EVIDENCE OF ELECTRON-ELECTRON SCATTERING FROM FIELD EMISSION

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A small tail above the Fermi energy has been found in the energy distribution of field-emitted electrons on three crystal planes of clean tungsten, at 20°K. The current in the tail is proportional to the square of total probe hole current, and is interpreted as resulting from electron-electron scattering in the metal.

In the course of energy-distribution measurements on clean tungsten we have observed an effect which is most readily explained as electron-electron scattering in the metal. The apparatus, to be described in detail elsewhere, was of the spherical retardation type.[1] Electrons from a W field emitter steered by an electrostatic quadrant lens passed through succesive holes in concentric spherical shells, were retarded by a fine Au-coated mesh covering an opening in the last electrode and then multiplied by a Channeltron operated in the dc mode with a gain of 1.3×10^4. The gain was constant over the range of currents used. The i vs ϵ curve was electronically differentiated by modulating the retarding voltage with a 10-mV peak-to-peak ac signal. Ultrahigh vacuum was maintained cryogenically, by immersing the entire apparatus in liquid H_2. The emitter temperature could be controlled electrically in the usual manner.[2] Resolution was estimated to be 25-40 mV, depending on the region probed, by the criterion of Young and Kuyatt,[3] from the energy spread between the 10 and 90% values of the leading edge of the $j(\epsilon)$ vs ϵ curve.

Experimental energy distributions conformed closely over most of the range to the values predicted by the free-electron model,[4]

$$j(\epsilon)\propto e^{\epsilon/d}/(1+e^{\epsilon/kT}), \tag{1}$$

where ϵ is the energy relative to the Fermi level, and $d=0.976F/\varphi^{1/2}$, F being the field in V/Å and φ the work function in eV. Thus Fig. 1, selected at random, shows experimental and predicted values for the (120) plane at 250°K; the experimental width at half-maximum is 178 mV, the predicted value being 168 mV. At 20°K where the effect of finite resolution becomes more critical the data could be fitted very precisely by the convolution of Eq. (1) with a Gaussian analyzer transmission function[3,5]; in the case of (120) a full width at half-maximum of 0.040 eV was required (Fig. 2). The slight deviation far below

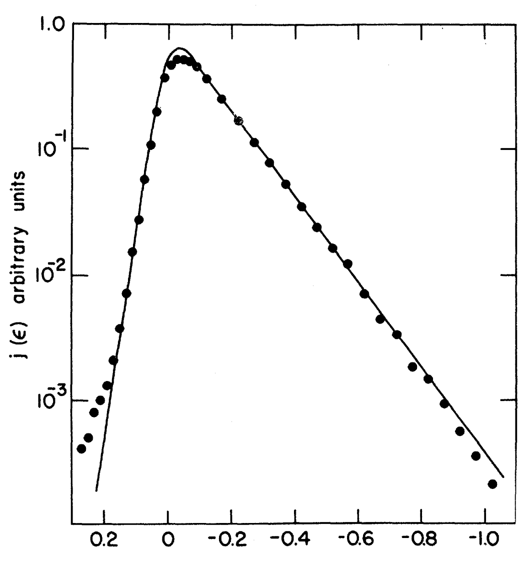

FIG. 1. Total energy distribution from the (120) plane at 250°K. The points are experimental, the line from Eq. (1). $F=0.274$ V/Å; probe current after multiplication is 1×10^{-9} A.

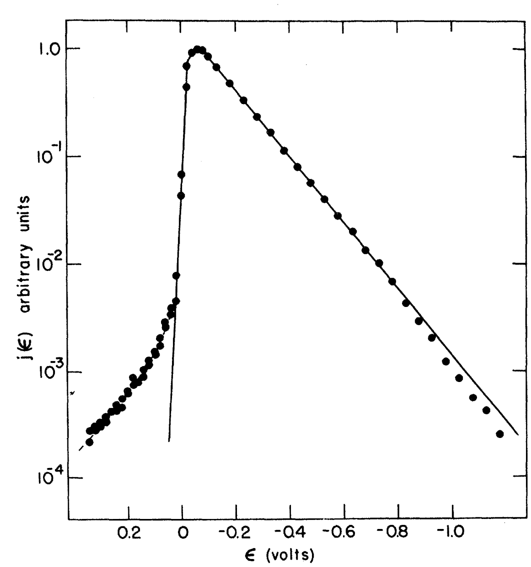

FIG. 2. Total energy distribution from the (120) plane at 20°K, with $F=0.300$ V/Å and a probe current after multiplication of 1×10^{-8} A. The points are experimental; the line, a convolution of Eq. (1) and a Gaussian analyzer transmission function with a full width at half-maximum of 40 meV.

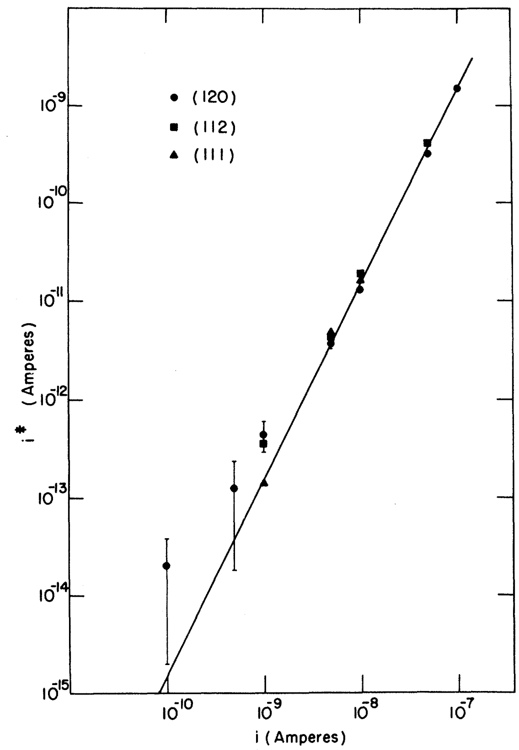

FIG. 3. Plot of i^*, probe current after multiplication arising from electrons emitted with $\epsilon > E_F$, versus i, total probe current after multiplication. Data obtained from the (120), (111), and (112) planes of tungsten at 20°K.

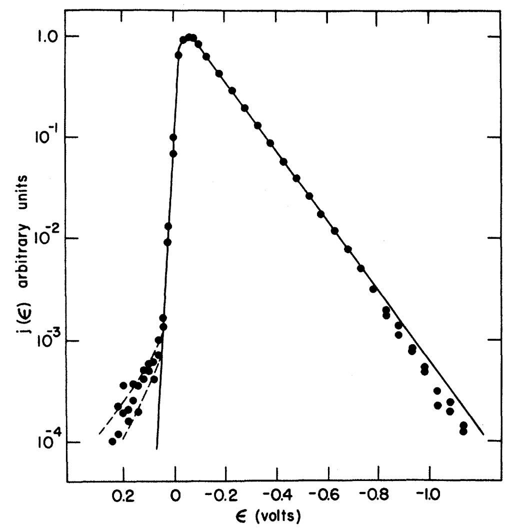

FIG. 4. Total energy distribution from the (120) plane at 20°K with $F = 0.266$ V/Å and a probe current after multiplication of 5×10^{-10} A. Dashed lines correspond to error bar shown in Fig. 3.

E_F has also been seen by Plummer and Young,[6] and attributed by them to deviations from the simple WKB tunneling probability used to derive Eq. (1).

Despite the excellent fit over most of the range, a tail in the leading edge at $\epsilon > E_F$ can be seen on all distributions, even at 20°K. This cannot be explained in terms of reasonable analyzer transmission functions, the more so since all apparatus errors except patchiness of the retarder tend to skew the transmission toward the low-energy side. That energies above E_F are involved follows also because a consistent collector work function for different emission directions, and for different fields at fixed orientation, can be obtained only by ignoring the tails and fixing the Fermi level by extrapolation from the upper portion of the leading edge. $j(\epsilon > E_F)$ and the current in the tail, i^*, turn out to be proportional to i^2 for all directions investigated, i being the total probe-hole current (Fig. 3). Deviations from the i^2 relation at very low i^* can almost certainly be explained on the basis of experimental uncertainties in this measurement (Fig. 4). The i^2 relation militates against trivial explanations since it is difficult to see how apparatus errors could give rise to anything but a linear relation between i^* and i. It is also clear why these tails are difficult to see without electron multiplication since even at the highest currents used here $i^* < 0.01i$. Tails of the kind described here have also been seen and discussed at meetings by Plummer.

The i^2 dependence of i^* rules out the possibility of a static distribution with levels $>E_F$ occupied, since an increase in field should then lead to a relative decrease in tail current. However, the i^2 relation is readily explained by electron-electron scattering. We first show that scattering on the vacuum side of the barrier could not possibly account for the observations under the conditions of the experiment: The total current emitted over the entire tip was $\leq 10^{-8}$ A, so that the average time between successively emitted electrons was $t \geq 1.6\times10^{-11}$ sec. For these times the distance traveled by an electron before emission of the next one is ≥ 0.03 cm, so that the average Coulomb interaction between two nearest electrons is $\leq 4\times10^{-6}$ eV. A simple space-charge calculation shows that the maximum average Coulomb interaction $\langle E\rangle$ of an electron with the space charge occurs at $r \simeq 2r_t$, r_t being the tip radius, and is given approximately by

$$\langle E\rangle \simeq I(me/2Fr_t)^{1/2}, \qquad (2)$$

I being total emitted current and F the applied field. Equation (2) leads to an average interaction energy of $\leq 9\times10^{-6}$ eV. Since the width of the tail is >0.1 eV, the space-charge fluctuations required to produce it would have to be unreasonably large.

The most probable explanation of the high-energy tail seems the following. Consider virtual-scattering events in the metal near the Fermi surface whose net result is to scatter one electron above E_F and another below by an equal energy. Because all states below E_F are filled at low T, this process remains virtual in the absence of an applied field. If the electron scattered downward can tunnel out of the metal, however, the process can become real; note that E, but not $\vec{k}$, is conserved for the electron pair. A fraction of the electrons scattered above E_F will also tunnel and it is these which give the observed tail. While all of the electrons scattered down must tunnel, the high value of $j(\epsilon)$ for $\epsilon < E_F$ makes them invisible in practice. The overall probability of observing an electron of energy $E_F + \Delta$ thus depends on a dynamical factor $f(\Delta)$ governing the scattering in the metal, multiplied by the probability that both electrons have tunneled. The latter is given in WKB approximation by

$$P_{\text{tunneling}} = \exp[-c(\varphi+\Delta)^{3/2}/F] \times \exp[-c(\varphi-\Delta)^{3/2}/F], \qquad (3)$$

where $c = 0.68$ in eV-Å units. To first order in Δ this is $\exp(-2c\varphi^{3/2}/F)$. However, the total current is proportional, from the Fowler-Nordheim equation,[2] to $\exp(-c\varphi^{3/2}/F)$, so that $j(\Delta)$ is proportional to i^2; the proportionality is preserved for $i^* = \int_0^\infty j(\Delta)d\Delta$.

The model predicts that i^* corresponds to emission of electron pairs correlated in time (within the lifetime of a hole at $E_F - \Delta$). Lack of sufficient space correlation, the small value of i^*, and its dependence on i^2 would make it difficult to find these, since at probe currents low enough to permit electron counting the number of space-correlated pairs would be very small indeed. It is not surprising therefore that Young, who looked for them for quite different reasons, did not find pairs.[7] It might be possible so to bias the retarder that only a narrow range about E_F could reach the electron counter in order to prevent its saturation; the steep rise of $j(\epsilon)$ below E_F might defeat the experiment even so.

To second order in Δ, $j(\Delta)$ contains the term $\exp(-\frac{3}{4}c\Delta^2/F\varphi^{1/2})$. It can easily be seen that this is practically constant over the range of Δ encountered here, so that the almost exponential decrease of $j(\Delta)$ with Δ must be explained in terms of the dynamical factor $f(\Delta)$. Since the final result, emission of an electron with $\epsilon = E_F + \Delta$ and of an electron with $\epsilon = E_F - \Delta$, is the product of all relevant diagrams in the metal, $f(\Delta)$ can only be evaluated by a full-dress many-body calculation. It is one purpose of this paper to show that this would be worthwhile since uninteresting effects like the second-order term in the tunneling probability do not mask it; further, both ~0°K and finite-T calculations could be checked against experiment.

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