Commission under contract with the Union Carbide Corporation.

†Oak Ridge Associated Universities Fellow from the University of Tennessee, Knoxville, Tenn.

‡Oak Ridge Associated Universities Fellow from the Ohio State University, Columbus, Ohio.

§Consultant, University of Tennessee, Knoxville, Tenn.

‖Student Guest from The University of Iowa, Iowa City, Ia.

[1]T. H. Jenson and F. R. Scott, Phys. Fluids 11, 1809 (1968); S. D. Fanchenko, B. A. Demidov, N. I. Elagin, and N. F. Perepelkin, Phys. Rev. Lett. 21, 789 (1968); S. M. Hamberger, A. Malein, J. H. Adlam, and M. Friedman, Phys. Rev. Lett. 19, 350 (1967); F. Kawabe and H. Eubank, private communication; M. V. Babykin, P. P. Gavrin, E. K. Zavoiskii, S. L. Nedoseev, L. I. Rudakov, and V. A. Skoryupin, Zh. Eksp. Teor. Fiz. 52, 643 (1967) [Sov. Phys. JETP 25, 421 (1967)]; M. S. Ioffe, R. I. Sobolev, V. G. Tel'kovskii, and E. E. Yushmanov, Zh. Eksp. Teor. Fiz. 39, 1062 (1960) [Sov. Phys. JETP 12, 1117 (1961)]; M. V. Nezlin, in *Proceedings of a Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, U. S. S. R., 1968* (International Atomic Energy Agency, Vienna, Austria, 1969), p. 257; E. K. Zavoisky, S. L. Nedoseev, L. I. Rudakov, V. D. Rusanov, V. A. Skoryupin, and S. D. Franchenko, *ibid.*, p. 235; M. Fumelli, R. Dei-Cas, J. P. Girard, and F. P. G. Valckx, Association EURATOM–Commissariat à l'Energie Atomique Report No. EUR-CEA-FC-503, 1969 (unpublished); C. B. Wharton, D. S. Prono, and F. Sandel, in *Proceedings of the Ninth International Conference on Phenomena in Ionized Gasses, Bucharest, Rumania, 1969*, edited by G. Musa, A. Popescu, and L. Nastose (Institute of Physics, Bucharest, Rumania, 1969), p. 649.

[2]R. V. Neidigh, I. Alexeff, G. E. Guest, W. D. Jones, D. C. Montgomery, D. J. Rose, and W. L. Stirling, in *Proceedings of the Third International Conference on Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, Austria, 1969), Vol. 2, p. 693; I. Alexeff, W. D. Jones, and R. V. Neidigh, Phys. Rev. Lett. 18, 1109 (1967).

[3]I. Alexeff, R. V. Neidigh, and W. R. Wing, Int. J. Eng. Sci. 7, 531 (1969). (Note: Our present correlators are greatly improved over that discussed in the above reference.)

[4]F. R. Scott, R. V. Neidigh, J. R. McNally, Jr., and W. S. Cooper, III, to be published. See also M. Baranger and B. Mozer, Phys. Rev. 123, 25 (1961); W. S. Cooper, III, and H. Ringler, Phys. Rev. 179, 226 (1969); H. J. Kunze and H. R. Griem, Phys. Rev. Lett. 21, 1048 (1968).

[5]A. B. Mikhailovskii and V. S. Tsypin, Zh. Eksp. Teor. Fiz.–Pis'ma Redakt. 3, 247 (1966) [JETP Lett. 3, 158 (1966)].

LINEAR MECHANISM FOR THERMAL ENERGY TRANSPORT IN CURRENT-CARRYING PLASMAS*

B. Coppi†
International Centre for Theoretical Physics, Miramare, Trieste, Italy
(Received 6 July 1970)

Collisionless modes extracting electron thermal energy and convecting it radially are considered as a possible explanation of the electron thermal energy radial transport, larger than the one predicted by the proper collisional theory, in confined high-temperature plasmas carrying a current along the magnetic field.

Recent experiments on plasma confined in toroidal magnetic configurations in which electrons are resistively heated by a current flowing along the magnetic field have indicated that the electron thermal-energy transport across the field is much larger than that obtained by the proper collisional transport theory.[1-4] In other words, while it appears that the resistivity as predicted by previous experimental and theoretical work is anomalous[5] (that is, enhanced by a finite factor over its collisional value), the evidence seems to be that the resulting increased electron heating is accompanied by enhanced electron thermal-energy loss.

We propose for this a mechanism which relies on the excitation of current-driven drift modes of the same type as those which have been invoked to explain the observation of anomalous resistivity.[5] These modes have frequency proportional to the radial gradient of the electron pressure $p_{e\parallel}$ parallel to the magnetic field and tend to grow at the expense of longitudinal electron thermal energy. The existence of drift modes is consistent with the observation of a relatively flat electron pressure profile[2] in the plasma column of the T-3 Tokamak in the sense that their excitation is expected to reduce the electron pressure gradient.

We consider for simplicity a one-dimensional plane model of a confined plasma with density gradient in the x direction and the main magnetic field in the z direction. In the neighborhood of a point $x = x_0$ the magnetic field can be represented by $\vec{B} \approx B_0[\vec{e}_z + \vec{e}_y(x - x_0)]/L_s$, the latter term indicating the magnetic shear that is associated with the current $\vec{J}$ flowing along z. This current is carried by the electrons so that $J_z = neu_{e\parallel}$, n being the electron density and $u_{e\parallel}$ the average electron velocity. We consider a low-β plasma with $n(T_e + T_i) \ll B_0^2/8\pi$ and zero electric field at equilibrium. Electrostatic perturbations with $\vec{E} = -\nabla\varphi$ from the equilibrium can be expanded in normal-mode solutions of the form $\varphi = \tilde{\varphi}(x-x_0)\exp(i\omega t + ik_y y + ik_z z)$. We take $k_z = 0$ and notice that $k_\parallel = \vec{k}\cdot\vec{B}/B_0 = k_y(x-x_0)/L_s$. The ion-mass conservation equation gives

$$i\omega\tilde{n}_i - ik_y\frac{c}{B}\frac{\partial n}{\partial x}\left(1+\frac{\rho_i^2}{2}\nabla_\perp^2\right)\tilde{\varphi} - \frac{i\omega}{\Omega_i}\frac{c}{B}n\nabla_\perp^2\tilde{\varphi} + ik_\parallel n\tilde{u}_{i\parallel} = 0. \quad (1)$$

Here $\rho_i^2/2 = T_i/(m_i\Omega_i^2)$ and T_i, m_i, and Ω_i are, respectively, the ion temperature, mass, and gyration frequency. We have included the $\vec{E}\times\vec{B}$, the finite Larmor radius and polarization drift[6] for the motion across B. We obtain $u_{i\parallel}$ from the total momentum conservation equation along the magnetic field,

$$i\omega n m_i\tilde{u}_{i\parallel} = -ik_\parallel T_i\tilde{n}_i - ik_\parallel n\tilde{\varphi}, \quad (2)$$

that is valid in the limit $v_{\text{th}i} < \omega/k_\parallel$.

For wavelengths longer than the Debye length we also have

$$\tilde{n}_i = \tilde{n}_e,$$

and $\tilde{n}_e$ is obtained from the perturbed electron distribution function in the drift approximation. This is simply[5]

$$\tilde{f}_e = -\left(\frac{e}{m_e}\right)\tilde{\varphi}\,\frac{k_\parallel\partial f_e/\partial v_\parallel + (k_y/\Omega_e)\partial f_e/\partial x}{-\omega + k_\parallel v_\parallel}, \quad (3)$$

where Ω_e is the electron gyro frequency and f_e the equilibrium electron distribution function proportional to $\exp(-2v_\parallel'^2 m_e/T_e)$ with $v_\parallel' = v_\parallel - u_{e\parallel}$. So the dispersion equation[7] that results, in the interesting limit $v_{\text{th}i} < \omega/k_\parallel < v_{\text{th}e} = (2T_e/m_e)^{1/2}$ and $\omega_{*n} = k_y(cT_e dn/dx)/(eBn)$, is

$$\left(1-\frac{\omega_{*n}}{\omega}\right)\tilde{\varphi} - \left(1+\frac{\omega_{*n}}{\omega}\frac{T_i}{T_e}\right)\left(\frac{T_e}{m_i}\right)\left[\frac{1}{\Omega_i^2}\left(\frac{\partial^2}{\partial x^2}-k_y^2\right)+\frac{1}{\omega^2}k_\parallel^2\right]\tilde{\varphi} - i\pi^{1/2}\frac{\omega-\omega_{*n}-k_\parallel u_{e\parallel}}{|k_\parallel|v_{\text{th}e}}\tilde{\varphi} = 0. \quad (4)$$

We assume for simplicity that $k_y < \partial/\partial x$ and $(T_e/m_i\Omega_i^2)\partial^2/\partial x^2 < 1$. Therefore $\omega = \omega_{*n} + \delta\omega$ with $\delta\omega \ll \omega$ and, to lowest order, Eq. (4) becomes

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{x^2}{\delta^4} - \left(\frac{\delta\omega}{\omega_{*n}} + i\pi^{1/2}\frac{u_{e\parallel}}{v_{\text{th}e}}\right)\frac{m_i\Omega_i^2}{T_e+T_i}\right\}\tilde{\varphi} = 0, \quad (5)$$

where $\delta^4 = \omega^2 L_s^2/\Omega_i^2 k_y^2 = [L_sT_e/(r_n m_i\Omega_i^2)]^2$ and $r_n^{-1} = -(dn/dx)/n$, and we have replaced $x-x_0$ simply by x. Now we can see, adopting the point of view of Pearlstein and Berk[8] rather than the one of looking for convective wave packets,[9] that solutions growing in time, for instance, of the form $\exp(i\sigma x^2/2 + i\omega t)$ with σ real, can be found. These solutions are significant in that they carry energy outward and their stability is determined by the rate at which energy is extracted from the particles versus the rate at which it is carried away.

The existence of these waves, therefore, gives a natural mechanism for thermal energy transport towards the outer regions.

In particular, taking $\sigma = -k_y\Omega_i/\omega L_s = \delta^{-2}$, the condition that the energy be outgoing from $x = x_0$ implies that

$$\mathfrak{D} \equiv \frac{\partial\omega}{\partial\sigma} = -\frac{\partial}{\partial\sigma}\left(\frac{k_y}{L_s}\frac{\Omega_i}{\sigma}\right) = \frac{k_y}{L_s}\frac{\Omega_i}{\sigma^2} < 0. \quad (6)$$

So $k_y < 0$, $\omega_{*n} > 0$, and $\sigma > 0$.

The resulting dispersion relation is

$$\frac{\delta\omega}{\omega_{*n}} = -i\pi^{1/2}\frac{u_{e\parallel}}{v_{\text{th}e}} + i\left(1+\frac{T_i}{T_e}\right)\frac{r_n}{L_s}, \quad (7)$$

and if

$$\frac{u_{e\parallel}}{v_{\text{th}e}} > \pi^{-1/2}\frac{r_n}{L_s}\left(1+\frac{T_i}{T_e}\right),$$

the mode will be linearly unstable.

The instability amplitude will be limited by nonlinear effects such as coupling with modes which are linearly damped. In this case or when the mode is marginally stable (i.e., with zero linear growth rate), the rate of energy convection to the outside can be represented by

$$\nu_T = \frac{\mathfrak{D}}{r_n^2} = \frac{\omega^2}{\Omega_i}\frac{L_s}{k_y r_n^2} = \left(\frac{v_s}{\Omega_i r_n}\right)^3\frac{L_s}{r_n}k_y v_s, \quad (8)$$

where $v_s = (T_e/m_i)^{1/2}$ is the ion sound-wave velocity.

Then, if we take $k_y \sim \delta^{-1}$,

$$\nu_T \approx \frac{v_s}{r_n}\left(\frac{v_s}{\Omega_i r_n}\right)^2\left(\frac{L_s}{r_n}\right)^{1/2}, \quad (9)$$

and the energy transport is

$$q_T \approx \sum_k \nu_T \frac{\partial}{\partial\omega}(\epsilon\omega)\left(\frac{k^2\tilde{\varphi}_k^{\,2}}{8\pi}\right)$$
$$\simeq \frac{v_s}{r_n}\left(\frac{L_s}{r_n}\right)^{1/2}\left(\frac{v_s}{\Omega_i r_n}\right)^2\left(\frac{e\tilde{\varphi}_k}{T_e}\right)\mathfrak{N} n T_e,$$

where $\mathfrak{N}$ is the number of excited modes and ϵ is the dielectric constant. Notice that ν_T is bounded since this analysis implies $L_s/r_n \lesssim \sqrt{2} r_n T_i/(\rho_i T_e)$ and thus nonconvective normal-mode solutions are excluded from consideration.[10]

If we compare q_T with the energy loss rate that is due to electron collisions[11] and can be written as $\nu_e \rho_e^2 n T_e G/r_T^2$, where G is a geometrical factor taking into account the effect of toroidal geometry and r_T is the temperature-gradient scale distance, we have

$$\frac{\lambda_e}{2r_n}\left(\frac{L_s}{2r_n}\frac{m_i}{m_e}\right)^{1/2}\left(\frac{r_T}{r_n}\right)^2\frac{\mathcal{E}}{G} \text{ compared with } O(1),$$

where $\mathcal{E} \approx (e\tilde{\varphi}_k/T_e)^2\mathfrak{N}$ and $e\tilde{\varphi}_k/T_e \approx \tilde{n}_k/n$. For densities and temperatures typical of the T-3 Tokamak ($T_e \approx 1$ keV, $n \approx 2\times 10^{13}$) and magnetic field $B\approx 25$ kG, we have $\lambda_{ei}\approx 10^5$ cm and $v_s/\Omega_i \approx 0.1$ cm. So if we take $r_n \approx 5$ cm and $L_s \approx 300$ cm, it appears that even with density fluctuations of the order of 1%, a convection of thermal energy larger than the collisional energy transport can be obtained.

It is a pleasure to thank E. K. Maschke for several discussions and Professor Abdus Salam and the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, Italy.

*Work supported in part by the U. S. Atomic Energy Commission.

†On leave from the Massachusetts Institute of Technology, Cambridge, Mass.

[1]L. A. Artsimovich, G. A. Bobrovskii, E. P. Gorbunov, D. P. Ivanov, V. D. Kirillov, E. I. Kuznetsov, S. V. Mirnov, M. P. Petrov, K. A. Razumova, V. S. Strelkov, and D. A. Shcheglov, in *Proceedings of the Third International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, U. S. S. R., 1968* (International Atomic Energy Agency, Vienna, Austria, 1969), p. 17.

[2]N. J. Peacock, D. C. Robinson, M. J. Forrest, P. D. Wilcock, and V. V. Sannikov, Nature 224, 488 (1969).

[3]D. C. Robinson, private communication.

[4]L. A. Artsimovich, lecture delivered at the International Centre for Theoretical Physics, Trieste, Italy, 1970 (unpublished).

[5]B. Coppi and E. Mazzucato, Princeton University Plasma Physics Laboratory Report No. MATT-720, 1969 (unpublished).

[6]G. Schmidt, *Physics of High Temperature Plasmas* (Academic, New York, 1966), p. 292.

[7]B. B. Kadomtsev, J. Nucl. Energy, Pt. C 5, 31 (1963).

[8]L. D. Pearlstein and H. Berk, Phys. Rev. Lett. 23, 220 (1969).

[9]B. Coppi, G. Laval, R. Pellat, and M. N. Rosenbluth, Nucl. Fusion 6, 261 (1966).

[10]N. A. Krall and N. M. Rosenbluth, Phys. Fluids 8, 1488 (1965).

[11]S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. 1, p. 205.

DETERMINATION OF 2e/h BASED ON THE ac JOSEPHSON EFFECT

I. K. Harvey, J. C. Macfarlane, and R. B. Frenkel
Commonwealth Scientific and Industrial Research Organization, Division of Applied Physics, National Standards Laboratory, Sydney, Australia
(Received 13 August 1970)

A value for $2e/h$ has been determined by the ac Josephson effect using niobium point-contact junctions. The value with respect to the international voltage standard as maintained at the Bureau International des Poids et Mesures, Sèvres (V_{BIPM69}), is $2e/h = 483.593\,84 \pm 0.000\,10$ MHz/μV.

Recent measurements of $2e/h$ using the ac Josephson effect have been reported by Parker et al.,[1] Taylor et al.,[2] Petley and Morris,[3] and Finnegan, Denenstein, and Langenberg.[4] Finnegan, Denenstein, and Langenberg,[4] using thin-film tunnel junctions, report a precision approaching 0.1 ppm and an accuracy limited to that which obtains for the comparison of voltage standards. We have determined a value of $2e/h$ with respect to the National Standards Laboratory (NSL) voltage standard to a comparable high precision, using niobium point-contact junctions.

Contrary to previous reports,[1,5] the point-contact junctions provided (within the resolution of the present experiments) steps of constant voltage, with a current range of some 200 μA being regularly observed. The junctions were formed by pressing a sharpened niobium wire against