi_0(Z)$, the total strength to the monopole excitation is then

$$M_{am}^2=\frac{1}{2}\frac{2T_\zeta-1}{T_\zeta(2T_\zeta+1)}\sum_{nl}\langle nl|\varphi_0(Z)|n+1l\rangle^2N_{nl}. \tag{6}$$

We note the following description of this result: The factor $(2T_\zeta-1)/T_\zeta(2T_\zeta+1)$ is the square of the Wigner coefficient for coupling a $t=1$, $t_3=0$ state to a $T=T_\zeta$, $T_3=T_\zeta-1$ state to form a $T=T_\zeta-1$, $T_3=T_\zeta-1$ state, while the factor $\frac{1}{2}$ is the probability of the proton particle-hole excitation being in a $t=1$ state. $\langle nl|\varphi_0(Z)|n+1l\rangle$ is the radial matrix element describing the excitation of a proton from the orbit nl to the orbit $n+1, l$ through the one-body potential $\varphi_0(Z)$, and N_{nl} is the number of protons in the level nl that can make such transitions. Now, the sum is restricted by the exclusion principle to extend over those states nl for which the state $n+1, l$ is above the proton Fermi sea. Moreover, since the proton particle-hole excitation must couple to $t=1$, the sum is further restricted to those states nl for which $n+1, l$ lies above the Fermi level of the neutrons. The exclusion principle also acts to restrict the contribution of the F term to the total transition strength by limiting the sum in Eq. (5) to levels α and β of the excess neutron shells in the parent. Including these effects, the values of M_{am}^2 can be obtained, and they vary rather smoothly from 2 MeV^2 for ^{48}Ca to 4 MeV^2 for ^{208}Pb with a slight maximum of 5 MeV^2 around $A=90$. In contrast, the transition strength to the configuration states fluctuates violently because of shell effects between 0 MeV^2 and 0.150 MeV^2 for $A \leq 100$.[5] For $A>100$ this transition strength begins to lose its violent fluctuations and varies rather smoothly from 0.150 MeV^2 for $A\approx 100$ to 0.200 MeV^2 for $A\approx 200$.

Having obtained the coupling strengths, we now calculate the energies of these states. First, the isospin-splitting energy of the analog state from the configuration states, E_a-E_c, can be obtained from the symmetry potential[6] $V_{\text{sym}}=\vec{t}_1\cdot\vec{T}_0V_1/A$. Here, $\vec{t}_1$ is the isospin operator of an extra nucleon, $\vec{T}_0$ is the isospin operator of the core of neutrons, and $V_1\cong 110$ MeV. Calculating the expectation values of this potential with these states, we obtain $E_a-E_c=T_\zeta V_1/A$ which has characteristic values of 5, 7, and 11 MeV for $A=50$, 90, and 200, respectively. Next, the shift in energy of the monopole state due to its interaction with the symmetry potential, $V_{\text{sym}}=(\vec{t}_1+\vec{t}_2+\vec{t}_3)\cdot\vec{T}_0V_1/A+(\vec{t}_1\cdot\vec{t}_2+\vec{t}_1\cdot\vec{t}_3+\vec{t}_2\cdot\vec{t}_3)V_1/A$, is downward relative to its unperturbed position in the absence of V_{sym} and is $-(T_\zeta+2)V_1/2A$. Here $\vec{t}_2$ and $\vec{t}_3$ are isospin operators of the particle-hole excitation. Since the monopole excitation lies above the analog state because of the large energy necessary to create a $t=1$ particle-hole pair, the effect of the symmetry potential is to shift the analog state closer to the monopole excitation. Contrastingly, when calculating ground-state impurities, the symmetry potential pushes the higher isospin states away from the ground state. Our zero-order estimate of the energy of the monopole state above the analog state is then $E_m-E_a=2\hbar\omega-(T_\zeta+\frac{3}{4})V_1/A$. Using the relation $2\hbar\omega=82A^{-1/3}$, we see that for $A=208$ the energy splitting is only ~3 MeV. However, the single-particle shift $2\hbar\omega$ does not include correlations in the particle-hole states produced by the nuclear force.[7] A calculation[7] of the monopole polarizability based on the two-fluid hydrodynamic model results in an increased separation by a factor of 2 for the $t=1$ excitation. Such an increase is also familiar from giant-dipole resonance studies. Using this increased separation, we have $E_m-E_a=4\hbar\omega-(T_\zeta+\frac{3}{4})V_1/A$ which is ~40 MeV for $A=50$, ~30 MeV for $A=90$, and ~15 MeV for $A=200$. From these results, we see that as we go from light to heavy nuclei, the analog state moves away from the configuration states and shifts closer to the monopole excitation.

Since the configuration and monopole states have normal isospin, they couple to other states of normal isospin by the nuclear force. Consequently, these states mix with the dense spectrum of such states near the analog, and it is just this damping which we believe is, in turn, responsible for the damping of the analog state. This latter damping is then determined by the mechanism for the $T=T_\zeta-1$ mixing. Two models suggest themselves for this mixing, namely the strong-coupling model with a uniform spreading picture, and the giant-resonance model. In the former model, the Coulomb mixing strength becomes uniformly distributed and we can approximate $\langle V^2\rangle$ by $M_V(T_\zeta-1)/N$ and ρ by $N/\langle E\rangle$, where N is the number of states coupled to the analog state and $\langle E\rangle$ is the energy region over which this mixing extends. The width $\Gamma_a^\downarrow$ of Eq. (1) is then

$$\Gamma_a^\downarrow \cong 2\pi M_V(T_\zeta-1)/\langle E\rangle, \tag{7}$$

and is numerically a factor of 5-10 times larger than the experimental values listed in Table I. Also, the spreading width is predicted to have a Z^3/T_ζ dependence for $A<100$, a result which contradicts the experimental data. On the other hand, if the damping of the $T=T_\zeta-1$ modes is governed by a giant-resonance picture, the width $\Gamma_a^\downarrow$ is given by[8]

$$\Gamma_a^\downarrow=\frac{\Gamma_c}{(E_a-E_c)^2+(\Gamma_c/2)^2}\sum_c M_{a,c}^2+\frac{\Gamma_m M_{am}^2}{(E_a-E_m)^2+(\Gamma_m/2)^2}=\Gamma_{a,c}^\downarrow+\Gamma_{a,m}^\downarrow. \tag{8}$$

Here Γ_c and Γ_m are the damping widths of the configuration and monopole excitation states, respectively, into the $T = T_\zeta - 1$ states which surround the analog state. In Lane[9], Γ_c has been estimated to be 5 MeV for $A = 50$ and 10 MeV for $A = 200$. Since both $E_a - E_c$ and $\sum_c M_{a,c}^2$ increase from $A = 50$ to $A = 200$, the configuration-state contribution to $\Gamma_a^\downarrow$ is nearly constant for $A > 100$ with a value $\Gamma_{a,c} \cong$ 10-15 keV. For $A < 100$ this contribution fluctuates with a maximum value of $\simeq$30 keV. In Table I, we list the contributions of this term to $\Gamma_a^\downarrow$. In order to evaluate the monopole term, we must know the value of Γ_m at the analog energy. Now, little is known both theoretically and experimentally about this width. Nevertheless, a consistent picture can be obtained if Γ_m is taken to be 3 MeV for all nuclei. Using this value, the monopole excitation contributions, $\Gamma_{a,m}^\downarrow$, are given in Table I. From this table we see that the monopole contribution increases fairly regularly with increasing A and is of the order of 5 keV for $A < 70$. This result combined with the configuration-state contribution predicts a minima in $\Gamma_a^\downarrow$ around $A = 50$.

In conclusion, the contributions of the configuration states and monopole excitation to $\Gamma_a^\downarrow$ are consistent with the experimental data if we assume a giant-resonance picture for the damping of these states. In turn, this assumption means that the distribution of Coulomb mixing strength has essentially two peaks with the analog state lying between the two. Furthermore, with this hypothesis, the decrease in the spreading width of an analog state from $A = 37$ to $A = 50$ and the increase again for $A > 60$ is explained by the motion as a function of A and T_ζ of the analog-state position between these two peaks coupled with the fluctuating contribution of the lower peak to this spreading width and the systematically increasing contribution of the upper peak to it with increasing A.

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